Simulation and Characterization of a Novel Nonlinear Magnetic Shock Absorber Element

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Abstract—The aim of this research is to propose a novel magnetic nonlinear shock absorber and study the feasibility of the system in real world applications. This shock absorber consists of an array of identical repelling magnets with an electromagnetic coil energy harvesting system. The nonlinear dynamics of the shock absorber were developed analytically and are implemented in a simulation environment. The transient response of the system was obtained for various inputs where the magnets demonstrate solitary wave behaviour in their response. Moreover, the nonlinear characteristics of the shock absorber enables its stiffness to increase with higher compression, which offers the possibility for it to act as a bump stop without the usual discontinuity in the force-displacement characteristic. A parametric study was conducted to study the effects of the magnet mass, repelling force, inter-lattice equilibrium distance, and coil damping on the response of the system.

Keywords—repelling magnetic chain, nonlinear dynamics, shock absorber, solitary wave

I. INTRODUCTION

Energy harvesting devices, utilizing magnets and electromagnetic coil setups, have received increasing attention over the years. Various design concepts have been proposed where, through single [1]–[3] or multi magnet arrangements [4, 5], the vibration of a system is converted into electric energy. Some concepts incorporate springs to aid in the levitation of the magnets [6] and some with proposed applications in vehicle suspension systems [7, 8].

However, the proposed devices only act as an energy harvesting system. In this paper, we propose a novel magnetic nonlinear shock absorber that serves as a spring-damper element while incorporating energy harvesting capabilities. This novel shock absorber consists of a 1D nonlinear lattice of equal-repelling permanent magnets that are confined between two fixed magnets with a multi-coil setup as an energy dissipation source.

Nonlinear latices have been studied extensively and various applications have been proposed such as wave control [9, 10], vibration control [11], and shock mitigation [12], to name a few. The dynamics of solitary waves in a 1D magnetic lattice was studied by Molerón et al [13] and the possibility of shock mitigation using such a design was suggested. Despite the vast literature on nonlinear lattices, no previous work has addressed the feasibility of shock absorption and energy harvesting through utilization of a 1D magnetic lattice.

In this paper, the nonlinear dynamics of the shock absorber element is developed and the properties of the system are analyzed through a parametric study. This is organized as follows. Following the introduction in Section I, Section II describes the derivation of equations of motion (EOM) for the magnetic shock absorber. The EOM are then utilized to simulate the transient time response of the system for various inputs in Section III. Section IV describes the force vs. displacement relationship, which provides insight into the stiffness of the system. Lastly, a parametric study is conducted to analyze the effect of different design parameters on characteristics of the system in Section V.

II. DYNAMICS

With reference to Fig. 1, the system consists of a number of ring magnets arranged in a 1D lattice configuration with axial magnetic polarization and similar physical properties so that the initial distance between

![Fig. 1. Repelling magnetic chain diagram.](image-url)
adjacent magnets is the same and assumed to be \( d_0 \). The magnets at each end are fixed, confining the range of motion of the sliding magnets. With \( n \) sliding magnets, the external force is applied to the \( n \)th magnet. For each sliding magnet there is a corresponding electromagnetic coil, thus there are \( n \) coils as well. Each sliding magnet experiences a force due to its interactions with the adjacent sliding or fixed magnets and an additional force due to its interaction with the coils.

A. Magnetic Force

Based on the research conducted by Molerón et al. [13], the repelling force between two adjacent magnets is given by

\[
F_{\text{repelling}} = Ad^p
\]  
(1)

where \( A > 0 \) and \( p < 0 \) are empirically-derived constants and \( d \) is the distance between the two adjacent magnets. Note that the constants depend on the magnet’s properties, such as grade, size, etc. Since the force diminishes rapidly as the distance increases, the effects of non-adjacent magnets are negligible and ignored [13].

B. Electromagnetic Damping

The repelling magnetic chain alone acts as a nonlinear spring element; however, implementing electromagnetic coils enables the dissipation of energy from the system and provides the ability to harvest the kinetic energy of the magnets into electrical energy. This is due to Faraday’s law of induction, which states that whenever a conductor loop experiences a change in magnetic flux, a voltage will be induced in that loop. The current in the loop in return creates a magnetic field which opposes changes in the initial magnetic field according to Lenz’s law [14].

According to Faraday’s induction law, the electromagnetic force is proportional to the relative speed of the magnet with respect to the coil. Therefore, electromagnetic energy dissipation is velocity dependent and can be modelled similar to a viscous damper as:

\[
F_{e_{ij}} = C_{e_{ij}} \ddot{x}_i
\]
(2)

where \( F_{e_{ij}} \) is the electromagnetic damping force exerted on magnet \( i \) by coil \( j \), \((1 \leq i, j \leq n)\), \( C_{e_{ij}} \) is the damping coefficient due to interaction between magnet \( i \) and coil \( j \), and \( \ddot{x}_i \) is the velocity of \( i \)th magnet with the assumption of fixed coils.

Unlike a simple viscous damper, the damping coefficient is not constant. According to [15]–[17] it is dependant on the magnetic field density as given by

\[
C_{e_{ij}} = \frac{(NB_{ij}lc)^2}{R_{\text{load}} + R_{\text{coil}}} \]
(3)

where \( N \) is the number of coil turns, \( B_{ij} \) is the average magnetic flux density of magnet \( i \) interacting with coil \( j \), \( lc \) is the coil length, \( R_{\text{coil}} \) is the internal electrical resistance of the coil, and \( R_{\text{load}} \) is the connected resistive load. For a conventionally-wound coil, \( R_{\text{coil}} = \rho_w \frac{L_w}{\pi R_i} \), where \( \rho_w \) is the wire resistivity, \( L_w = \frac{V_c}{f_i V_i / A_w} \) is the wire length, \( V_c = \pi (r_o^2 - r_i^2) l_c \) is the coil volume, \( A_w \) is the wire cross-sectional area, \( r_o \) is the outer radius of the coil, and \( r_i \) is the inner radius of the coil [18]. Therefore,

\[
R_{\text{coil}} = 16 \rho_w f_i \left( \frac{r_o^2 - r_i^2}{l_c} \right) \frac{l_c}{\Phi^4}
\]
(4)

where \( \Phi \) is the wire diameter and \( f_i \) is the coil fill factor which is the percentage of conductive material occupying the coil volume. The fill factor of wound coils will vary, but a figure of 50–60% can be reasonably assumed [18].

Fig. 2. Interaction of a magnet with a single turn of coil.

On the other hand, with reference to Fig. 2, the \( x \)-component of magnetic flux density on the axis of symmetry for a ring magnet is given by [19]

\[
B_{ij}(x) = \frac{B_r}{2} \left[ \frac{L + a_{ij}}{\sqrt{R_o^2 + (L + a_{ij})^2}} - \frac{a_{ij}}{\sqrt{R_o^2 + a_{ij}^2}} \right] \left[ \frac{L + a_{ij}}{\sqrt{R_i^2 + (L + a_{ij})^2}} - \frac{a_{ij}}{\sqrt{R_i^2 + a_{ij}^2}} \right]
\]
(5)

where \( a_{ij} \) is the distance from the pole face of the magnet \( i \) to first turn of the coil \( j \), \( 2L \) is the length of the magnet, \( R_i \) is the inner radius of the ring, \( R_o \) is the outer radius of the ring, and \( B_r \) is the residual magnetism which is independent of the magnet’s geometry. For instance, for a N52 neodymium magnet, the residual magnetism ranges from 1.42 to 1.47 Tesla. Note that for a relative motion between the magnet and the coil, only the \( x \)-component of the magnetic induction \( B_x \) will cause a magnetic flux change in the coil turns [14]. Therefore, the total magnetic flux density is given by [20]

\[
B_{ij} = \frac{1}{l_c} \int_0^{l_c} B_{ij}(x) \ dx
\]
(6)

Note that this magnetic flux density calculation does not take into account the interactions of adjacent magnets and, according to [21], the existence of other magnets noticeably increases the flux density. Derivation
of the exact analytical electromagnetic damping force for a multi-magnet-coil setup is beyond the scope of this paper. Finite element methods can also be used to determine the flux density [2]. The coil damping coefficient also can be obtained through identification of damping force experimentally and analysis of the voltage outputs of the coils [22]. However, this simplification allows for simulation efficiency and offers sufficient information to study the feasibility of the shock absorber in real-world applications. Ultimately, the total electromagnetic damping coefficient of the $i^{th}$ magnet can be obtained as

$$C_{ei} = \sum_{j=1}^{n} C_{eij}$$

(7)

Note that unlike the repelling force case where the effect of non-adjacent magnets was ignored, in this case the effect of all coils is considered since a single magnet throughout its travel due to a perturbation can come within range of any coil where the force due to their interaction is significant. Lastly, the total damping coefficient for each magnet is $C_i = C_{ei} + C_{vi}$, where $C_{vi}$ is the viscous friction coefficient of $i^{th}$ magnet obtained experimentally. For a more comprehensive friction model, a LuGre model could be used to incorporate stiction force [24], which could play an important role in the back and forth type motion of the magnets.

Based on the developed relations for the repelling and the damping forces, the EOM of the system were obtained using Newtonian dynamics as,

$$\begin{bmatrix}
    m & 0 & \cdots & 0 \\
    0 & \ddots & \ddots & \vdots \\
    \vdots & \ddots & m & \vdots \\
    0 & \cdots & 0 & m
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_1 \\
    \vdots \\
    \ddot{x}_n \\
\end{bmatrix}
= \begin{bmatrix}
    x_1 \\
    \vdots \\
    x_n \\
\end{bmatrix}
$$

(8)

where $m$ is the mass of each magnet, $f_n$ is the external force applied to the $n^{th}$ magnet, and $x_i$ is the displacement of magnet $i$ from the initial equilibrium position.

### III. SIMULATION

Based on the developed EOM, a Simulink model was created. Parameters used to simulate the response of the system to an external force are summarized in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet Mass, $m$</td>
<td>$10.3 \times 10^{-3}$</td>
<td>kg</td>
</tr>
<tr>
<td>Repelling Force Exponent, $p$</td>
<td>-2.73</td>
<td></td>
</tr>
<tr>
<td>Repelling Force Constant, $A$</td>
<td>$6.25 \times 10^{-5}$</td>
<td>N/m$^2$</td>
</tr>
<tr>
<td>Viscous Damping Coefficient, $C_v$</td>
<td>0.03</td>
<td>m/s/m</td>
</tr>
<tr>
<td>Inter-Lattice Equilibrium Distance, $d_0$</td>
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<td>m</td>
</tr>
<tr>
<td>Magnet Inner Radius, $R_i$</td>
<td>4.25</td>
<td>mm</td>
</tr>
<tr>
<td>Magnet Outer Radius, $R_o$</td>
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<td>mm</td>
</tr>
<tr>
<td>Magnet Length, $2L$</td>
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<td>mm</td>
</tr>
<tr>
<td>Coil Length, $l_c$</td>
<td>15.64</td>
<td>mm</td>
</tr>
<tr>
<td>Number of Coil Turns, $N$</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>Total Resistance, $R_{tot}$</td>
<td>10</td>
<td>$\Omega$</td>
</tr>
</tbody>
</table>

TABLE I: Magnetic shock absorber parameters.

Fig. 3 shows the displacement of a four-magnet setup, (i.e. $n = 4$), to a constant force of magnitude 0.3 N, applied to the $n^{th}$ magnet at $t = 0.5s$. The $n^{th}$ magnet is settled at a final displacement of approximately 102 mm due to the external force, but the displacement of the magnets decreases as they get further away from the $n^{th}$ magnet.

#### A. Wave-like Behaviour

As demonstrated in Fig. 3, various “modes” are detectable in the response of the system. For nonlinear systems, the modes are called nonlinear normal modes (NNMs). However, the principle of superposition of modes does not apply to nonlinear systems [25]. Thus, the complex response of the magnets cannot be decomposed into modes as in a linear system; however the compound motion can be better understood as a wave.

A closer look at the displacement reveals the wave-like behaviour of the repelling chain and gives better insight into the reason behind the observed complex oscillations. Fig. 4 demonstrates the initiation of the wave. As the wave starts to propagate, each magnet begins to move with a slight delay, with magnet number one being the last one to displace. On the other hand, Fig. 4 shows that magnet one peaks earlier than the other magnets with magnet four being the last one to peak. Magnet one then bounces back and causes a second peak. This second wave travels down the lattice to magnets two and three. However, due to the existence of damping forces, the wave dissipates before reaching magnet 4. This is a sign of the solitary wave-like behaviour of the system.

#### IV. SYSTEM STIFFNESS

To comprehend the stiffness characteristics of the system, the force vs. displacement curve was generated. To
do so, a constant force was applied to the \( n \)th magnet and the displacement of each magnet was recorded after the system had reached static equilibrium. This process was repeated for an array of external forces, the result of which is summarized in Fig. 5. Note that this is a hardening type of system, meaning the higher the displacement, the higher the stiffness.

Using an exponential curve fitting algorithm, the force-displacement relation was estimated as:

\[
F(x_n) = (5.34 \times 10^{-3}) e^{18x_n} + (2.36 \times 10^{-9}) e^{62.12x_n} \tag{9}
\]

where \( F(x_n) \) is the external force as a function of displacement. This estimation resulted in a minor sum of squares due to error (SSE) equal to 0.055. As demonstrated in the zoomed-in view provided in Fig. 5, the slight deterioration of the fitness is mainly confined to the low external force range. Equation 9 can be used to quickly obtain the stiffness of the system for a given input and shows its exponential nature.

V. Parametric Study

The parametric studies in this subsection are carried out to understand the effect of each parameter on the response of the system.

A. Magnet count

The number of magnets that were simulated ranged from 1 to 8, excluding the fixed magnets at each end. Every other aspect of the shock absorber was kept constant, even the overall dimension, implying that as the number of magnets was increased, the inter-lattice equilibrium distance of the magnets, \( d_0 \), decreased. The simulation was run for a step input force of magnitude of 0.3 N, as well as a sinusoidal one with the same magnitude and a frequency of 5 rad/s. Fig. 6 shows the displacement of the \( n \)th magnet, the magnet to which the external force is applied, for the step and sinusoidal inputs.

It is observed that a lower magnet count in the lattice results in higher amplitudes of the response and longer settling time. Fig. 7 shows spikes in inter-lattice force amplitude for one and two magnet setups. This can be explained by the relatively large equilibrium distance in lattices with low count of magnets, which allows for the magnets to attain higher velocities and lower inter-magnet distances, and hence higher forces, when perturbed. Depending on the application, higher magnet counts might be desirable, since they do not demonstrate spikes in inter-lattice forces.

Also, as shown in Fig. 6, the response of a single magnet system is similar to a damped harmonic oscillator in the transitory phase for a step input and more complex for a harmonic input.

B. Mass and Repelling Force

The mass and repelling force coefficient (the constant \( A \) in Eq. 1) of the magnets are coupled to each other, i.e. stronger magnets will be heavier. However, their proportional relation was unknown, so it is assumed to be linearly proportional in this investigation. Meaning, if a magnet is twice as heavy, it can exert twice the repelling force. Future more accurate investigations will determine this relationship experimentally.

Note that as the coefficient \( A \) increases, the amplitude of the displacement and total force decreases, as demonstrated in Figs. 8 and 9. However, the settling times for the stronger magnets are longer, which could be rectified through damper tuning.

C. Inter-lattice Equilibrium Distance

The effect of overall dimension of the shock absorber, \( D \), was also studied. Changing the overall dimension affects the inter-lattice equilibrium distance between the magnets, \( d_0 \). Larger shock absorber length allows the
magnets to reach higher velocities before being stopped due to the repulsive force of the adjacent magnets. The higher velocities attained by the perturbed magnets in the lattice are reasoned to cause the spikes in the registered forces in Fig. 10 since the magnets get closer to each other for the same magnet count and increasing lattice length. Also, the ability to adjust the inter-lattice equilibrium distance opens up the possibility to fine tune the shock absorber for various applications, such as meeting the suspension requirements of an off-road vs. on-road vehicle.

D. Coil Damping

Coil damping can be adjusted by changing the electric load resistance, which provides the possibility to actively tune the shock absorber. The coil damping, unlike a simple viscous damping, is velocity and displacement dependant; therefore as the magnet is perturbed along the axis of the lattice, it experiences damping forces that range from high to vanishing. As seen in Fig. 11, as the electrical resistance of the coil decreases, which increases its effective damping according to Eq. 3, the displacement of the magnet along the axis of the lattice is increasingly damped as expected; however, the oscillations in the response persist. Also, the displacement profile is dependant on the applied force magnitude as it would affect the velocity of the magnets.

VI. Conclusion

In this paper, a novel shock absorber element with energy harvesting capabilities was proposed. The nonlinear dynamics of the element were studied and the Simulink simulation software was utilized to obtain the
The stiffness characteristics of the device were obtained revealing a hardening behaviour. Lastly, a parametric study was conducted to analyze the effects of the number of magnets, magnet mass, repelling force amplitude, inter-lattice equilibrium distance, and coil damping on the response of the system. The adjustability of the stiffness of the element via equilibrium distance, magnet grades, and size demonstrate the tunability. This element can be implemented in road and rail vehicle suspension systems with the capability of energy harvesting. The element also has the potential for developing active suspension systems through energization of the electromagnetic coils.

**Fig. 11.** Displacement of the $n^{th}$ magnet vs. time for step input and various coil damping magnitude.

**Fig. 10.** Total Force experienced by the $n^{th}$ magnet vs. time for step input and various element dimensions ($D$).

References


