THERMODYNAMIC PATHS ALONG STREAMLINES OF A SINGLE-PHASE TRANSCRITICAL CO$_2$ VORTEX TUBE

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Abstract—The Ranque-Hilsch vortex tube is at the center of a renewed interest as a robust expansion device and a useful energy supply for transcritical CO$_2$ vapor compression systems. This paper gives insights into thermodynamic paths along streamlines in a transcritical CO$_2$ vortex tube employing computational fluid dynamics (CFD). A pressure-based coupled solver along with a low-Reynolds number turbulence model and the Span-Wagner equation of state constitutes the present numerical framework. Transcritical expansions are carried out at several cold mass fractions and both outer and inner vortex flow streamlines are tracked in the pressure-specific volume and pressure-specific enthalpy diagrams. The results confirm that the transcritical CO$_2$ vortex tube is the seat of energy and temperature separations but they are inverted. A correlation between the flow transcriticality and vortex tube temperature and energy separation inversions is observed.

Keywords-component—: CFD; Vortex Tube; Transcritical CO$_2$; Single-Phase; Thermodynamic Path

I. INTRODUCTION

The search for the best refrigerant in vapor compression systems has led carbon dioxide (CO$_2$) to become one of the most promising candidates to meet the upcoming environmental standards [1]. Indeed, CO$_2$ is not only a greenhouse gas, but one of few natural refrigerants to combine zero ozone depletion and very low global warming potentials to non-flammability and non-toxicity [2]. Although the use of CO$_2$ as a refrigerant could solve the effect of harmful systems leaks on the environment, the specific properties of CO$_2$ makes cycles operating at much higher pressures than conventional ones, hence hazardous to use with existing technical solutions. Consequently, enhancement of system design and components is necessary to reach the best energy efficiency. In the Ranque-Hilsch vortex tube (RHVT), a tangentially injected pressurized gas into a cylindrical tube allowed to exit from both ends produces energy separation. This phenomenon, arising in a highly turbulent and complex swirling flow structure, has not found a comprehensive explanation so far. Although intriguing, owing to its simple construction and operation, the vortex tube has found numerous applications. It can be used as a spot cooler [3], as a mass separator in the oil and gas industry [4], or as an expansion valve in compressed air energy storage [5] as well as in the refrigeration industry, especially for transcritical CO$_2$ heat pumps [6], which is the context of the present research.

In the literature, several authors used thermodynamic models of vortex tube-based transcritical CO$_2$ refrigeration and heat pump cycles. Nevertheless, these models often treat the vortex tube as a black box. Attempts to account for the RHVT physics were made by few authors who developed vortex tube-specific one-dimensional (1D) thermodynamic models based on simplified compressible Navier–Stokes equations. Relying on the ideal gas equation of state (EOS), the momentum equations and the isentropic flow assumption in the inlet nozzle and the cold tube, [7] predicted within 1.1% the experimental RHVT outlet temperature values using air as the working fluid. Afterwards, [8] markedly improved these results by accounting for friction losses in the inlet nozzles, the Bödewadt boundary layer and additional degree of freedom on the hot side pressure. Additionally, [9] extended the latter model to real gases at high pressures. Their validation against experimental data showed significant improvement of exit temperature predictions in real-gas fluid regions for CO$_2$ and R134a. Thus, characterization of the non-ideal gas behavior effect on the transcritical CO$_2$ vortex tube temperature and energy separation has recently been performed by [10]. As encountered with ejector-improved CO$_2$ heat pumps, flash condensation can produce substantial liquid phase that affects the vortex tube performance, and in turn, influence the performance and layout of the transcritical CO$_2$ heat pump cycle. The comprehension of supercritical carbon dioxide expansion in the Ranque-Hilsch vortex tube and the effect of the operating conditions on temperature and energy separations is then paramount before any integration in a heat pump cycle.

In the present paper, an investigation of the operation of a single-phase transcritical CO$_2$ Ranque-Hilsch vortex tube is
carried out through a CFD analysis. First the numerical setup is described, then results including temperature and energy separations as a function of the cold mass fraction ($\mu_c$) as well as thermodynamic paths taken by peripheral and core fluid layers in the pressure-specific volume ($p - v$) and the pressure-specific enthalpy ($p - h$) diagrams are presented and discussed. Finally, concluding remarks are drawn.

II. NUMERICAL MODELING

In this section, the numerical solver and models are described as well as the computational domain, boundary conditions and simulation procedure.

A. Flow Solver

The steady-state 3D Reynolds-Averaged Navier-Stokes (RANS) governing equations for the conservation of mass, momentum and total energy for a compressible turbulent flow inside the vortex tube were solved using ANSYS Fluent v.2020 R2, based on the finite volume method. Second-order discretization schemes were adopted for advective and diffusive terms and the PRESTO! scheme for pressure. The coupled pressure-based solver was used to solve for the algebraic system of equations for its stability and convergence due to an implicit discretization of pressure gradient terms in the momentum equations and of mass fluxes [11]. Then, turbulence and energy equations were solved and density was computed through the equation of state $\rho = f(p, T)$.

B. Equation of State

The Span-Wagner equation of state (SW EOS) [12], as the international reference for CO$_2$ and following the numerical benchmark in [13], was selected. In Fluent v.2020 R2, the REFPROP v9.1 equation database was called by activation of the NIST real-gas model. The former employs the formulation based on the Helmholtz free-energy calibrated on a multiparameter fit of extensive CO$_2$ experimental data. The equation is valid in fluid regions up to temperature of 1100 K and pressure up to 800 MPa, which is sufficient for industrial applications involving transcritical CO$_2$.

C. Turbulence Modeling and Meshing

The $k - \omega$ SST turbulence model was selected as it has proven to be the most accurate two-equation eddy-viscosity modeling approach to predict outlet total pressure and temperature in a previous work [13]. The $k - \omega$ SST model combines the formulation of the Standard $k - \epsilon$ model in the far-field domain with the expression of the Standard $k - \omega$ model in the near-wall region through a blending function. Contrary to the latter model, the $k - \omega$ SST involves a damped cross-diffusion derivative term in the transport equation of $\omega$ and the turbulence viscosity is modified to account for the transport of the turbulent shear stresses. The wall-resolved mesh includes 1.75 million unstructured elements with 12 prismatic layers a growth rate of 1.2. It ensures $y^+ < 5$ at the walls, with values less than 1 in most parts of the vortex tube, a prerequisite for low-Reynolds number turbulence closure.

D. Geometry and Boundary Conditions

The vortex tube geometry is based on the experimental set-up developed by [14] using an improved commercial vortex tube including six inlet nozzles. Figure 1 gathers the principal dimensions of the experimental vortex tube. While only subcritical CO$_2$ was covered in [14], the supercritical inlet operating point is fixed and prescribed at $p_{in} = 8$ MPa and $T_{in} = 336$ K in the present work, ranging eight representative cold mass fractions from 0.14 to 0.84 as displayed in Table I. The computational domain consists of a 1/6 portion of the vortex tube using a periodic symmetry with a 60-degree offset. Pressure-inlet and pressure-outlet boundary conditions are prescribed at the inlet and outlets, respectively. The pressure-outlet conditions are supplemented by a target mass flow rate $\dot{m}_{tar}$, which allows the boundary pressure to adjust to match the corresponding mass flow rate via the Bernoulli equation to compute the pressure change:

$$dp = \frac{\dot{m}_2^2 - \dot{m}_1^2}{2 \rho A^2}$$

where $\dot{m}$ is the outlet mass flow rate at the current iteration, $A$ is the area of the boundary patch and $\rho$ the fluid density. At the walls, no-slip and adiabatic conditions are set. Finally, a 5% turbulence intensity at the inlet is applied to reflect a medium level of velocity fluctuations [15]. To start the calculations, a hybrid initialization based on boundary interpolation methods is employed to compute the pressure and velocity fields from the specified boundary conditions. First-order, then second-order upwind spatial schemes are applied with a constant density to limit numerical instabilities specific to high-speed compressible flows. Afterwards, the ideal-gas, then the SW EOS are enabled once the vortex flow is fully developed. Parallel computations were run on 96 CPUs with 187 GB of RAM and 2.1 GHz CPUs on Compute Canada clusters. Convergence is reached when stable values of the cold mass fraction and of the outlet temperatures and pressures are obtained over the last 200 iterations while the residuals sank below $10^{-4}$.

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III. RESULTS AND DISCUSSION

A. Single-Phase Transcritical Temperature Separation

Figure 2 displays the predictions of the cold and hot exit temperature differences \( \Delta T_{out} = T_{out} - T_{in} \) as a function of the cold mass fraction. The results show that the RHVT produces a significant temperature drop regardless of the considered outlet conditions. Specifically, the hot end temperature is observed to be lower than the inlet temperature. This phenomenon can be explained by the shape of the isotherms close to the critical point. At the present inlet operating point, CO\(_2\) behaves like a real-gas and has a close-to-horizontal isothermal line in the \( p-h \) diagram. Therefore, the hot end temperature is lower than the inlet temperature as any isenthalpic pressure reduction will result in a temperature drop. At low cold mass fractions, the vortex tube produces a temperature separation, between both exits, in the same order of magnitude as for the subcritical CO\(_2\) vortex tube [13], [16]. As the cold mass fraction rises, the temperature drop intensifies due to the cold end pressure reduction. The temperature separation between outlets vanishes between 0.6 and 0.7, because both outlets reached the same isotherm, the critical isotherm, even though at different pressures. Afterwards, temperature separation inversion, characterized by a hot end temperature below the cold end one, occurs between 0.64 and 0.84.

B. Single-Phase Transcritical Energy Separation

Figure 3 depicts the computed cold and hot exit specific enthalpy differences, respectively defined as \( \Delta h_{0c} = h_{0c} - h_{0i} \) and \( \Delta h_{0h} = h_{0h} - h_{0i} \). Contrary to the subcritical case, at the current transcritical conditions, the enthalpy differences are negative. This result means that the RHVT cold exhaust is actually supplying a heating power, and the hot exhaust a cooling power, as noticed in [10] using a 1D thermodynamic model. The results further reveal that at low cold mass fractions, the absolute heating power on the cold side is higher than the absolute cooling power on the hot side, but as the cold mass fraction increases beyond 0.5, the relationship is flipped. This is correlated with the reduction of the hot exit enthalpy while the cold exit one remains quasi-constant, at constant inlet enthalpy, when \( \mu_c \) rises. Temperature and energy separation inversions do not occur at the same cold mass fraction, contrary to temperature and power inversions, which was not represented here for the sake of brevity. This can be due to the effect of the cold mass fraction on the flow transcriticality. The higher the cold mass fraction, the deeper the incursion of the flow under the critical temperature, which inhibits the classical energy separation phenomenon.

C. Thermodynamic Paths Along Streamlines

Fig. 4 displays different seed locations in the inlet nozzle along the vortex tube axis. It can be seen that the seed location...
discriminates the outlet selection. For instance, particles are more likely to exit via the hot outlet if they are initially close to the vortex chamber hot-side wall and vice versa for the cold-ended particles. Then, as showed in Fig. 5, one can select the bounding seed location to follow hot-ended (h-stream) or cold-ended particles on their way from the inlet to the selected outlet. There exists a variety of paths for cold-ended particles however, two bounding ones generally occur (Fig. 5):

- Through the Bödewadt boundary layer (c-stream B),
- Through the vortex breakdown (c-stream VB).

Figure 6 also confirm that not only inlet and outlet boundaries are in supercritical state but also the bounding hot-ended and cold-ended streamlines. Consequently, the entire flow remains supercritical at low cold mass fractions at the present supercritical CO₂ inlet operating conditions.

Figure 6 also presents the $p−h$ diagram of CO₂ including the three vortex streamlines at cold mass fractions 0.14 (c) and 0.84 (d). The temperature and energy separation inversions are clearly observed on Figure 6 d), as the hot end temperature of the gas sinks below the Joule-Thomson temperature. The cold mass fraction rise actually tends to increase the cooling power on the hot side at quasi-constant pressure and to reduce the cold end temperature at quasi-constant enthalpy. The isothermal lines in the $p−h$ diagram being slower to cross by an isenthalpic pressure drop than by an isobaric enthalpy reduction in this region, this is the reason why the cold end temperature drop is slower than the hot end temperature drop on Figure 2. In addition, while the distinction of cold-ended particle paths is not obvious at $\mu_c = 0.14$, it is remarkable at $\mu_c = 0.84$ in both $p−v$ and $p−h$ diagrams (Fig. 6 b) and d). Indeed, as mentioned before, while c-stream B and c-stream VB follow similar thermodynamic paths at low $\mu_c$, the distinction is enhanced at high $\mu_c$, where c-stream VB gets rather similar to h-stream. At $\mu_c = 0.84$, c-stream VB first undergoes an expansion in the vortex chamber, then a compression as it enters the main tube peripheral fluid layers and gets thrown to the wall. Along the main tube, its pressure drops at a lower pace than at $\mu_c = 0.14$. When the hot end valve is reached, a second expansion occurs on its way to the core, due to the radial pressure gradient, and continues along its reverse way towards the cold exit. As this stream goes back through the vortex chamber it warms up after the final temperature drop in the cold tube. The cold-ended stream undergoing vortex breakdown could even match the hot outlet state before deflection, Fig. 6 d) clearly highlights the departure of c-stream VB from c-stream B, supporting the vortex breakdown role on the temperature and energy separation phenomena in the Ranque-Hilsch vortex tube. Moreover, it is observed from Fig. 6 d) that c-stream VB is the seat of supercritical-to-vapor-state phase transition between cold mass fractions 0.14 and 0.84. More insights into the localization of transcritical phase changes is given in Figure 7 where c-stream VB at $\mu_c = 0.84$ is colored by the total temperature. It should be noted that only two transcritical phase transitions appear on Figure 7 due to the area-averaged value at the cold outlet on Figure 6. Starting from the supercritical inlet nozzle at $T_{0\text{in}} = 336$ K, transcritical phase change is triggered in the peripheral fluid layers in the second half of the main tube before the subcritical gas, following deflection on the hot end valve, is subjected to flow reversal and to heating by the inlet stream followed by cooling in the cold tube. Therefore, high cold mass fractions promote the transcriticality of cold-ended streams undergoing vortex breakdown.
Figure 6. CO$_2$ $p$–$v$ and $p$–$h$ diagrams at $\mu_c$ = 0.14 and $\mu_c$ = 0.84. Vortex flow streamlines for c-stream B (blue), c-stream VB (cyan) and h-stream (red) with boundary conditions at the inlet (black dot), cold (blue dot) and hot (red dot) outlets, saturation curve (solid line) and critical isotherm (dash dotted line).

Figure 7. Supercritical-to-vapor phase transition at $\mu_c$ = 0.84 delimited by brackets on c-stream VB, colored by the total temperature.
IV. CONCLUSIONS

In this work, numerical simulations of a single-phase transcritical CO\textsubscript{2} Ranque-Hilsch vortex tube were carried out using a pressure-based coupled solver with the $k-\omega$ SST turbulence model and the Span-Wagner equation of state. The results revealed that temperature and energy separations happen, but the hot outlet temperature is lower than the inlet temperature at low cold mass fractions, due to the shape of the isotherms close to the critical point, and even lower than the cold side temperature at high cold mass fractions. Consequently, the vortex tube cold exit provides a heating power and the hot exit a cooling power, and the temperature separation between both ends is reduced. Further analysis of internal flow streamlines showed that supercritical outlet conditions are not a guarantee of the absence of transcritical flow. Indeed, the vortex breakdown, being the seat of transcritical phase change as the cold mass fraction rises, fosters both temperature and energy separation inversions. Further experimental and numerical works in two-phase transcritical flow regime will be done to validate and extend the present conclusions.

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