1D MODELLING OF THE LIQUID-GAS JET PUMP PERFORMANCE

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Abstract—The performance of a liquid-gas jet pump, which uses a high-velocity liquid flow to compress and entrain a gas flow, can be divided into two modes; on-design and off-design. The present paper investigates the mentioned modes of performance using a 1D model based on the conservation equations of mass, momentum and energy. Comparisons in terms of compression ratio and efficiency between the present model and experimental data show that the 1D model is capable of predicting the behavior of the liquid-gas jet pump for both modes. The effects of the primary flow velocity head, and areas of the mixing throat and diffuser on the performance of the jet pump are also investigated through a sensitivity study.

Keywords: jet pump; 1D numerical model; liquid-gas flow; compression ratio; efficiency

I. INTRODUCTION

Jet pumps are devices without any moving parts, that transfer energy from a primary (motive) fluid to a secondary (driven) fluid [1]. As it is schematically shown in Fig. 1, the jet pump consists of four main components, namely: nozzle, suction chamber, mixing throat and diffuser. The high-pressure primary fluid with certain energy is discharged into the mixing throat by the nozzle at a high speed, and the air is taken away to form a vacuum negative pressure state near the nozzle. The low-pressure forces the secondary fluid into the pump through the suction pipe, and then the secondary fluid enters the mixing chamber together with the high-speed primary fluid. In the mixing throat, the primary fluid with high-pressure will transfer part of its kinetic energy to the secondary fluid with low-pressure while the two flows are mixing together. Then the mixed fluid is introduced into the diffuser where its pressure is gradually increased [2]. The condition in which the primary and secondary fluid will not mix in the mixing throat is termed as off-design mode, as will be explained later in this paper.

Jet pumps have been widely utilized because of their simplicity and high reliability, absence of lubricants or bearings [3] and low installation costs [4] in broad areas including thermal energy refrigeration systems [5] and city central heating systems [6]. Jet pumps can be employed for transportation of liquids which contain solid particles or fish [7] and also hazardous liquids [8].

1D analysis of the jet pump performance using the conservation of mass, momentum and energy equations is an efficient method in terms of computational costs. Additionally, the reliability of this method has been confirmed by previous studies. Winoto et al. [9] performed a combined theoretical and experimental analysis of the jet pump efficiency of jet pumps, using liquid water for both the primary and secondary streams. It was concluded that using non-circular nozzles for liquid-liquid jet pumps increases energy losses and lowers the efficiency of the pump. They also concluded that the performance of the water jet pump can be adequately described using 1D theoretical formulation. De Oliveira Marum et al. [10] performed axisymmetric CFD simulations to calculate the friction loss coefficients of each part of the water jet pump by fitting the CFD data to a quasi-1D mathematical model adapted from [9]. The efficiency, static pressure profile along the length of the jet pump and static pressure at different radial cross sections were calculated by CFD model coupled with various turbulence models. By comparing the numerical data obtained using k-ε, k-ω and k-ω SST turbulence models with experimental results, it was concluded that k-ω SST model is the most suitable to capture the ejector’s flow characteristics in all operational
The following assumptions are considered in the developed 1D mathematical model:

- Water is the primary fluid and gas is the secondary fluid.
- Gas is compressed isothermally from section $s$ to section $d$ (cross sections are shown Fig. 1).
- There is no pressure or temperature change in the gas flow at the jet pump entry (i.e., from section $s$ to $o$).
- Any change in water temperature is neglected.
- Once the gas and water flows are mixed, a homogeneous bubbly mixture flow is created in which there is no slip between the two phases.
- The distance between the nozzle outlet and the mixing section inlet is zero.
- The pressure is uniform at any cross section along the jet pump.
- Gas flow is assumed to behave as an ideal gas.

Application of conservation equations considering the mentioned assumptions in each component of the jet pump leads to an expression for the calculation of the liquid-gas jet pump efficiency.

### A. On-design mode

When the liquid and gas phases mix in the mixing throat, the jet pump is operating in the on-design mode. The governing equations for the flow inside the device are shown below.

1) **Nozzle**

Energy equation for the nozzle can be expressed as:

$$
P_{1i} + \frac{p_1 V_{1i}^2}{2} = P_{1o} + \frac{p_1 V_{1o}^2}{2} + \frac{K_{nz} p_1 V_{1o}^2}{2}$$

(1)

where $P_1$ and $P_o$ are the static pressures at the nozzle inlet and outlet, respectively. $V_{1i}$ is the velocity of the primary flow at the nozzle inlet, $V_{1o}$ is the velocity of the primary flow at the cross-section $o$ and $K_{nz}$ is the friction loss coefficient of the nozzle. $p_1$ is the density of the primary fluid.

By replacing the total pressure of the primary fluid at the nozzle inlet $P_{1i}$ in the equation:

$$
\bar{P}_{1i} = P_{1i} + p_1 V_{1i}^2/2
$$

(1), the nozzle equation becomes:

$$
\bar{P}_{1i} - P_{1o} = Z (1+K_{nz})
$$

(2)

where $Z = p_1 V_{1o}^2/2$ is the jet velocity head.

2) **Suction chamber**

The assumption of negligible pressure drop in the suction chamber for the gas flow leads to:

$$
P_{1s} = P_{1o}
$$

(3)

Additionally, due to the assumption of pressure uniformity in all cross-sections, the pressure at the mixing chamber inlet is denoted by $P_o$ in the following.

3) **Mixing throat**

The momentum equation for the mixing throat (from sections $o$ to $l$) can be express as:
\[ (P_o - P_1)A_t - \tau A_w = (m_1 + m_2)V_{3t} - [m_1V_{1o} + m_2V_{2o}] \]  

(4)

where \( m_1 \) and \( m_2 \) are the mass flow rates of the primary and secondary flows, \( V_{3t} \) is the velocity of the mixed flow at the mixing throat outlet, \( P_t \) is the static pressure at the mixing section outlet, \( \tau \) is the shear stress, \( A_t \) and \( A_w \) are the cross-sectional area and internal wall area of the mixing throat, respectively.

Considering \( m = \rho AV \), Eq. (4) can be written as:

\[ (P_o - P_1)A_t - \tau A_w = \rho_3 V_{3t}^2 A_t - \rho_1 V_{1o}^2 A_n - \rho_2 V_{2o}^2 A_{2o} \]  

(5)

where \( A_n \) and \( A_{2o} \) are the nozzle area and the area occupied by the secondary flow at \( o \) section, respectively. The density and velocity of the mixture flow at the end of the mixing throat are calculated by:

\[ \rho_{3t} = \left( \frac{m_1 + m_2}{m_1 + m_2} \right) \rho_1 + \left( \frac{m_1 + m_2}{m_1 + m_2} \right) \rho_2 \]  

(6)

\[ V_{3t} = \frac{Q_{3t}}{A_t} = \frac{Q_1(1+\varphi_1)}{A_n + \varphi_1} = V_{1o}(1+\varphi_1) \]  

(7)

where \( \varphi = Q_2/Q_1 \) is the volumetric flow ratio (at various sections: \( o, t, d \), \( \gamma = \rho_{2o}/\rho_1 \) is the density ratio and \( b = A_n/A_t \) is the nozzle to mixing throat area ratio. With \( \varphi_1 = P_o/\rho_o/\rho_1 \) and \( \tau_{4L/D_t} = k_d \rho_3 V_{3t}^2/2 \), the pressure rise in the mixing chamber (Eq. (5)) can be written as:

\[ P_t^2 - \left[ 2(2 - b^2) + 2k_d \right] (1 + \varphi_1) + 2\varphi_2 \rho_0^2 \frac{2 + b}{1 - b} + P_o \]  

(8)

A homogeneous mixture enters the diffuser at \( P_t \) and \( V_{3t} \), and decelerates to \( V_{3d} \) and pump discharge pressure \( P_d \). The Euler equation of motion for the mixture flow in the diffuser writes:

\[ \frac{dP}{\rho_3} + VdV + d \left( K_{\text{diff}} V_{3d}^2 \right) = 0 \]  

(9)

where \( K_{\text{diff}} \) is the friction loss coefficient of the diffuser. Using the ideal equation of state, the density of the mixture through the diffuser can be expressed as a function of pressure as:

\[ \rho_3 = \rho_1 \left( \frac{1 + \varphi_0}{1 + \varphi_2} \right) \]  

(10)

The integration of the motion equation along the diffuser (from cross-sections \( t \) to \( d \)) yields:

\[ P_4 - P_t = Z \left( 1 + \gamma \varphi_0 \right) [b^2 + (1 + \varphi_1)^2 + a^2 b^2 (1 + \varphi_2)^2 - K_d b^2 (1 + \varphi_1)] - P_0 \varphi_o \ln \left( \frac{P_d}{P_t} \right) \]  

(11)

5) Jet pump efficiency

The efficiency of the liquid-gas jet pump is defined as:

\[ \eta = \frac{W_{\text{out}}}{\varphi} \]  

(12)

where \( W_{\text{out}} \) is the useful work output rate (isothermal compression of an ideal gas from \( P_t \) to \( P_d \)), and \( \varphi \) is the input energy rate. These terms are defined by:

\[ W_{\text{out}} = Q_{2s} \left( \frac{P_d}{P_t} \right) \]  

(13)

\[ \varphi = Q_1 \left( \frac{P_d}{P_t} \right) \]  

(14)

B. Off-design mode

When the liquid jet leaves the mixing throat without mixing with the gas flow, the liquid-gas jet pump operates in the off-design mode. The governing equations of the flow inside the device are written below. It must be noted that equations for the nozzle, suction chamber and jet pump efficiency are similar to the equations mentioned in the previous section and are not repeated here for brevity.

1) Mixing throat

Conservation of mass, energy and momentum for the water jet in the mixing throat can be expressed by Eqs (15), (16) and (17), respectively:

\[ V_{1o} A_n = V_{1t} A_{1t} \]  

(15)

\[ P_o + \left( \frac{P_{1o} V_{1o}^2}{2} \right) = P_1 + (1 + k_d) \left( \frac{P_{1t} V_{1t}^2}{2} \right) \]  

(16)

\[ (P_o A_{1o} - P_1 A_{1t}) - k_d \left( \frac{P_{1t} V_{1t}^2}{2} \right) A_{1t} = (P_{1o} V_{1o} A_n) [V_{1t} - V_{1o}] \]  

(17)

2) Diffuser

Fig. 3 schematically shows the fluid flow and exerted forces in the diffuser when the liquid-gas jet pump operates in the off-design mode.
The conservation of momentum in the diffuser can be expressed by:

\[(P_t A_t - P_d A_d) - \tau A_w = (m_1 + m_2 V_{3d}^2 - [m_1 V_{in} + m_2 V_{2i}])\]  \(\text{(18)}\)

Considering \(m = \rho A V\) and \(\tau A_w = k_{th} \rho_{3d} V_{3d}^2 A_d\), Eq. (18) can be written as:

\[(P_t A_t - P_d A_d) - k_{th} \rho_{3d} V_{3d}^2 A_d = (\rho_{3d} V_{3d}^2 A_d) - (\rho_{1t} V_{1t}^2 A_{1t}) - (\rho_{2t} V_{2t}^2 A_{2t})\]  \(\text{(19)}\)

where \(\rho_{3d}\) and \(V_{3d}\) are the density and velocity of the liquid-gas mixture at the outlet of the diffuser and can be defined by Eqs (20) and (21), respectively.

\[\rho_{3d} = \rho_1^{1 + \gamma \phi_d}\]  \(\text{(20)}\)

\[V_{3d} = Q_d^{1/(1 + \phi_d)}\]  \(\text{(21)}\)

where \(A_o = A_d\) and \(\phi_d = \frac{P_p}{\rho_d}\).

III. VALIDATION

Experimental results for a liquid-air jet pump reported by Cunningham and Dopkin [12] are used to validate the 1D mathematical model. Fig. 4 shows the pressure ratio in the mixing throat \(R_{on} = P_t / P_a\) and diffuser \(R_{off} = P_d / P_a\) as a function of the flow ratio \(\phi\), which is an independent variable. In Ref. [12] various flow ratios were achieved by adjusting the discharge pressure \(P_d\).

The entrained air flow in the jet pump is compressed in two stages; i.e., in the mixing throat and diffuser as shown in Fig. 4a and Fig. 4b. The jet pump operates in the on-design mode up to the point when there is a sharp change in the slope of \(R_{on}\) and \(R_{off}\). Further reducing the value of \(P_d\) beyond this point leads to less air compression and more entrained air flow.

By looking at Figs 4 and 5, it can be concluded that the 1D numerical model is in fairly good agreement with the experimental data of Cunningham and Dopkin [12] for both on- and off-design modes. The maximum discrepancy between the experimental and numerical values of the pressure ratio in the mixing throat and the diffuser are 7% and 9%, respectively. The maximum difference in terms of jet pump efficiency is 23%. The stated errors are related to the off-design mode. The discrepancy of all parameters in the on-design mode is less than 5% in comparison to the experimental data.
IV. RESULTS AND DISCUSSION

This section focuses on the influence of some design parameters, including the jet velocity head, mixing throat diameter and exit diameter of the diffuser on the performance of a water-air jet pump. In the following sensitivity analysis, every parameter is kept constant except the parameter of interest.

The efficiency curve and the pressure ratio of the water-air jet pump operating with three values of the jet velocity head i.e., $Z = 345$, 666 and 1034 kPa, are displayed in Figs 6 and 7, respectively. It shows that discharging a primary jet with higher velocity head causes more compression and less efficiency. Higher compression ratio is due to the ability of the jet with higher velocity head to transfer more momentum to the secondary phase. Increasing the velocity head of the primary fluid can be interpreted as increasing the numerator and denominator of Eq. (12) at the same time. Since the increase in $e_{in}$ is higher than in $W_{out}$ while increasing $Z$, the jet pump with more velocity head is less efficient (Fig. 6). Consequently, if more compression ratio is needed, it can be achieved at the expense of operating with less efficiency.

![Figure 6](image6.png)

**Figure 6.** Jet pump efficiency as a function of the flow ratio for three values of the primary flow velocity head. Results obtained by the 1D model.

![Figure 7](image7.png)

**Figure 7.** Pressure ratio of the jet pump as a function of the flow ratio for three values of the primary flow velocity head. Results obtained by the 1D model.

The efficiency curve and the pressure ratio of the liquid-air jet pump operating with three values of the nozzle-to-throat area ratio i.e., $b = A_n/A_t = 0.3, 0.2$ and 0.1, are shown in Figs 8 and 9, respectively. It must be noted that changing the value of the parameter $b$ when the area of the mixing throat is constant can be interpreted as changing the outlet area of the diffuser, $A_d$.

![Figure 8](image8.png)

**Figure 8.** Efficiency of the jet pump as a function of the flow ratio for three values of the nozzle-to-throat area ratio. Results obtained by the 1D model.

![Figure 9](image9.png)

**Figure 9.** Pressure ratio of the mixing throat as a function of the flow ratio for three values of the nozzle-to-throat area ratio. Results obtained by the 1D model.

As shown in Fig. 8, increasing the diameter of the mixing throat causes efficiency drop with the same flow ratio or back pressure, $P_d$. The reason behind this phenomenon can be seen in Fig. 9 where increasing the diameter of the mixing throat leads to less compression ratio in the mixing throat. In other words, when there is more space around the primary flow, the gas flow will be less compressed by the momentum transfer process.

![Figure 10](image10.png)

**Figure 10.** Efficiency of the jet pump as a function of the flow ratio for three values of the throat-to-diffuser area ratio. Results obtained by the 1D model.
Figure 11. Pressure ratio of the jet pump as a function of the flow ratio for three values of the throat-to-diffuser area ratio. Results obtained by the 1D model.

As shown in Fig. 10, changing $a=0.235$ (the value used in [11]) to $a=0.05$ leads to marginal efficiency gain. However, decreasing the outlet area of the diffuser to $a=0.7$ causes considerable efficiency loss for the same air flow entrained by the jet pump. The underlying reason for this behavior could be the modification of the compression ratio due to a change in the diffuser area as displayed in Fig. 11. It is obvious that by decreasing the outlet area of the diffuser, the air flow is less compressed in the diffuser. On the other hand, changing $a=0.235$ to $a=0.05$ does not change the compression ratio considerably. It can be due to the fact that the momentum transfer process from the primary flow to the secondary flow is almost completed when $a=0.235$, and increasing further the outlet area of the diffuser cannot compress the air flow noticeably. It also shows that Cunningham [11] chose the optimum value of the throat-to-diffuser for the experimental setup.

V. CONCLUSIONS

The present study investigated the performance of a liquid-gas jet pump using a 1D model for the on-design and off-design operating regimes. The following conclusions can be drawn:

- The 1D model based on the conservation equations of mass, momentum and energy can predict the performance of liquid-gas jet pumps operating in the on-design and off-design modes.
- The utilized 1D model predicts the jet pump performance in the on-design mode with less than 5% error.
- In the off-design mode, the maximum discrepancy between the experimental and numerical values of the pressure ratio in the mixing throat and the diffuser are 7% and 9%, respectively. The maximum difference in terms of jet pump efficiency in the off-design is 23%.
- Increasing the velocity head of the primary flow leads to more compression of the secondary flow and a lower efficiency of the jet pump.
- Increasing the area of the mixing throat decreases the efficiency of the jet pump since it reduces the ability of the device for compression of the gas flow around the liquid jet.
- Increasing the area of the diffuser beyond an optimum value does not improve the efficiency of the jet pump.

Experimental investigation of the internal flow field of the liquid-gas jet pump using Particle Image Velocimetry method will be conducted in future works. Additionally, the 1D theoretical model will be validated by advanced 3D CFD simulations.

ACKNOWLEDGEMENTS

The authors acknowledge the NSERC chair on industrial energy efficiency established in 2019 at Université de Sherbrooke with the support of Hydro-Québec, Natural Resources Canada and Emerson Commercial and Residential Solutions.

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