Numerical investigation of flow structures development of the slat cove of the flow past a three-element high-lift wing

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Abstract—Three-dimensional simulations of the flow past a 30P30N three-element high lift wing are performed at low Reynolds numbers [O(10⁴)] using a high-order spectral element method. The study focuses on investigating flow structures of the slat cove and Görtler vortices that appear on the top of the main airfoil. The spanwise velocity component is measured in the slat cove and shows a sinusoidal form with a fixed wavelength corresponding to the spanwise velocity component. The amplitude of the spanwise velocity grows exponentially in time for approximately 6 orders of magnitude before a saturation is observed. Proper Orthogonal Decomposition (POD) modes are next used to analyze the flow and Görtler vortices.

Keywords-component—30P30N airfoil, Görtler vortices, Direct Numerical Simulation, vortex dynamics.

I. INTRODUCTION

Multi-element airfoils have been an area of active research in aerospace engineering due to their ability to provide high lift coefficients and improve aircraft performance, particularly during takeoff and landing. The 30P30N three-element high-lift airfoil, in particular, has shown promising results in wind tunnel tests and simulations, making it an attractive design for aircraft wings. The significane of multi-element airfoil research goes beyond the improved lift and performance, as these designs also play a crucial role in reducing aircraft noise and emissions, and enhancing safety during takeoff and landing. Furthermore, the use of multi-element airfoils has been of great interest for unmanned aerial vehicles (UAVs) and other small-scale applications, where low Reynolds numbers pose challenges for airfoil design. These applications frequently include low-Reynolds-number flows with airfoil-chord Reynolds numbers of $Re_c = cU_{∞}/ν < 5 \times 10^5$, where $c$ is the chord length, $U_{∞}$ is the free-stream velocity and $ν$ is the kinematic viscosity [1], [2]. The laminar boundary layer above an airfoil’s suction side is extremely prone to separation due to the low $Re_c$. A laminar-to-turbulent transition can result from the instability of the separated boundary layer, affecting the aerodynamic performance as airfoil performance tends to deteriorate as the chord Reynolds number $Re_c$ gets below 500,000 [1], [8], leading to numerous research studies to better understand the impact of the laminar-to-turbulent transition in multi-element airfoils [3]–[7]. At low Reynolds numbers, the effects of the angle of attack $\alpha$ and $Re_c$ on vortical structures interactions of a 30P30N multi-element airfoil were studied [9], [10]. For $Re_c$ below a critical value between $1.27 \times 10^4$ – $1.38 \times 10^4$, the top of the main airfoil is primarily characterized by the formation of Görtler vortices, which are counter-rotating vortices evolving in the streamwise direction. These vortices are thought to arise due to a virtual curved wall formed by the upper right boundary layer of the separation bubble located in the slat cove. The Görtler vortices, remain localized above the separated shear layer present on the main element, inducing the formation of streaky structures within the separated shear layer. As $Re_c$ increases to $3.05 \times 10^4$, the dominating vortices in the slat wake switched from Görtler vortices to roll-up vortices that leads to a simultaneous existence of spanwise and streamwise vortices in the slat wake. Shed spanwise vortices from the slat cove are distorted upon impact with the slat trailing edge and the accelerated flow in the gap region. They subsequently transform into hairpin-like vortices, for $Re_c$ larger than $1.27 \times 10^4$.

The complex disturbances generated by the slat can significantly impact the aerodynamic performance of the multi-element airfoil [11], particularly with regard to boundary...
layer interactions. The noise generated by the slat is an additional concern [12], making it important to understand the fundamental fluid mechanics of the slat’s vortex dynamics. Improving our understanding of the vortex dynamics in the slat wake could lead to the development of more efficient and quieter low-Reynolds-number aircraft.

The formation of Görtler vortices is a common feature in boundary layer flows over concave walls. Despite a clear understanding of the linear evolution of these vortices, their breakdown mechanisms remain not fully understood [13], [14]. Previous studies mainly investigated the wake/shear layer interactions over the main element, which revealed that Görtler vortices can interact with the separated shear layer above the main element and induce streaky structures [9], [10], further investigations are needed to understand the vortex dynamics within the slat cove at low Reynolds numbers.

This study presents an early-time investigation of the development of flow structures in the slat cove of a three-element high-lift airfoil operating at a low critical Reynolds number of $1.27 \times 10^4$. Three-dimensional numerical simulations were performed to analyze the flow in the slat cove and understand key factors contributing to the formation of Görtler vortices.

II. METHODOLOGY

The initial configuration and the coordinate system $(x,y,z)$, where $x$ is the streamwise, $y$ the normal and $z$ the spanwise directions are shown in a sketch in Fig. 1. The flow direction is from the left entry to the right exit.

The incompressible Navier-Stokes equations are solved using a high-order spectral element method [16] developed in the Nek5000 solver [15], with constant density $\rho$ and viscosity $\mu$: 

$$\nabla \cdot \mathbf{u} = 0,$$

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u},$$

where $\mathbf{u}$ is the velocity vector and $p$ the pressure. The governing equations are discretized in the weak form using a Galerkin method, where the computational domain is divided into $K$ hexahedral spectral elements. For each element, the solution is approximated using tensor products of $N$ th-order Legendre polynomial Lagrangian interpolants [15].

The present study employs a polynomial order of $N = 7$ with a total of 132,820 spectral elements. In a previous study [17] a mesh convergence is achieved on the same mesh using a polynomial order $N = 7$. Increasing the polynomial order up to $N = 11$ has shown no significant difference in the flow dynamics. A 30P30N three-element high lift airfoil that incorporates three separate elements—slat, main, and flap—is featured in the computational domain. In the present simulations, the airfoil’s stowed chord length $c$ corresponds to 1m. The lengths of the slat and flap are 15%c and 30%c, respectively. The computational domain extends three chords ahead, 4.75 chords behind, and 2.5 chords above and below the airfoil. The domain is rectangular with a length of nine chords and a height of five chords, while the span is 20% of the chord length. Neumann boundary conditions are applied at the top and bottom boundaries of the domain, as depicted in Fig. 1. These boundary conditions enforce a zero $y$-derivative of the vertical velocity, and are translated in Nek5000 as symmetrical boundary conditions. At the inlet boundary, uniform velocity boundary condition is applied. On the walls of the airfoil, a no-slip condition is mandated. The outflow boundary at the exit is defined with a no-stress condition, while a sponge-forcing function [18] is implemented to dampen outgoing structures with a length over 0.1c.

To appropriately resolve the boundary layer, the mesh density is high close to the airfoil surface and gradually decreases while moving farther away. Ten equal-sized elements are used to resolve the spanwise direction; the length of the spanwise direction should be sufficiently large to allow the development of three-dimensional instabilities. A side view of the spectral

Figure 1. A sketch of the Computational domain of the 30P30N airfoil showing the side view of the $x-y$ plane with a focus on the slat position.

Figure 2. 2D slice showing the computational domain and the 30P30N airfoil elements mesh distribution.
element grid for each of the three airfoil elements is illustrated in the insets of Fig. 2. The flow analysis below is focused on the early-time development of vortical structures formed in the slat cove.

III. RESULTS

Three-dimensional simulations are run at $Re_c = 1.27 \times 10^4$. As shown in Fig. 3, where vorticity magnitude contours are displayed, regular streamwise structures can be seen above the main element. The inset image of this figure shows a 2D slice in the $z-y$ plane. The traditional “Mushroom shape” of Görtler vortices are shown. Secondary counter-rotating vortices, rotating in the opposite direction, are located beneath the Görtler vortices and illustrated in the $z-y$ plane of Fig. 3. This Mushroom shape persists with the flow in the streamwise direction and is in a good agreement with previous experimental observations [9], [10].

To investigate the origin of the Görtler vortices, we analyze the early-time flow development of the slat cove. The spanwise velocity component, $w$, is measured at $x/c = 0.02$ and $y/c = -0.01$, which corresponds to a spanwise line located inside the slat cove. The obtained results reveal a distinct sinusoidal shape of the $w$-velocity at time $tU_\infty/c \lesssim 8$. The amplitude of the $w$-velocity is computed using a fit with a sinusoidal function. An exponential growth is observed and the corresponding wavelength of the perturbation is found to be constant during the early-time dynamics over a range of $tU_\infty/c = 3.2 - 8$, with a value of $\lambda/c = 0.04$, see Fig. 4. This value is consistent with the reported wavelength of streaky structures observed at the same $Re_c = 1.27 \times 10^4$ [17]. Note that Görtler vortices are observed on the main airfoil after an initial amplification of approximately four orders of magnitude of the slat cove perturbation. A nonlinear regime is observed at time $tU_\infty/c > 8$. The growth rate of the linear amplification is estimated by fitting an exponential function to the amplitude plot. The resulting growth rate is found to be $\omega = 1.45U_\infty/c$ during the time interval between $tU_\infty/c = 3.2 - 8$. Vortical structures evolving in the slat cove prior to the nonlinear regime are observed using 2D slices of the $x$-vorticity contours in $z-y$ planes covering the space between the slat and the main airfoil, see Fig. 5. At the early-time instance of $tU_\infty/c = 6$, vortical structures are observed at $x/c = -0.01$ in the slat cove region as shown in Fig. 5(a). These vortices are accompanied by secondary counter-rotating vortices. The secondary vortices are located below the primary vortices as observed for the Görtler vortices in Fig. 3. The presence of these vortices at the trailing edge and within the slat cove clearly indicates the spatial origin of the Görtler vortices. The separation flow from the concave wall formed by the bottom part of the slat is therefore the key element of the formation of Görtler vortices. As the flow progresses in time, a nonlinear evolution of the vortical structures leads to the formation of distinct bell-shaped structures at $x/c = 0.02$ in Fig. 5(b). The bell-shaped structures indicate a complex interaction between the primary vortices and the secondary counter-rotating vortices. At a later stage, mushroom-shaped structures are observed at $x/c = 0.06$, as depicted in Fig. 5(c), persisting in the streamwise direction. The observed vortices at the flow separation of the trailing edge of the slat undergo nonlinear transformations from their initial shape at early-time to the development of Görtler vortices. The presence of secondary counter-rotating vortices, bell-shaped structures, and mushroom-shaped structures at different time instances highlights the intricate interplay between vortical structures in the slat cove.

Next, we use a two-point correlation analysis of the streamwise velocity to examine similarities and coherent structures present in the flow. Fig. 6 illustrates the two-point correlation of the streamwise velocity taken at $tU_\infty/c = 7.5$ within the slat cove in a $z-y$ plane at $x = 0.02$. It can be seen that there are specific regions where the coherence between velocity fluctuations is strong, highlighting the presence of organized flow structures. These coherent structures appear to be well-defined and maintain their organization over the spatial extent with a clear wavelength $\lambda/c = 0.04$ as found in the linear amplification of Fig. 4. Conversely, there are areas where the correlation values are low, suggesting that the velocity fluctuations are less organized in these regions.

In the study of flow dynamics, it is of paramount importance to comprehend the most dominant structures and their contribution to the overall flow behavior. Proper Orthogonal Decomposition (POD) [22], [23] serves as a valuable tool for this purpose, as it allows us to decompose complex flow fields into a set of ordered modes that capture the most energetic structures. The application of POD helps to identify these dominant structures and shed light on their interaction with the flow dynamics [24]. POD works by transforming the original dataset into a new coordinate system, with the aim of capturing...
the maximum variance in the flow field with the smallest number of modes. The main procedure for POD calculation involves computing the covariance matrix of the dataset, followed by obtaining the eigenvalues and eigenvectors of this matrix. The eigenvalues represent the energy content of each mode, while the eigenvectors (also known as POD modes) describe the spatial structure of the corresponding mode. The mathematical expression for the POD modes can be written as:

$$\phi_i(x) = \frac{1}{\sqrt{\lambda_i}} \sum_{n=1}^{N} a_{ni} u_n(x),$$

where $\phi_i(x)$ is the $i$-th POD mode, $\lambda_i$ is the corresponding eigenvalue, $a_{ni}$ is the time coefficient for the $i$-mode at the $n$-th time step, $u_n(x)$ is the velocity fluctuation at the $n$-th time step, and $N$ is the total number of time steps. The energy content of each mode is given by the square of the singular values from the POD analysis:

$$E_i = \sigma_i^2$$

where $E_i$ is the energy of mode $i$, and $\sigma_i$ is the singular value associated with mode $i$. The relative energy contribution of each mode is calculated as the ratio of its energy to the total energy of all modes:

$$\frac{E_i}{\sum_{j=1}^{N} E_j} \times 100$$

where $N$ is the total number of modes. The first POD mode has the highest eigenvalue and captures the most significant structures in the flow, while the lower-energy modes, represent the finer-scale structures and their interactions with the dominant flow features.

Figures 7, 8 and 9 show the first four POD modes at different time intervals. Both mode 1 and mode 2 captured most of the total energy in each of the three time span shown in Fig. 7, 8 and 9. For the three figures analyzed, the combined energy contribution of mode 1 and mode 2 was 72.3 %, 78.92 %, and 61.12 %, respectively. On average, these two modes account for approximately 70.78 % of the total flow energy, indicating that they are the most dominant modes in the flow field. In Fig. 7, it is evident that no coherent structures are present in the flow. In this initial time range $tU_\infty/c = 3.2$ to 6, the flow is dominated by shear layers and linear amplification of spanwise perturbations. As time increases to the interval represented by Fig. 8, $tU_\infty/c = 6$ to 8, we observe wave structures in the first POD mode and well defined coherent structures in the second and third POD modes. Five different cores of these structures are shown in the second POD mode and correspond to the coherent structures exhibit in the two-point correlation of Fig. 6. For this time range, the flow is still dominated by shear layers interactions which are the most energetic modes and the primary flow features and dynamics. Coherent structures are secondary for this time range and do not significantly impact the overall flow field. This explains why the Görtler vortices are observed above the main airfoil only at long time. For time between $tU_\infty/c = 8$ to 10, the coherent structures become dominant and are shown in the first POD mode of Fig. 9. These structures exhibit characteristics similar to Görtler vortices in terms of their wavelength and spatial distribution. The remaining POD modes display nonlinear patterns indicating a transition to turbulent flow in the slat cove.

IV. CONCLUSION

Three-dimensional numerical simulations of the flow past a 30P30N airfoil are performed using a high-order spectral element method at a critical Reynolds number of $Re_c = 1.27 \times 10^4$ with a fixed angle of attack $\alpha = 4^\circ$. Görtler vortices are observed on the top of the main airfoil and agree with previous experimental work [9], [10]. The early-time flow dynamics of the slat cove shows an exponential amplification of a spanwise perturbation with a fixed wavelength corresponding to that of the structures later observed above the main airfoil. Vortical structures with similar configurations as Görtler vortices, namely bell-shaped structures consisting of pairs of counter-rotating vortices accompanied by underneath secondary vortices, are observed in the early-time dynamics near the lower trailing edge of the slat. This suggests that the origin of Görtler vortices formation lies in the separation
Figure 5. 2D slices of streamwise vorticity contours of the slat cove in three different \((y-z)\) planes \(x/c = -0.01, 0.02, 0.06\). (a) Initial vortical structures and counter-rotating vortices at \(tU_\infty/c = 6.0\); (b) Nonlinear evolution of vortical structures forming distinct bell-shaped structures at \(tU_\infty/c = 7.0\); (c) Mushroom shape of Görtler vortices and underneath counter-rotating vortices at \(tU_\infty/c = 8.0\).

Figure 6. Visualization of spatial coherent structures in the slat cove at \(tU_\infty/c = 7.5\) using the two-point correlation of the streamwise velocity fluctuation.

Figure 7. POD modes of the slat cove for the time range \(tU_\infty/c = 3.2\) to 6. Top: mode 1 (left) and mode 2 (right); Bottom: mode 3 (left) and mode 4 (right).

The flow from the concave wall formed by the bottom part of the slat. Cores of coherent structures and bell-shaped vortices are well captured and represented by the two-point correlation of the streamwise velocity fluctuation. Proper Orthogonal Decomposition (POD) analysis is used to further investigate the dynamic development of the flow structures. It is shown that the flow is initially dominated by shear layers which develop into spanwise perturbations. Later, vortical structures emanating from the flow separation at the lower trailing edge of the slat evolve and interact with spanwise perturbations. The resulting bell-shaped structures consisting of pairs of counter-rotating vortices become dominant in the slat cove dynamics at later time. These structures are advected by the flow to the top
of the main airfoil and formed the observed Görtler vortices.

REFERENCES