Efficient Solution-Adaptive Finite-Volume Scheme for
Time-Invariant Multi-Dimensional Solutions of
Maximum-Entropy-Based 14-Moment Closure for
Non-Equilibrium Gases

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Abstract—A computationally efficient solution-adaptive finite-volume scheme is proposed and developed for obtaining time-invariant multi-dimensional solutions of a novel maximum-entropy-based, interpolative, 14-moment closure which provides a fully hyperbolic description of non-equilibrium transport phenomena in monatomic gases, including heat transfer. Unlike the maximum-entropy closure on which it is based, the interpolative closure provides approximate closed-form expressions for the closing fluxes. A Godunov-type finite-volume with piecewise limited linear solution reconstruction is combined with an adaptive mesh refinement (AMR) algorithm permitting local refinement to obtain solutions to the governing hyperbolic system of moment equations on two-dimensional, body-fitted, multi-block grids consisting of quadrilateral cells. Time-invariant solutions of the spatially-discretized moment equations are obtained by using an inexact Newton’s method combined with a preconditioned Krylov subspace iterative method. In particular, the GMRES (Generalized Minimal RESidual) iterative method is combined with a Schwarz-type preconditioning strategy based on the multi-block grid in the iterative solution solution of linear equations at each Newton step. The application of the 14-moment closure is considered for some canonical non-equilibrium flow problems, including subsonic flow around a circular-cylinder and a lid driven cavity flow. The predictive capabilities of the 14-moment interpolative closure are shown to surpass those of the regularized Gaussian closure, which models heat transfer through the addition of elliptic terms using a regularization technique applied to the low-order Gaussian closure. The 14-moment closure is also found to predict interesting non-equilibrium phenomena, such as counter-gradient heat transfer, a highly non-equilibrium phenomenon.

Keywords-component—Kinetic-theory; Non-equilibrium gas dynamics; Transition-regime gas dynamics; Hyperbolic moment closures.

I. INTRODUCTION AND MOTIVATION

The prediction of transition-regime non-equilibrium flows has proven to be a challenging branch of study in computational fluid dynamics (CFD). Transition-regime flows are encountered in a variety of engineering scenarios including: upper atmosphere flight and orbital reentry [1], flows in micro-electromechanical systems (MEMS) [2], chemical vapour deposition in semi-conductor manufacturing, and the study of internal shock structure [3]. These high-Knudsen-number flows cannot be modelled using typical continuum approaches, such as the Euler and Navier-Stokes-Fourier (NSF) equations. Traditional methods for modelling non-equilibrium flows, such as direct simulation Monte Carlo (DSMC) methods [4] and techniques involving direct discretization of the Boltzmann equation [5], are limited by their high computational cost, especially at low Mach numbers.

The method of moment closures [6]–[9] offers an alternative technique for accurately treating transition-regime flows with the potential of greater robustness and a significantly reduced computational cost. The moment-closure method yields an approximation to the Boltzmann equation that consists of a finite set of partial differential equations (PDEs). These equations are of lower dimensionality as compared with the Boltzmann equation, and thus computational cost is reduced. A hierarchy of moment closures having a number of desirable properties has been proposed based on the maximization of thermodynamic entropy [10]–[12]. Not only are these closures physically intuitive and mathematically elegant, but they have several practical advantages over traditional closures, such as those resulting from the classical Chapman-Enskog method.
They also lead to hyperbolic transport equations, which result in finite speeds for the propagation of solution information as expected based on classical laws, but not found in mixed-type descriptions involving elliptic-hyperbolic moment equations such as the Navier-Stokes equations.

The purely hyperbolic and first-order quasilinear nature of the maximum-entropy closures also presents several numerical advantages which extend into both the transition and continuum regimes [6], [15]. These hyperbolic systems are less sensitive to grid irregularities, making them well suited to adaptive mesh refinement (AMR) and complex geometries. They also only require evaluating first derivatives, which means that an extra order of spatial accuracy, relative to a mixed hyperbolic-parabolic system, can often be gained using the same stencil.

Unfortunately, complications encountered when considering maximum-entropy closures with treatment for higher-order (i.e., super-quadratic) moments, such as heat transfer, have severely limited the use of these maximum-entropy closures for general non-equilibrium flows. Nevertheless more recently, interpolative-type, maximum-entropy-based, 5-moment (one-dimensional gas) and 14-moment (three-dimensional gas) closures, initially investigated by McDonald and Groth [6], and expanded upon by McDonald and Torrilhon [16], have been proposed that successfully navigate the aforementioned problems. This has allowed the successful application of the interpolative variants of the super-quadatic maximum-entropy closures to a number of two-dimensional non-equilibrium flows, demonstrating their promise for transition-regime non-equilibrium flows [17]–[19].

In this study, a computationally efficient solution-adaptive finite-volume scheme is proposed and developed for obtaining time-invariant multi-dimensional solutions of the interpolative-based maximum-entropy 14-moment closure. A Godunov-type finite-volume with piecewise limited linear solution reconstruction is combined with an AMR algorithm [15], [20]–[22] permitting local refinement to obtain solutions to the governing hyperbolic system of moment equations on two-dimensional, body-fitted, multi-block grids consisting of quadrilateral cells. Time-invariant solutions of the spatially-discretized moment equations are obtained by using an inexact Newton’s method combined with a preconditioned Krylov subspace iterative method [20], [23], [24]. In particular, the GMRES (Generalized Minimal RESidual) iterative method is combined with a Schwarz-type preconditioning strategy based on the multi-block grid in the iterative solution solution of linear equations at each Newton step. The application of the 14-moment closure is examined for several canonical flow problems and the predictive capabilities of the closure is assessed for steady non-equilibrium gaseous flows of monatomic gases.

II. 14-MOMENT INTERPOLATIVE-BASED MAXIMUM-ENTROPY MOMENT CLOSURE

A. Kinetic Theory for a Monatomic Gas

Gas kinetic theory, as pioneered by Maxwell [25] and Boltzmann [26], provides a statistical-based description of the non-equilibrium behaviour of gases in terms of a number density (NDF), \( F(x_i, v_i, t) \), which is related to the probability of finding a single particle or molecule of the gas having a velocity, \( v_i \), at position, \( x_i \), at a given time, \( t \). The evolution of the distribution function is fully described by the Boltzmann equation [7], [26] which, for a monatomic gas in the absence of external forces, is given by

\[
\frac{\partial F}{\partial t} + v_i \frac{\partial F}{\partial x_i} = \frac{\delta F}{\delta t},
\]

where \( \delta F/\delta t \) Boltzmann collision integral and represents the influence of inter-particle collisional processes on \( F \). For the purposes of this numerical study, the Boltzmann collision integral is modelled by using the extension of the original relaxation time approximation or Bhatnagar-Gross-Krook (BGK) model [27], commonly referred to as the Ellipsoidal-Statistical (ES-BGK) [28], which allows for realistic Prandtl numbers while at the same time remaining computationally tractable for practical simulations. The ES-BGK approximation has also been shown to satisfy the Boltzmann H theorem [29].

B. Macroscopic Moments

Knowledge of the NDF, \( F \), provides a full and complete description of a non-equilibrium gas, including the prescription of all macroscopic quantities. Conventional macroscopic quantities of practical and/or engineering interest can be defined in terms of so-called “moments” of the NDF. The macroscopic moment quantities considered herein are as follows

\[
\rho = \iint_{-\infty}^{\infty} mF \, d^3c_i = m \langle F \rangle, \quad \frac{m}{\rho} \langle c_i F \rangle = 0,
\]

\[
P_{ij} = m \langle c_i c_j F \rangle, \quad p = \frac{1}{3} P_{ii} = \frac{m}{3} \langle c^2 F \rangle,
\]

\[
Q_{ijk} = m \langle c_i c_j c_k F \rangle, \quad q_i = \frac{1}{2} Q_{iij} = \frac{m}{2} \langle c_i^2 F \rangle,
\]

\[
R_{ijkl} = m \langle c_i c_j c_k c_l F \rangle, \quad 15r = R_{ikkk} = m \langle c^4 F \rangle,
\]

\[
S_{ijkm} = m \langle c_i c_j c_k c_m F \rangle, \quad s_i = S_{ijll} = m \langle c_i c^4 F \rangle,
\]

where \( \rho \) is the density, \( c_i = v_i - u_i \) is the random component of the molecular velocity, \( p \) is the usual hydrostatic pressure, \( P_{ij} \) is the generalized pressure tensor with \( p = P_{ii}/3 \), \( q_i \) is the usual heat-flux vector, \( Q_{ijk} \) is the generalized heat-flux tensor with \( q_i = Q_{iij}/2 \), \( R_{ijkl} \) and \( r \) are the fourth-order and contracted fourth-order moments, respectively, and \( S_{ijkm} \) is the fifth-order moment. The lower-order moments, \( \rho, u_i, p, P_{ij}, \) and \( q_i \) are familiar fluid-dynamic quantities. Recall that \( P_{ij} = \delta_{ij} \rho - \tau_{ij} \), where \( \tau_{ij} \) is the fluid stress tensor. Other higher-order moments can be thought of as fluxes of lower-order quantities produced by the random molecular motion.

C. Maximum-Entropy Approximation for NDF

In maximum-entropy moment approximations, approximate solutions to the Boltzmann equation are sought by assuming the most likely form for the NDF corresponding to a finite set of
Boltzmann entropy, is given by
\[ \mathcal{F}(x_i, v_i, t) \approx \mathcal{F}^{(N)}(x_i, v_i, t; \alpha^{(N)}) = \exp \left[ \alpha^T \mathbf{V}^{(N)}(v_i) \right], \tag{3} \]
where \( \mathbf{V}^{(N)} = [1, v_i, v_iv_j, v_iv_jv_k, v_iv_jv_kv_l, \ldots] \) are the \( N \) total velocity weights for the closure, \( \alpha^{(N)} \) is the set of corresponding coefficients associated with each weight that ensures agreement with the \( N \) moment constraints, \( \mathbf{M}^{(N)} \). The values of \( \alpha^{(N)} \) can be obtained in terms of the selected moments by solving a constrained entropy-maximization problem.

Unfortunately, issues with maximum-entropy closures arise for approximations involving higher-order, super-quadratic, velocity weights, such as those that describe heat transfer. Firstly, the entropy-maximization problem ceases to have closed-form analytic solution and relatively expensive iterative numerical solution methods are required to relate the closure coefficients, \( \alpha^{(N)} \), to the known moments, \( \mathbf{M}^{(N)} \). Furthermore and more limiting, regions in physically realizable moment space (i.e., the phase space of physically realistic moments corresponding to a strictly positive definite NDF) are present as first identified by Junk [30] within which the maximum-entropy problem cannot be solved as the approximate distribution does not remain bounded. This subspace of non-realizable moment is commonly referred to as the Junk subspace. The lack of analytical and bounded solutions to the entropy-maximization problem is particularly challenging to the closure problem and evaluation of the closing fluxes in terms of the known moments, \( \mathbf{M}^{(N)} \).

**D. Hyperbolic Moment Equations of 14-Moment Closure**

Considering the maximum-entropy hierarchy proposed by Levermore [11], the fourth-order or quartic 14-moment closure involves the velocity weights \( \mathbf{V} = [1, v_i, v_iv_j, v_iv_jv_k, v_iv_jv_kv_l, v_i^2, v_i^2v_j, v_i^2v_jv_k, v_i^2v_jv_kv_l, \ldots] \) and includes a treatment for heat transfer. The corresponding set of 14 macroscopic moments are

\[
\mathbf{M}^{(N)} = \begin{bmatrix}
\rho \\
\rho u_i \\
\rho u_i u_j + P_{ij}
\end{bmatrix}, \tag{4}
\]

\[
\begin{bmatrix}
\rho u_i u_j + u_i P_{ij} + u_j P_{ij} + Q_{ijj}
\rho u_i u_j u_j + u_i u_j P_{jj} + 2u_i P_{jj} + 2u_j P_{jj} + Q_{ijj}
\rho u_i u_j u_j + u_i u_j P_{jj} + 2u_i u_j P_{jj} + 4u_i u_j P_{ij} + 4u_i u_j Q_{ijj} + R_{ijijj}
\end{bmatrix}
\]

Transport equations for these macroscopic quantities can be obtained by taking appropriate velocity moments of the Boltzmann equation. The resulting hyperbolic set of moment equations for the 14-moment maximum-entropy closure are

\[
\frac{\partial}{\partial t} \mathbf{M} + \frac{\partial}{\partial x_k} (\mathbf{F}_k) = \mathbf{S}, \tag{5}
\]

where the moment flux dyad, \( \mathbf{F}_k \), is given by

\[
\mathbf{F}_k = \begin{bmatrix}
\rho u_i \\
\rho u_i u_j + P_{ij} \\
\rho u_i u_j u_j + u_i u_j P_{jj} + u_j P_{jj} + u_i P_{ij} + u_j P_{ij} + Q_{ijj}
\end{bmatrix}, \tag{6}
\]

and the source vector, \( \mathbf{S} \), of collision terms associated with the ES-BGK collision operator is given by

\[
\mathbf{S} = \begin{bmatrix}
0 \\
0 \\
\frac{P_{ij} - \delta_{ij} P}{\tau}
\end{bmatrix}, \tag{7}
\]

and where \( \tau \) is a characteristic relaxation time scale and \( \Pr \) is again the Prandtl number. It is readily apparent from the preceding set of moment equations that there are a number of high-order moment fluxes (i.e., \( Q_{ijj}, R_{ijijj}, \) and \( S_{ijjjkk} \)) appearing in the flux dyad, \( \mathbf{F}_k \), which are not part of the known moment set, \( \mathbf{M} \). For closure, these fluxes must be specified in terms of the known moments. Lack of an analytically integrable approximation for \( \mathcal{F} \) as well as the issues with singular nature of solutions of the entropy-maximization problem on the Junk subspace discussed previously prevent the formulation of closed-form analytical expressions for the closing fluxes. Nevertheless, a way forward is offered by an interpolation procedure for the closing fluxes.

**E. Closing Fluxes Via Interpolation**

As described by McDonald and Torrillhon [16], approximate expressions for the closing fluxes \( Q_{ijj}, R_{ijjj}, \) and \( S_{ijjjkk} \) can be obtained via an interpolation procedure in which a parabolic mapping of the region of physically realizability moments is first identified in terms of a mapping parameter, \( \sigma \in [0, 1] \), with \( \sigma = 0 \) on the singular Junk subspace and \( \sigma = 1 \) on the lower parabolic boundary of realizability space. Expressions for the closing fluxes are then sought that are consistent with the values of the fluxes at the boundaries and transition to the interior of moment realizability space with approximate agreement to numerically computed solutions of the maximum-entropy and
moment inversion problem, while importantly preserving the singular behaviour of the closure at the Junk subspace. In the case of the 14-moment closure for three-dimensional kinetic theory of a monatomic gas, the parabolic surface mapping can be defined by the following expression for the fully contracted fourth-order velocity moment, \( R_{ijkk} \):

\[
R_{ijj} = \frac{1}{\sigma} Q_{kkl}(P^{-1})_{kl} Q_{lij} + \frac{2(1 - \sigma) P_{jil} P_{ij} + P_{ij} P_{jlk}}{\rho},
\]

with \( \sigma \in [0, 1] \). The corresponding approximate expressions for the closing fluxes \( Q_{ijk} \), \( R_{ijkk} \), and \( S_{ijjkk} \) are then taken to have the forms

\[
Q_{ijk} = K_{ijklm} Q_{mmn},
\]

\[
R_{ijkk} = \frac{1}{\sigma} Q_{ijj}(P^{-1})_{lm} Q_{nkk} + \frac{2(1 - \sigma) P_{ik} P_{kj} + P_{ij} P_{kk}}{\rho},
\]

and

\[
S_{ijjkk} = \frac{Q_{npp} Q_{mjj} Q_{ikl}}{\sigma^2 P_{kn} P_{lm}} + 2\frac{\sigma}{\rho} \frac{P_{ij} Q_{kk}}{\rho} + (1 - \sigma) W_{ij} Q_{mmn},
\]

where

\[
K_{ijklm} = \frac{P_{il}(P^2)_{jk} + P_{kl}(P^2)_{ij} + P_{jl}(P^2)_{ik}}{P_{lm}(P^2)_{\alpha\alpha} + 2(P^3)_{lm}},
\]

and

\[
W_{ij} = \left[ P_{il}(P^2)_{\alpha\alpha} + 6 P_{il}(P^3)_{\alpha\alpha} + 7(P^2)_{\alpha\alpha}(P^2)_{il} + 10(P^4)_{il} + 10(P^4)_{il} - (P^2)_{\alpha\alpha} P_{il} P_{il} \right] \rho P_{lm}(P^2)_{\alpha\alpha} + 2(P^3)_{lm}^{-1}.
\]

The preceding interpolative-based 14-moment closure avoids extremely expensive iterative numerical solution procedures for the moment inversion problem and evaluation of the closing fluxes via numerical quadrature. As evident from the parabolic mapping procedure of Eq. (10), the interpolative closure retains the singular behaviour of the underlying maximum-entropy 14-moment closure along the Junk subspace, which yields very large propagation speeds and therefore yields accurate solutions of stationary, one-dimensional, shock structure having smooth transitions, without undesirable sub-shocks, even for high-Mach numbers [16], [19].

**F. Boundary Conditions**

Determining the boundary conditions at a solid wall to preserve non-equilibrium phenomena is achieved by assuming that a Knudsen layer of infinitesimal thickness forms adjacent to the wall [31]. In the Knudsen layer, the NDF, \( \mathcal{F}_{Kn} \), is assumed to be a linear combination of the distribution function of particles/molecules from the interior flow field and particles reflected and/or arising from the solid wall. The NDF for particles reflected to/from arising from the solid wall is estimated based on an accommodation coefficient, \( \mathcal{A} \), indicating the degree on diffuse or specular reflection from the wall. Particles fully accommodated at the wall, \( \mathcal{A} = 1 \), are described by the Maxwellian-Boltzmann equilibrium distribution at the wall. Particles that undergo elastic reflection at the wall, \( \mathcal{A} = 0 \), are described by a Grad-like truncated power series expansion based on the Gaussian NDF [32], [33]. The latter allows the derivation of analytical expressions for the non-equilibrium moments in the Knudsen layer.

**III. NUMERICAL SOLUTION METHOD FOR STEADY FLOWS**

**A. Upwind Finite-Volume Method and Semi-Discrete Form**

The hyperbolic moment equations of the interpolative 14-moment closure of Eq. (5) are very well-suited to solution via Godunov-type upwind finite-volume schemes [34]. In the present study, a high-resolution Godunov-type upwind finite-volume scheme with limited piece-wise linear solution reconstruction is applied to the moment equations of the 14-moment closure for two-dimensional flows on multi-block body-fitted meshes consisting of quadrilateral cells. For a two-dimensional quadrilateral computational cell \((i, j)\) and adopting a second-order accurate mid-point quadrature rule for the flux evaluation at the face of each cell, application of the upwind finite-volume scheme to the integral form of the governing equations results in the following semi-discrete form of the 14-moment equations:

\[
d\mathbf{M}_{ij} = -\frac{1}{A_{ij}} \sum_k \left( \mathbf{F} \cdot \mathbf{n} \Delta \ell \right)_{i,j,k} + \mathbf{S}_{ij} = -\mathbf{R}_{ij}(\mathbf{M}),
\]

where \( \mathbf{M}_{ij}^n \) and \( \mathbf{S}_{ij}^n \) are the vectors of conserved solution variables and collisional source terms, respectively, associated with cell \((i,j)\), \( \mathbf{F} \) is the flux dyad, \( \Delta \ell \) is the length of cell face \( k \), \( \mathbf{n} \) is the outward unit vector normal to cell face \( k \), \( A_{ij} \) is the area of cell \((i,j)\), and \( \mathbf{R}_{ij}(\mathbf{M}) \) is the so-called solution residual. The upwind numerical flux, \( (\mathbf{F} \cdot \mathbf{n} \Delta \ell)_{i,j,k} \), associated with the \( k \)th cell face is evaluated here by using a combination of Riemann-solver based flux function with limited least-squares piece-wise linear reconstruction within each computational in conjunction with the slope limiter of Venkatakrishnan [35]. The HLLE Riemann-solver-based flux function [36], [37] is used to approximate solutions to the Riemann problem.

**B. Block-Based Adaptive Mesh Refinement (AMR)**

Computational grids that automatically adapt to the solution of the governing partial differential equations are very effective in treating problems with disparate length scales, providing the required spatial resolution while minimizing memory and storage requirements. A flexible block-based hierarchical data structure has been developed and is used here in conjunction with the preceding upwind finite-volume scheme to facilitate automatic solution-directed mesh adaptation on multi-block body-fitted quadrilateral mesh according to physics-based refinement criteria [15], [20]–[22]. In the proposed AMR scheme, the solution of the moment equations via the finite-volume method described above are obtained for multiple quadrilateral computational cells and these cells are embedded with the structured blocks of a multi-block grid.
C. Inexact Newton Method

For steady flows, the semi-discrete form of the 14-moment equations reduces to a large coupled nonlinear system of algebraic equations. A Newton iterative method is a common, robust, and efficient iterative technique for the solution of nonlinear systems of this type and is used here. The present implementation of Newton’s method [20], [23], [24], make use of right-preconditioned generalized minimal residual (GMRES) iterative method [38]–[41] for solution of the linear problem. A domain-based additive Schwarz preconditioning technique is used as the global preconditioner and incomplete lower-upper factorization with fill is used as the local preconditioner to improve the convergence rate. The Schwarz preconditioner provides the advantage of using the same domain decomposition procedure as used by the block-based AMR scheme, making parallel implementation rather straightforward.

IV. Numerical Results

The application of the proposed combination of upwind finite-volume scheme and NKS iterative procedure for the solution of the interpolative-based 14-moment closure is now considered for several steady flows, both in the near-equilibrium or continuum limit and under non-equilibrium conditions in the transition regime.

A. Transition-Regime Subsonic Flow Past a Circular Cylinder

The case of subsonic flow past a circular cylinder is examined first and for this case, solutions are obtained in the transition regime at $Kn = 1$, for a speed ratio of $S = 0.027$. The gas considered is argon at a temperature of 397.37 K and pressure of 101.325 kPa. For the 14-moment solution, the heat-flux streamlines are similar to that of the regularized Gaussian solution, however, the temperature contours differ. Refer to Figure 1. In the case of the regularized Gaussian solution, the temperature decreases at the front of the cylinder and increases at the back. For the 14-moment solution, the temperature increases at the front of the cylinder and decreases at the back, such that the 14-moment closure predicts heat flux in the direction opposite to the temperature gradient. This counter-gradient heat transfer has previously been observed for transition regime flows in analytical predictions by Torrillon [42], demonstrating the capability of the 14-moment closure to predict highly non-equilibrium flow phenomena. The non-gradient transport effect appears in the analytical solution for the bulk temperature [42], which includes terms proportional to Kn and are not dependent on the heat-flux. The regularized Gaussian cannot predict the counter-gradient heat transfer due to the nature of the first-order correction for the generalized heat flux tensor, $Q_{ijk}$, introduced by the perturbative expansion procedure applied to the Gaussian closure. For the Gaussian closure, $Q_{ijk} = 0$. The resulting first-order correction to the heat-flux tensor is estimated to be directly proportional to the negative of the gradient of the anisotropic temperature tensor, $P_{ij}/\rho$, such that the direction of the heat flux is always opposite to that of the temperature gradient, as in the NSF equations.

B. Lid-Driven Cavity Flow

The second case considered here is that of lid-driven cavity flow. In this case, a square computational domain is considered with the lid moving in the positive $x$-direction at a velocity of 50 m/s. The cavity contains argon gas at a pressure of 101.325 kPa and temperature of 273 K. The wall temperatures are maintained at the initial gas temperature of 273 K. Solutions are computed at Knudsen numbers of Kn $= 0.001$ and Kn $= 0.1$. In the continuum regime, Kn $= 0.001$, the solution is compared to the Navier-Stokes Fourier (NSF) solution and regularized Gaussian. In the transition regime, Kn $= 0.1$, the results are compared to available DSMC solutions [43], [44] and the regularized Gaussian. The Mach number profiles for Kn $= 0.001$ are in fairly good agreement show improvement for all three solutions. Refer to Figure 2. The temperature profiles for Kn $= 0.1$ are shown in Figure 3 to demonstrate the capabilities of the 14 Moment closure to predict highly non-equilibrium flow phenomena. Both the DSMC and 14-moment solutions predict counter-gradient heat flux at the top of the cavity which is associated with highly non-equilibrium flow.

V. Concluding Remarks

An efficient iterative numerical procedure has been proposed for the solution of steady, non-equilibrium, two-dimensional flows of a monatomic gas governed by a novel 14-moment, interpolative-based, maximum-entropy closure. The 14-moment closure is shown to accurately describe a range of non-equilibrium non-equilibrium phenomena, including counter-gradient heat transfer.


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