FINITE ELEMENT MODELLING OF EXTERNALLY SHEAR-STRENGTHENED BEAMS USING FIBRE REINFORCED POLYMERS

MODÉLISATION PAR ÉLÉMENTS FINIS DU RENFORCEMENT EXTERNE EN CISAILLEMENT DES POUTRES EN BÉTON ARMÉ EN UTILISANT LES POLYMÈRES RENFORCÉS DE FIBRES

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Le besoin en réhabilitation des structures en béton est bien connu. Un grand nombre de recherches sont dans ce domaine. L'utilisation de polymères renforcés de fibres dans la réhabilitation a montré que cette solution est compétitive du point de vue de sa performance structurale et de son aspect économique. Le renforcement au cisaillement des poutres en béton est nécessaire quand la poutre est déficiente en cisaillement, ou quand sa capacité au cisaillement devient insuffisante après de son renforcement en flexion. Une technique qui a fait ses preuves pour le renforcement des poutres en béton est de coller des lamelles de composites additionnelles.

Au cours des dernières années, une grande quantité de travaux de recherche a été conduite sur le renforcement au cisaillement avec des composites et cela a mené à une meilleure compréhension du comportement. Plusieurs équations de design ont été proposées pour le calcul de poutres de béton armé renforcées au cisaillement avec des composites. La plupart des paramètres qui contrôlent le comportement des poutres renforcées au cisaillement ont été identifiés. Les équations de design, qui décrivent le comportement des poutres renforcées au cisaillement, ne sont pas suffisantes pour évaluer la contribution au cisaillement des composites PRF utilisés. Ceci peut être attribué à l'absence d'un modèle numérique précis, dont l'utilisation est plus économique que l'expérimentation, pour tenir compte des complexités du comportement des poutres renforcées au cisaillement et pour atteindre une meilleure compréhension des mécanismes de rupture.

Des analyses limitées par élément finis ont été effectuées sur les poutres renforcées en cisaillement. Comme contribution pour remplir ce manque, un modèle numérique versatile est développé dans cette étude pour prédire le comportement des poutres renforcées au cisaillement par des composites, avec une emphase sur le comportement de l'interface et le problème du délamination. Cette recherche est divisée en trois parties : (1) le développement d'un modèle numérique capable de capturer le comportement réel des poutres renforcées en cisaillement; (2) le modèle numérique proposé appliqué à différents cas de configurations de renforcement, tel que, poutres avec des lamelles verticales ou des lamelles inclinées, des poutres avec des enveloppes en forme de U, ainsi que des poutres avec des lamelles ancrées aux extrémités et; (3) une étude paramétrique faite pour évaluer l'influence sur
Le comportement au cisaillement du taux d'armature des étriers, de la résistance à la compression du béton, du module élastique du composites, ainsi que son épaisseur, et du rapport entre la largeur du composites et celle de la poutre.


Des équations de régression ont été développées, sur la base d'une approche statistique (RSM). De nouvelles équations de design ont été proposées pour les cas de lamelles collées et pour les enveloppes en forme de U. Les équations proposées peuvent être utilisées dans un guide de conception de la contribution du composites au cisaillement. Quelques résultats de ce travail de recherche peuvent être trouvés dans Godat et al. [2007a,b].
Abstract

The need for structural rehabilitation of concrete structures all over the world is well known. A great amount of research is going on in this field. The use of fibre reinforced polymer (FRP) plate bonding has been shown to be a competitive solution regarding both the structural performance and the economical aspects. Shear strengthening of reinforced concrete beams is required when the beam is deficient in shear, or when its shear capacity falls below its flexural capacity after flexural strengthening. An accepted technique for the shear strengthening of reinforced concrete beams is to provide an additional FRP web reinforcement in the form of externally bonded FRP sheets.

Over the last few years, a considerable amount of research has been conducted on shear strengthening with FRP composites and that has led to a better understanding of the behaviour. Hence, many design equations have been proposed to design shear-strengthened beams. Most of the parameters that control the behaviour of shear-strengthened beams have been addressed. However, the design equations describing the behaviour of shear-strengthened beams are not sufficient to properly evaluate the shear contribution of the FRP composites. This might be attributed to the absence of an accurate numerical model, which is more economical than the experimental tests, to capture the complexities of shear-strengthened beams and to lead to a better understanding of the failure mechanisms.

Limited finite element analyses have been carried out on FRP shear-strengthened beams. As a contribution to fill this need, a versatile numerical model is developed in this study to predict the response of reinforced concrete beams strengthened in shear with bonded FRP composites, with a particular emphasis on the interfacial behaviour and debonding phenomena. This research consists of three phases. They are: (1) the development of a reliable numerical model that can capture the real behaviour of FRP shear-strengthened beams; (2) the use of the proposed numerical model to verify various cases having different strengthening configurations: beams with vertical and inclined side-bonded FRP sheets, the U-wrap scheme, as well as anchored FRP sheets and; (3) a parametric study conducted to identify design variables that have the greatest influence on the behaviour of shear-strengthened beams such as the steel stirrup reinforcement ratio,
concrete compressive strength, FRP elastic modulus, FRP thickness, and ratio between FRP width to beam width.

The proposed numerical model is validated against published experimental results. The predicted results are shown to compare very well with test results. It is shown that the formulation of the FRP/concrete interfacial behaviour is essential to analyses utilizing finite element models. The implementation of interface elements produces accurate predictions of the response of shear-strengthened beams. Furthermore, the numerical analysis provides useful information on the slips and propagation of debonding along the FRP/concrete interfaces. Predicted strain profiles along the FRP sheet depth are also presented.

Regression equations based on the statistical approach of the response surface methodology (RSM) are developed. New design equations to describe the FRP axial effective strain at the state of debonding are proposed for both side-bonded and U-wrap strengthening schemes. The proposed design equations can be used to provide simple design guidelines to predict the FRP shear contribution. Some of the results of this thesis research can be found in Godat et al. [2007a,b].
To my mother and father...

to my brothers and sisters...

to those gave me their hearts...

and their hearts are always with me...
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\[ A_f \quad = \quad \text{Area of FRP sheets} \]
\[ A_e \quad = \quad \text{Flexural reinforcement ratio} \]
\[ A_{sv} \quad = \quad \text{Cross sectional area of shear steel stirrups} \]
\[ a \quad = \quad \text{Shear span length} \]
\[ a_0 \quad = \quad \text{Inner shear span length} \]
\[ a/d \quad = \quad \text{Shear span length to effective depth ratio} \]
\[ b_c \quad = \quad \text{Spacing between FRP strips} \]
\[ b_f \quad = \quad \text{Width of FRP sheets} \]
\[ b_f/b_c \quad = \quad \text{Width ratio between FRP sheets to concrete member} \]
\[ b_w \quad = \quad \text{Width of concrete beam at the web} \]
\[ C \quad = \quad \text{Matrix of concrete modulus of elasticity} \]
\[ d \quad = \quad \text{Effective depth of concrete section} \]
\[ d_f \quad = \quad \text{Effective depth of FRP stirrups} \]
\[ E_c \quad = \quad \text{Concrete modulus of elasticity} \]
\[ E_f \quad = \quad \text{FRP Tensile modulus of elasticity} \]
\[ E_{pi} \quad = \quad \text{Equivalent multiaxial modulus of elasticity in the principal directions} \]
\[ E_s \quad = \quad \text{Secant modulus of elasticity} \]
\[ E_{fp} \quad = \quad \text{Axial rigidity of FRP sheets} \]
\[ f_c \quad = \quad \text{Concrete compressive strength} \]
\[ f_e \quad = \quad \text{Effective stress of FRP sheets} \]
\[ f_{fu} \quad = \quad \text{Ultimate stress in FRP sheets} \]
\[ f_t \quad = \quad \text{Concrete tensile strength} \]
\[ G_c \quad = \quad \text{Concrete shear stiffness} \]
\[ G_f \quad = \quad \text{Interfacial fracture energy} \]
\[ G_f \quad = \quad \text{Concrete fracture energy} \]
\[ h \quad = \quad \text{Height of concrete section} \]
\[ k \quad = \quad \text{Reduction factor for the characteristics of FRP sheets} \]
\[ k_1 \quad = \quad \text{Factor of concrete strength} \]
\[ k_2 \quad = \quad \text{Factor of strengthening scheme} \]
LIST OF SYMBOLS

\( k_v \) = Bond-reduction factor  
\( L \) = Beam length  
\( L_e \) = Effective bond length  
\( n \) = Number of FRP layers  
\( R_{Lr} \) = Ratio between the remaining bonded length and the initial bonded length  
\( s_0 \) = Corresponding interfacial slip to the maximum interfacial shear stress  
\( s_f \) = Central spacing between FRP strips  
\( s_{max} \) = Maximum interfacial slip  
\( t_f \) = Thickness of FRP sheets  
\( V_c \) = Shear strength of concrete  
\( V_n \) = Shear strength of the section  
\( V_r \) = Required shear capacity  
\( V_s \) = Shear strength of steel stirrups  
\( w_f \) = Width of FRP sheets  
\( w_{fe} \) = Effective width of FRP sheets  
\( \alpha \) = Inclination of FRP orientation  
\( \beta \) = Angle between the principal tensile stress and fibre directions  
\( \beta_w \) = Interfacial width ratio between the FRP sheet to spacing between FRP strips  
\( \gamma_f \) = Partial safety factor  
\( \gamma_{m,F} \) = Partial safety factor  
\( \varepsilon \) = Incremental strain  
\( \varepsilon_{fe} \) = Effective FRP axial strain  
\( \varepsilon_{fed} \) = Design FRP effective axial strain  
\( \varepsilon_{fu} \) = Ultimate strain in FRP sheets  
\( \varepsilon_m \) = Concrete maximum strain  
\( \varepsilon_{u} \) = Concrete ultimate strain  
\( \zeta \) = Tension stiffness factor  
\( \eta_m \) = Shear reduction factor  
\( \theta \) = Inclination of diagonal crack  
\( \nu \) = Poisson ratio  
\( \rho_f \) = FRP reinforcement ratio  
\( \delta \) = Incremental stress  
\( \sigma_m \) = Concrete maximum stress  
\( \sigma_{p} \) = Principal stress  
\( \sigma_u \) = Concrete ultimate stress  
\( \tau_{max} \) = Maximum bond stress
Chapter 1

Introduction

1.1 General

Changing social needs, more stringent design standards, increasing safety requirements, and the deterioration of existing reinforced concrete infrastructure are steadily increasing the demands for structural strengthening. Worldwide, many concrete highway bridges suffer from chloride-induced deterioration, which is the result of using de-icing salts during severe winter conditions. Bridge authorities have the common problem of an ageing bridge infrastructure subjected to increasing traffic volumes and loads. Additionally, other problems include design and construction errors, changes in design requirements and damage due to accidents. From the economic point of view, it is obviously untenable to replace all deficient structures. As an alternative, strengthening is an option to keep such structures safe.

Strengthening methods have been developed for flexural strengthening and used quite widely. Historically, steel has been the primary material used to strengthen concrete bridges and buildings [Swamy et al., 1987]. The bonding of steel plates to the tension face of the reinforced concrete is presented as an effective method for increasing the load carrying capacity, thereby reducing deflections and controlling cracking. However, using steel as a strengthening material faces some difficulty in the handling of the plates and in cutting to the shape, adds additional dead load to the structure and sometimes the installation time may be the critical issue [Li et al., 2001]. Furthermore, the main shortcoming
with the steel plate strengthening techniques is the danger of corrosion at the epoxy-steel interface in a highly corrosive environment, which could adversely affect the bond strength between the steel plate and the beam. Researchers have been searching to eliminate the corrosion problem and to replace the steel plate by a corrosion-resistant material. FRP composites are emerging as a popular and attractive material for the strengthening of reinforced concrete structures. This material is successfully being used in the automotive, marine and aerospace sectors.

When two or more distinct materials are combined on a macroscopic scale to form a useful material, the resulting product is referred to as a composite material [Agarwal and Broutman, 1990]. The basic constituents of such a material are usually combined in order that the composite exploits their best qualities. As a result, the composite material exhibits overall properties that are superior to those of the individual constituents. The FRP is a composite material generally consisting of carbon, aramid, or glass fibres in a polymeric matrix (e.g., thermosetting resin). Among many options these composites may be in the form of laminates or flexible sheets. The laminates are installed by bonding them to the concrete interface with a thermosetting resin. The sheets are either dry or pre-impregnated with resin (pre-preg) and cured after installation onto the concrete surface.

Compared to the other strengthening methods, FRPs show an outstanding performance when they are used for strengthening reinforced concrete structures. The advantages offered by the FRP sheets are the high strength/weight ratio, the lower weight that makes the handling and installation significantly easier, corrosion resistance, and formability. The latter property is important when sheets need to be installed in certain locations. Technological advancements in manufacturing and processing methods make it economically viable to be used in the construction industry. Some disadvantages of FRPs are their low fire resistance (unless they are protected), long-term durability is not yet guaranteed and their brittle failure mode has to be addressed. The FRPs have poor fire performance compared to the concrete. Finally, a significant disadvantage is the scarcity of accepted design standards.

It is often necessary to pay great attention to the shear capacity of reinforced concrete beams because of the catastrophic nature of shear failures, which typically occur without any advance warning [Teng et al., 2004]. Shear strengthening can be required for various
reasons, such as to remedy design and construction errors or because of functional changes or environmental attacks. Sometimes, the need for shear strengthening can also arise as a consequence of flexural strengthening, which may result in a shear capacity that is less than the enhanced flexural strength. Various strengthening schemes including full wrapping, U-jacketing, and side bonding of the beam have been used where both continuous FRP sheets and strips are employed. However, the option of complete wrapping is not likely to be adopted in the field since most beams are cast monolithically with the slabs. The FRP sheets contribute to the shear resistance of the beam in the same way as that of internal steel stirrups. One of the major difficulties of externally shear-strengthened beams is the debonding of the FRP plates when an access for complete wrapping is not available. Some anchorage systems have been proposed to ensure the full utilization of the FRP sheets.

The finite element method is a powerful computational tool, which allows complex analyses of the nonlinear response of RC structures to be carried out efficiently and accurately. With this method, the importance and interaction of various parameters affecting the response of FRP shear-strengthened beams can be studied analytically. An outcome of such analyses is the development of reliable numerical models that can reduce the number of costly test specimens required for investigations of a given problem. However, experimental research remains necessary to validate the essential information for the numerical models, such as material properties. Experimental results are required to evaluate the accuracy of numerical results. In comparison to analyses on FRP flexural strengthening, theoretical investigations concerning the behaviour of reinforced concrete beams strengthened in shear with FRP composites are rather limited. The numerical studies on FRP shear-strengthened beams are those of Kaliakin et al. [1996]; Arduini et al. [1997]; Malek and Saadatmanesh [1998a,b]; Al-Mahaidi et al. [2001]; Lee et al. [2001]; Wong [2001]; Lee [2003]; Santhakumar et al. [2004]; Elyasian et al. [2006]. In general, the numerical simulations provided quite satisfactory predictions of the overall behaviour of the shear-strengthened beams, in particular in terms of the overall load-deflection curves. However, most of these analyses did not explicitly address the details of the FRP/concrete interface and less attention has been paid to investigate the slip profiles along the FRP sheet depth. This thesis addresses this research gap.

Different parameters were observed to control the response of shear-strengthened beams. Experimental work is slowly increasing to build up a database of results of externally
CHAPTER 1. INTRODUCTION

shear-strengthened beams using FRP composites. However, the theoretical design models proposed in the literature are often contradictory. In order to determine the FRP shear contribution, the FRP effective strain is the most important and difficult term because it is the key factor in expressing the FRP shear contribution. Some researchers considered the FRP effective strain to be the ultimate strain in the FRP plies, others have taken it as a fraction of the bond stress between the FRP and concrete, and many researchers have related it to the axial rigidity of the FRP sheets and concrete compressive strength.

1.2 Scope

In this study, a three-dimensional model is developed to accurately simulate the behaviour of FRP shear-strengthened beams; it is based on the finite element package ADINA. In this model, the complex behaviour of the beams such as the bond-slip behaviour, concrete nonlinearity, and the different failure modes of the concrete and FRPs are taken into account. Also, the scope of this dissertation includes the development of reliable elements that are able to simulate the response of FRP composites as well as the FRP/concrete interfacial behaviour. In order to demonstrate the validity of the finite element model, different shear-strengthened beam applications such as side-bonded laminates and U-shaped wrapping configurations are investigated. Modelling simplifications and assumptions developed during this research are presented. The numerical predictions are compared to published test data. Such a model is used to investigate the characteristics of bond-slip behaviour and the various parameters influencing a shear-strengthened beam. Finally, statistical analyses are carried out using response surface methodology (RSM) and nonlinear regression analysis to develop an appropriate model for the effective FRP axial strain.

The accuracy of these models is evaluated by comparing the numerical predictions to the experimental data. Then, having verified the accuracy of the numerical models, parametric analyses are performed in order to gain better insight into the mechanisms governing the behaviour of such shear-strengthened beams. The significance of the present findings is discussed.
1.3 Research Objectives

The principal objective of this study is to increase our knowledge about the behaviour of externally shear-strengthened beams using FRP sheets, to establish the role of the various parameters controlling the behaviour, and to resolve the points of conflict. The specific objectives of this investigation are:

- To develop and validate an accurate numerical model for the simulation of the behaviour of externally shear-strengthened beams.
- To use this model to examine the role of the various parameters affecting such behaviour.
- To deliver an appropriate design expression that describes the effective FRP axial strain for the case of debonding.

The limitations of this research are:

- A reinforced concrete beam without any level of pre-stressing is assumed.
- Only static loads are considered.
- The research is limited to slender beams \( (a/d > 2; \text{ where } a \text{ is the shear span, and } d \text{ is the effective depth [CSA-A23.3-04, 2004]}) \).
- The effect of long term deflections and temperature effects are not taken into account.

1.4 Outline

The research presented herein is organized into different chapters. In this chapter, the objectives, backgrounds and scope of the study have been outlined. Previous studies relevant to the shear strengthening of reinforced concrete beams are summarized in Chapter 2. The development, analyses and specimens investigated to develop a reliable numerical
model are the focus of Chapter 3. The comparisons between the numerical and experimental data are given in Chapter 4. Chapter 5 describes the experimental and numerical program of the size effect of shear-strengthened beams. The validation of the numerical predictions against various configurations of FRPs is presented in Chapter 6. In Chapter 7, parametric studies and a design expression for the effective FRP axial strain are reported. Concluding remarks and recommendations for further studies are outlined in Chapter 8.
Chapter 2

Literature Review

2.1 Introduction

The purpose of this chapter is to provide a review of FRP shear-strengthened reinforced concrete beams. A brief description of the behaviour, factors controlling the behaviour, design algorithms and finite element models will be presented. In addition, this survey covers the use of anchorages at the ends of the FRP composites to avoid debonding failure modes.

2.2 Techniques for the Shear Strengthening of Beams

The strengthening of concrete members is usually accomplished by the construction of external reinforced concrete jackets, by epoxy bonding of steel plates to the side faces of the members, by external post-tensioning, or by fibre reinforced polymers (Figure 2.1). A suitable strengthening scheme needs to be selected for a given situation based on a number of factors, which may include:

- economical considerations
- the amount of increase in shear capacity required
2.2.1 Traditional Techniques

For the shear strengthening of existing concrete structures, a number of techniques have been developed to satisfy the demands for the increase of the shear bearing capacity. Among these techniques that have been developed, certain ones have experienced reasonable levels of use and success. For example, external post-tensioning, epoxy bonding of steel plates to the tension faces of the members, and addition of reinforced concrete outside the existing section are all recommended strengthening procedures. The most common problem with these techniques is that they are quite inconvenient, often requiring extensive equipment, time consuming and labour intensive.

Post-tensioning is a technique that has been used effectively to increase the shear capacity of both reinforced and prestressed concrete members either by internally or externally placed post tensioning to an existing beam web. With this type of upgrading, active external forces are applied to the structural member using post-tensioned cables externally or internally to resist new loads, as shown in Figure 2.2. The main advantages of this technique are that it provides immediate active strengthening, which relieves the overstressed conditions of the beam web, and its minimal additional weight of the repair system [Emmons, 1993; Deniaud, 2000; Suntharavadivel and Aravinthan, 2006; Shamsaia
2.2. TECHNIQUES FOR THE SHEAR STRENGTHENING OF BEAMS

Figure 2.2: Post-tensioning shear-strengthening technique [Deniaud, 2000]

et al., 2007]. The disadvantages of this technique are that the deck overlay needs to be removed and large numbers of holes must be drilled through the member. There is also some inconvenience for the user since one part of the bridge must be closed to facilitate the drilling process. The amount of steel weight added to a structure can also increase the dead load to the concrete girders.

Strengthening of concrete structures by means of bonding steel plates to the side faces of the beam was found to increase the shear capacity of the section [Taljsten, 1994]. Steel plate bonding found its way relatively quickly into practice [Barnes et al., 2001; Adhikary et al., 2000; Adhikary and Mutsuyoshi, 2006]. External steel plates are attached to the concrete with epoxy and/or mechanical anchorages, which are placed perpendicular to the shear cracks or to the beam longitudinal direction, as shown in Figure 2.3. Steel plate bonding was popular because of its various advantages, which include a minimal increase in the beam size, the simplicity, cost-effectiveness and efficiency. Although this method technically performs well for shear strengthening, it suffers from some drawbacks. The deterioration of the bond at the steel plate-concrete interface in a harsh environment by corrosion requires a lot of maintenance. The steel plate bonding operations require heavy lifting gear and full scaffolding for handling. During the curing operations, the plates require external pressure. The steel plates are always delivered in limited lengths (in the case of flexural strengthening of long elements) and they are difficult to apply to curved surfaces. Another drawback is the transportation problem [An et al., 1991].

Another technique of shear strengthening is to enlarge the cross section of the member. An overlay can be cast either around the web or over the top slab or by a combination of both (Figure 2.4). If adhesion between the new and old concrete can be assured, this technique is good from a technical standpoint. A new wider section will give a higher shear
capacity for the structure and reduce the deflections. Obviously, this technique increases the dead load and in some cases it is not permissible due to the lack of the capacity of substructure [Deniaud, 2000].

2.2.2 Fibre Reinforced Polymer Technique

Fibre reinforced polymers have found their way as strengthening materials for reinforced concrete structures in applications where conventional strengthening techniques may be problematic [Triantafillou, 1998]. The rapidly expanding body of literature in this area, along with the corresponding increase in the level of activity, confirms the fact that these new materials are progressively gaining wider acceptance by the civil engineering community. Nowadays, FRP systems are used in several applications to strengthen existing RC structures instead of the traditional systems using steel, as illustrated in Figure 2.5. FRPs may be attached on a beam or a slab tension surface to provide additional flexural strength, on the sides of a beam to provide additional shear strength, or wrapped
2.2. TECHNIQUES FOR THE SHEAR STRENGTHENING OF BEAMS

Figure 2.5: Examples of FRP strengthening for concrete structures [Sika Carbo Dur 2005]

around columns to provide confinement and additional ductility. Furthermore, concrete and masonry walls may be strengthened to resist seismic and wind loads, concrete pipes may be wrapped with FRP sheets to resist higher internal pressures, and tanks may be strengthened to withstand higher pressures [Neale, 2000].

The existing shear capacity of reinforced concrete beams can be enhanced by bonding FRP composites to the web, typically with the dominant fibre direction perpendicular to the shear crack or to the length of the member. There are a variety of FRP systems and arrangements reported in the literature. The type of arrangement used strongly influences the contribution of the externally bonded FRP to the shear load capacity. Apart from the common advantages of FRPs such as corrosion resistance and high strength-to-weight ratios, the versatility of FRP in coping with different sectional shapes and corners is also a benefit for shear strengthening applications [Teng et al., 2002]. Figure 2.6 illustrates a concrete bridge shear-strengthened with FRPs.

Various plate schemes have been used to increase the shear resistance of reinforced concrete beams. These schemes include bonding the plates to the sides of the beam only, bonding U-jackets to both sides and the tension face, or wrapping the plates around the whole cross-section of the beam. Different bonding schemes are shown below in Figure 2.7. The complete wrapping of the entire cross section is the most effective method of shear strengthening with FRP, as shown in Figure 2.7(a). Typically, this is not practical from a constructability standpoint. The presence of monolithic slabs or other supported elements
CHAPTER 2. LITERATURE REVIEW

Figure 2.6: Concrete bridge shear-strengthened with FRPs

Figure 2.7: Various schemes for wrapping FRP shear strengthening: (a) complete wrapping, (b) U-wrapping, (c) side-bonding

often prevents wrapping the sheet around the top of the section. One option might be to drill holes through the slab and wrap FRPs around the section. However, this method is complicated and costly.

The most common method of shear strengthening is to wrap the sides and bottom of the section. This method is referred to as a U-wrap and shown in Figure 2.7(b). The U-wrap is practical and is effective in increasing the shear carrying capacity. In some situations, it may not be possible to wrap the full height of the section. Shear strengthening is still possible by placing the reinforcement on both sides of the section (Figure 2.7c).

Shear repair schemes have been examined either by using strips or continuous sheets as illustrated in Figure 2.8. The advantages of the strips include the ability to select their number based on the shear strength requirements, and the ease of achieving a uniform epoxy thickness. The strips may be spaced at an equal distance throughout the shear span or at a different spacing. The plates may also be oriented at different angles to meet different reinforcing requirements as shown in Figure 2.9. The best way for increasing
2.2. Techniques for the Shear Strengthening of Beams

Figure 2.8: Various FRP shear strengthening distributions: (a) continuous reinforcement, (b) FRP strips

Figure 2.9: Sheets with their fibres oriented in various primary directions: (a) inclined sheets, (b) vertical sheets

The shear capacity of the section is bonding the plates parallel to the principal tensile stresses. This is achieved by the use of inclined sheets, Figure 2.9(a). However, vertically oriented sheets are easier to install and may reduce the total length of the wrap, as shown in Figure 2.9(b).

Bi-axial FRP reinforcement is achieved by placing two unidirectional FRP plies in mutually perpendicular directions as shown in Figure 2.10. The sheet in the primary direction acts to provide most of the reinforcement, while the sheet in the secondary direction limits shear crack openings and provides anchorage for the sheet in the primary direction.

Figure 2.10: Beams with bi-axial shear reinforcement: (a) vertical bi-axial sheet, (b) inclined bi-axial sheets
In addition, shear strengthening can also be achieved using near-surface mounted (NSM) FRP bars, see Figure 2.11. This method is relatively new with limited research and more studies are needed [Lorenzis and Nanni, 2001].

2.3 Concept of Shear Strengthening using FRP composites

The shear strengthening of reinforced concrete structures using FRP plates has now been used for over a decade. Most researchers have idealized the external bonding of the plates in a manner analogous to internal steel stirrups, assuming that the contribution of the plates to the shear capacity emanates from the capacity of the plates to carry tensile stresses at a constant strain. However, the effectiveness of the external shear strengthening plates and their contribution to the shear capacity of the reinforced concrete beam depends on the mode of failure, which may occur either:

- by peeling-off
- by tensile fracture.

The failure modes and the degree of strength enhancement are strongly dependent on the details of the bonding schemes and the anchorage methods. The problem of the anchorage arises from a practical difficulty. This difficulty is associated with the technique
of strengthening or repairing the existing structure and does not allow for bonding the plate in the form of a closed continuous loop around the whole cross section of the beam. Based on the available experimental data, side-bonded plates are very vulnerable to premature failure. Full wrapping is the most effective technique but it is not adopted in the field since most beams are cast monolithically with a slab, and U-jacket is lying in between. Therefore, full wrapping should be used whenever is possible, followed by U-jacketing.

Since shear strengthening often forms a key part of an effective strengthening strategy for reinforced concrete structures, numerous shear strengthening studies have been carried out since the 1990s (e.g. Uji [1992]; Al-Sulaimani et al. [1994]; Chajes et al. [1995]; Alexander and Cheng [1996]; Araki et al. [1997]; Arduini et al. [1997]; Sato et al. [1997b]; Hutchinson et al. [1997]; Triantafillou [1997]; Chaallal et al. [1998a,b]; Khalifa et al. [1998]; Malek and Saadatmanesh [1998a,b]; Fanning and Kelly [1999]; Deniaud and Cheng [2001a]; Kachlak and McCurry [2000]; Khalifa and Nanni [2000]; Triantafillou and Antonopoulos [2000]; Khalifa and Nanni [2002]; Pellegrino and Modena [2002]; Taljsten [2003]; Wong and Vecchio [2003]; Adhikary and Mutsuyoshi [2004]; Bousselham and Chaallal [2004a]; Teng et al. [2004]; Cao et al. [2005]; Carolin and Taljsten [2005]; Zhang and Hsu [2005]; Bousselham and Chaallal [2006b]; Pellegrino and Modena [2006]; Leung et al. [2007]; Monti and Liotta [2007]; Mosallam and Banerjee [2007]; Lee and Al-Mahaidi [2008]).

2.4 Shear Behaviour of Reinforced Concrete Beams

2.4.1 Shear Behaviour of RC Beams without FRP Strengthening

The shear behaviour of un-strengthened reinforced concrete beams was reviewed in ASCE [1998] and Kong and Evans [1990]. Shear is an important but controversial topic in structural concrete design. In design, it is generally desirable to ensure that the ultimate strengths are governed by flexure rather than by shear; except in seismic design. Small deflections and little warning characterize shear failures before the occurrence of failure. It is now known that the shear resistance mechanism of a simply supported reinforced concrete beam is not a function of a single variable. The mode of diagonal failure has been found by experiment to be strongly based upon the shear-span/effective depth ratio,
as illustrated in Figure 2.12. The shear behaviour of a beam is governed by some secondary parameters, such as:

- concrete strength
- tension steel
- aggregate type
- beam size.

The shear strength of a beam is substantially increased by the use of shear reinforcement. The shear reinforcement increases the ductility of the beam and considerably limits the crack width and reduces the likelihood of a sudden and catastrophic failure. The general principle of design of shear assumes that concrete provides the primary shear resistance of the beam and that the shear force in excess of the concrete shear resistance has to be resisted by the internal shear reinforcement. Before diagonal cracking, it is assumed the external applied force produces few stresses in the web reinforcement. When the diagonal crack forms, any web bar which intersects the diagonal crack will suddenly carry a portion of the shear force. Web bars not intersected with the diagonal crack remain slightly stressed. As the external applied force is increased, the most stressed stirrups start to yield. If the crack continues to widen, the neighbouring stirrups reach their yield
limit and start to deform until all the stirrups crossing the cracks start to yield. The shear strength carried by the web reinforcement remains stationary at the yield value and the subsequent increase of the external force will be carried by the concrete shear strength. When the diagonal crack widens, the aggregate interlock becomes less effective and the dowel action of the tension reinforcement increases rapidly. Failure of the beam finally occurs either by the dowel splitting of the concrete along the longitudinal reinforcement or by crushing of the concrete in the compression zone. Reinforced concrete beams without transverse steel fail with one principal diagonal crack whereas those with steel stirrups fail with a diagonal cracked area.

2.4.2 Shear Behaviour of FRP Shear-Strengthened RC Beams

The available evidence from experiments indicates a basic difference in the mode of failure of a reinforced concrete beam strengthened in shear with externally bonded FRP plates from that of a beam reinforced with internal steel stirrups [Swamy et al., 1999; Khalifa et al., 1998; Triantafillou and Antonopoulos, 2000; Teng et al., 2002]. In the case of beams with internal steel stirrups, the shape and position of the latter ensure sufficient anchorage, and failure is determined by the tensile strength of the links. By contrast, for beams reinforced with externally bonded FRP plates, the failure criterion is governed primarily by the anchorage efficiency, rather than by the tensile strength of the FRP plates.

2.4.2.1 Shear Failure Controlled by FRP Rupture

This type of failure occurs most often with a diagonal shear tension crack. Vertical flexural cracks originating from the tensile face occur first. A crack near the support may propagate towards the loading point and may become an inclined crack. In some cases, a diagonal crack may appear abruptly. The sudden formation of a diagonal crack causes an abrupt load transfer to a localized region of the FRP intersecting the diagonal crack. As the width of the diagonal crack increases, the maximum strain in the FRP strips eventually reaches its maximum strain, which often occurs at a strain lower than the ultimate strain of the FRP [Triantafillou and Antonopoulos, 2000; Teng et al., 2002].
The failure is initiated at the most highly stressed point in the FRP strip intersected by the shear crack. When the FRP strip reaches its ultimate tensile strength, then rupture of the FRP plates propagates along the diagonal shear crack in the concrete, leading to total failure of the beam in a brittle manner. After rupture of the first strip the stresses redistribute to the other fibres and so on until all strips are broken. Figure 2.13 shows the rupture failure of FRP plates. Partial debonding of the FRP sheets from the beam sides often occurs prior to rupture in most cases whilst only the FRP plates remain bonded to the top and bottom face of the beam. However, the eventual failure is due to the rupture of the FRP strips. The results show that the FRP strips do not close the cracks, but help to delay the occurrence and propagation of cracks. All of the beams with full wrapping of the FRP composites around the beam and some with U-jackets fail in this mode. The tensile strength of the FRP can always be fully utilized if there is a sufficient bond length, which is called the effective bond length. Beyond this length there is no further transfer of load to the FRP. The effective bond length is based on the concrete strength, modulus of elasticity, and thickness of the FRPs [Chajes et al., 1995; Triantafillou, 1998; Triantafillou and Antonopoulos, 2000; Deniaud and Cheng, 2000; Li et al., 2001; Taljsten and Carolin, 2001; Teng et al., 2002; Carolin and Taljsten, 2005].

2.4.2.2 Shear Failure Controlled by FRP Debonding

A shear-strengthened reinforced concrete beam may fail due to the debonding of the FRP plates from the sides of the beam. This kind of failure occurs when the concrete
2.4. SHEAR BEHAVIOUR OF REINFORCED CONCRETE BEAMS

Figure 2.14: Shear debonding failure of the FRP sheets in: (a) U-wrap [Khalifa and Nanni, 2000], (b) side-bonded [Pellegrino and Modena, 2002]

surface strength is too low, the epoxy used has low shear strength or the length of the FRP strips is not sufficient to transfer the shear forces between the reinforcement and the concrete. After the formation of the shear crack along the depth of the beam and with the increase of the external applied load, the debonding initiates at the FRP plates crossing the crack. The debonding is initiated above and below the shear cracks at the areas where the development length is not sufficient, with a concrete layer attached to them. The bonded area progressively decreases as debonding of FRPs proceeds outwards away from the shear crack location towards both ends of the strip. The studies reveal that strips intercepting the shear crack close to the end have small areas of debonding. Those intercepting the shear crack near mid-height show greater areas of debonding. This results in an instantaneous increase in the load carried by the vicinity, which leads to a rapid propagation of the debonding of the FRP sheets over the shear cracks, combined with beam failure. In some cases, the load is not able to increase beyond the first peak reached at the failure of the first strip. In other cases, the maximum load may occur after a few strips have already failed. Consequently, first strip failure does not correspond to the ultimate shear capacity. The deflection of beams failing in this mode is usually very limited. Figure 2.14 presents shear-debonding failure of FRP sheets. Available test results indicate that all beams strengthened with side-bonded FRPs, and many beams strengthened with U-jackets, fail in this mode [Khalifa and Nanni, 2000; Taljisten and Carolin, 2001; Teng et al., 2002]. A diagram of the shear behaviour of a reinforced concrete beam is shown in Figure 2.15.


2.5 Parameters Influencing Shear-Strengthened Beams

The success of FRPs in shear strengthening is a result of a considerable research effort carried out in recent years. Results of these studies led to interesting conclusions, particularly regarding the parameters influencing the behaviour of shear-strengthened beams, such as the FRP strengthening scheme. The effectiveness of the FRP strengthening depends on its failure mechanism. Whether debonding or fracture will occur first depends on many factors such as the bond conditions, the available bond length, type of attachment of the FRP plates to the concrete substrate, the thickness and stiffness of the laminates and other factors [Khalifa and Nanni, 2000; Bousselham and Chaallal, 2004a]. Additionally, an exhaustive review of research studies on shear-strengthening revealed that other parameters linked to the shear resistance mechanism of shear-strengthened beams have not been sufficiently documented. The shear-span-to-depth ratio as well as the transverse steel reinforcement are among these parameters. Research studies that have been carried out to investigate these parameters are somewhat contradictory and to a certain degree controversial [Triantafillou and Antonopoulos, 2000]. Many aspects are still not fully verified due to the relatively large scatter observed in the research studies. The pa-
2.5. PARAMETERS INFLUENCING SHEAR-STRENGTHENED BEAMS

Parameters influencing an externally shear-strengthened beam are presented at Figure 2.16. The parameters are grouped according to their types to:

- beam dimensions
- strengthening schemes
- FRP dimensions and characteristics.

as discussed below.

2.5.1 Beam Dimensions

- An increase of the steel shear reinforcement is reported to reduce the FRP shear contribution [Uji, 1992; Sato et al., 1996; Taerwe et al., 1997; Aedy et al., 1999; Swamy et al., 1999; Deniaud and Cheng, 2000; Li et al., 2001; Neto et al., 2001; Wong, 2001; Chaallal et al., 2002; Pellegrino and Modena, 2002; Deniaud and Cheng, 2003; Lee, 2003; Li et al., 2003; Bousselham and Chaallal, 2004a; Carolin and Taljsten, 2005; Bousselham and Chaallal, 2006b,a; Elyasian et al., 2006; Pellegrino and Modena, 2006; Grande et al., 2007; Leung et al., 2007]. Malek and Saadatmanesh [1998a] conducted a model study using the FEA package ABAQUS to simulate the behaviour of shear-strengthened beams. From this model, the authors reported that the FRP sheets shear contribution does not depend on the presence of steel stirrups.

- The influence of the concrete strength to the effective strain of the FRP plies was only studied by Deniaud and Cheng [2000]. They reported that the effective strain of the FRP is observed to increase proportionally with the increase of the concrete strength. Khalifa and Nanni [2000], Triantafillou and Antonopoulos [2000] and Elyasian et al. [2006] confirmed the same trend.

- Khalifa and Nanni [2002] observed from their experimental works that the shear span to effective depth ratio (a/d) influences the FRP shear contribution, which appears to decrease with the decrease of the ratio. This is opposed to what was reported by Cao et al. [2005], who stated that the shear capacity increases with the decrease of shear span/depth ratio. From the tests conducted by Mitsui et al. [1998], it is
CHAPTER 2. LITERATURE REVIEW

mentioned that the shear contribution of the FRP plies has no distinct relation with this ratio. A literature review of the parameters influencing the shear behaviour of strengthened beams was carried out by Bousselham and Chaallal [2004a]. In their study, three zones of a/d were identified, such as: (a) the zone corresponding to a/d less than 2.5, where failure is predominately by fracture; (b) the zone corresponding to (a/d) greater than 3.2, where failure is by debonding; and (c) a transition zone is ranged between the above values, where the failure in this case can be either by debonding or by fracture.

- The best corner radius is found to minimize the effect of stress concentration produced at the sharp corners is when the corner radius about 0.25 of the width of the section [Yang et al., 2001]. Radii of 35 mm, 13 mm and 15 mm minimum were proposed by the ISIS [2001], ACI [2002] and the BS [2000], respectively.

- The size effect of shear-strengthened beams attracted a few researchers for investigation, particularly for specimens that have an effective depth greater than approximately 300mm. The gain in shear strength tends to decrease with increasing the effective depth [Deniaud and Cheng, 2000; Qu et al., 2005; Leung et al., 2007]. With increase of depth size, the failure mode changes from fracture to debonding of the FRP sheets [Bousselham and Chaallal, 2004a]. By contrast, [Triantafillou, 1998] and [Hassan et al., 2007] linked the size effect in the members with the bonded surface area. The deeper the beam, the more the FRP contribution to the shear resistance.

- [Li et al., 2001] found that the FRP sheets shear contribution is greatly influenced by the area of the longitudinal steel. The effective strain of FRP sheets increases when the area of the longitudinal steel decreases. The opposite of this was reported by Bousselham and Chaallal [2004a, 2006b]; Hassan et al. [2007]. The effective strain of FRP sheets is not proportional to the longitudinal steel.

2.5.2 Strengthening Schemes

- When there is no access to full wrapping of the beams, all the researchers confirmed that the repair of beams with U-jacketing is the best solution.

- No difference was recorded in the shear capacity when strips or continuous sheets were used [Al-Sulaimani et al., 1994; Malek and Saadatmanesh, 1998a]. Khalifa and
Nanni [2000] noticed that sometimes FRP strips give higher shear capacity than continuous sheets. The use of continuous sheets increases the shear capacity of the strengthened beams more than using FRP strips [Taerwe et al., 1997; Khalifa and Nanni, 2002; Abdel-Jaber et al., 2003; Cao et al., 2005; Monti and Liotta, 2007]. This was attributed to the smaller bond area of FRP strips.

- Strengthened beams with inclined fibres were found to be more effective in resisting crack propagation than vertical strips [Uji, 1992; Chajes et al., 1995; Norris et al., 1997; Chaallal et al., 1998a; Deniaud, 2000; Deniaud and Cheng, 2001a; Neto et al., 2001; Diagana et al., 2003; Li et al., 2003; Zhang and Hsu, 2005; Monti and Liotta, 2007]. The tests conducted by Carolin and Taljsten [2005] showed no difference in shear capacity between vertical and inclined fibre directions. The vertical orientation of the FRPs provides higher strength gain and a more ductile failure than the inclined FRPs [Elyasian et al., 2006].

- Biaxial or triaxial FRP plates provide the beam with more ductile failure than unidirectional plates and no increase in load carrying capacity was observed [Norris et al., 1997; Deniaud and Cheng, 2003]. The biaxial FRP plates have no influence on the shear strength [Alexander and Cheng, 1996; Khalifa and Nanni, 2000]. Khalifa and Nanni [2002] and Chaallal et al. [2002] observed a considerable increase in the beams strengthened with biaxial sheets.

- Strengthening with double FRP plies increases the load carrying capacity but reduces the ductility of the section [Neto et al., 2001; Chaallal et al., 2002; Pellegrino and Modena, 2002; Carolin and Taljsten, 2005; Bousselham and Chaallal, 2006b; Pellegrino and Modena, 2006].

- Taerwe et al. [1997] tried to strengthen beams with U-jackets around the compression face of the beam instead of the tension face. He found that this method of strengthening led to debonding unlike the usual method, therefore reducing the FRP shear contribution. Most of the researchers who studied horizontally oriented fibres concluded the same result [Alexander and Cheng, 1996; Carolin and Taljsten, 2005; Zhang and Hsu, 2005]. Normally the strengthening scheme with horizontal fibres has no significant contribution to the shear carrying capacity. By contrast, Adhikary and Mutsuyoshi [2004] and Khalifa and Nanni [2002] concluded that beams with horizontally aligned bonded fibres showed enhanced shear strength.
• Taljsten and Elfgren [2000] examined different ways of attaching the FRP sheets to the concrete beams, which included hand lay-up, pre-preg with vacuum and heat and vacuum injection. The hand lay-up technique was found to be the optimum way of applying the FRP sheets in terms of applied load/deflection. There was no difference between the other ways of strengthening.

2.5.3 FRP Dimensions and Characteristics

• The contribution of the FRP plates to the shear capacity and the ductility of the beam increased almost linearly with the increase of the plate thickness [Malek and Saadatmanesh, 1998a; Li et al., 2001; Carolin and Taljsten, 2005]. The increase of the plate thickness was reported by others to reduce the shear contribution of the FRP plies [Triantafillou, 1997; Chaallal et al., 1998a; Triantafillou and Antonopoulos, 2000].

• The maximum shear contribution of the FRP composites was found when the plate stops at a point near the neutral axis of the strengthened beam [Malek and Saadatmanesh, 1998a]. Alexander and Cheng [1996] stated a minimum length of 75mm for the FRPs in order to develop their full capacity. Li et al. [2001] proposed that the FRP strip height should be equal to three fourths of the beam height to obtain the optimum contribution of the FRP plies. For the U-wrap scheme studies by Abdel-Jaber et al. [2003], Adhikary and Mutsuyoshi [2004] and Zhang and Hsu [2005], the best FRP depth is when the FRP sheet is placed up to the maximum possible section depth.

• There is a threshold for the shear contribution with respect to the axial rigidity of the FRP composites, which is defined by the area times the modulus of elasticity expressed by the product \( \rho_{frp}E_{frp} \). It was observed that the shear capacity increases linearly as the axial rigidity of the FRP increases until the value of axial rigidity reaches a 0.4 GPa, then tends to stabilize thereafter [Triantafillou, 1997]. Bousselham and Chaallal [2004a] used a new indication called the shear rigidity in their survey of the published experimental results and they stated the same conclusion at a shear rigidity equal to 0.05. Teng et al. [2002] reported that the effective bond length increases linearly with an increase of \( \sqrt{E_f t_f} \); where \( E_f \) and \( t_f \) are the modulus of elasticity and the thickness of the FRP plate, respectively.
2.6 ANCHORAGE OF THE FRP PLATES

- The width ratio between the bonded plate to the concrete member was shown to have a significant effect on the ultimate bond strength [Teng et al., 2002]. The study carried out by Fanning and Kelly [1999] on various widths of CFRP strips showed only a slight increase in the shear capacity of strengthened beams with increase of the width of the strip.

- The contribution of the FRP strips to the ultimate capacity of a beam increases with the reduction of the FRP strip spacing, while the effective strain of the FRP strips decreased with increasing the number of FRP plies [Wong, 2001; Diagana et al., 2003; Lee, 2003; Li et al., 2003]. Decreasing the spacing between the FRP composites may not result in a significant increase in the beam’s shear strength especially if debonding of the FRP plies controls the failure [Mecilli et al., 2002; Cao et al., 2005].

- Adhikary et al. [2004] examined CFRP and AFRP sheets to strengthen concrete beams. The CFRP sheets showed a higher increase in shear capacity. Deniaud and Cheng [2000] stated that the woven fabric glass materials performed better than the unidirectional carbon FRP sheets.

- Mosallam and Banerjee [2007] evaluated three FRP systems to strengthen beams in shear, including the carbon/epoxy wet lay-up, E-glass wet lay-up and carbon epoxy procured strips. The study showed that the carbon/epoxy wet lay-up provided an appreciable increase in the FRP contribution compared to the other techniques.

2.6 Anchorage of the FRP plates

The effectiveness of FRP end anchorages depends on the strengthening schemes employed and geometry of the beam. The purpose of wrapping with FRP sheets is to provide a confining effect. When full wrapping is not possible, beams strengthened with side-bonded sheets only are expected to debond. The latter strengthening configuration is not usually used. For this reason, many beams are strengthened with FRP U-jacket. There exists a critical length allowing the development of sufficient bond strength to either resist or delay debonding of FRPs. Once the FRP sheets start to peel off, the beam will fail rapidly
Figure 2.16: Diagram of the parameters influencing shear strengthened beams
and the deflection of the failed beams is limited. Hence, FRP shear-strengthened beams need to be anchored. There are several options available to improve the anchorage for the various strengthening schemes. Some solutions have been presented by researchers to delay the debonding and to increase the effective strain of the FRPs. The following describes the different types used by various investigators in attempts to delay the premature peeling failure of FRPs.

Steel plates or bolts are typically used as a mechanical anchorage to delay the FRP debonding. The steel plates are placed at the FRP end, and held in place by bolting the plate into the concrete. However, corrosion of the steel may be of concern. Caution should be exercised when using mechanical anchors made of steel. Stress concentrations exist at locations where bolts are used. The performance of the anchorage remains questionable in the long term. Sato et al. [1997a] applied a mechanical anchorage to the U-jacket strengthening of the FRP at the corner of the web-flange with a steel plate width of 50 mm and 10 mm thickness. The bolts used were 10 mm diameter and 164 MPa shear strength distributed every 100 mm along the steel plate and the neutral axis was found to lie within the flange. The mechanically anchored shear-strengthened beams were found to increase the load carrying capacity higher than the unanchored beam. The mechanical anchorage was observed to delay the peeling-off of the FRP sheets and gave an increase of 15% over that of the unanchored beam.

A more in-depth look at the mechanical anchoring systems was conducted by Sato et al. [1997b], in which four different mechanical anchorages were tested. The different anchorage types are shown in Figure 2.17. Although the Type 1 anchorage was shown to change the failure mode from debonding to fracture of the FRP plies, this way of anchoring is not preferable because it requires simultaneous work at the top and the bottom of the floor. Type 3 is the most effective solution of mechanical anchoring since it seems to be more practical in repair work by changing the failure mode, most of the times, from peeling-off to fracture of the FRP plies and gave an increase to about 100% over that of the unanchored beam. The other two methods were found, compared to types 3 and 4, to be less effective in reducing the debonding mode of failure.

Another kind of anchorage of the FRP sheets was proposed by Khalifa and Nanni [2000]. The anchorage was attained by grooving the concrete at the web-flange corner along the entire length of the strengthened portion. The groove dimensions were about 15
Figure 2.17: Mechanical anchorages types of the FRP sheets by Sato et al. [1997b]

Figure 2.18: Details of the U-anchor by GFRP rod: (a) groove into the flange, (b) groove into the web [Khalifa and Nanni, 2000]

x 15mm for the depth and the width, respectively, and extended throughout the strengthened part. The FRP sheets were bonded to the concrete surface and to the walls of the groove. A 10 mm GFRP rod was fixed inside the groove by means of a high viscosity epoxy paste. The anchorage system used FRP rods to eliminate issues with corrosion. A cross section showing the details of the U-anchor system is shown in Figure 2.18. As a result of the use of a rod anchorage, a significant increase in the shear capacity was achieved. The failure mode of the anchored beam changed from debonding of the FRP plates to a flexural failure mode. The cutting of the groove will require specialised machinery that can potentially increase installation cost.

Melo et al. [2003] have studied the anchoring of U-wrapped FRP sheets with horizontal FRP plates. The horizontal FRP plates were applied beneath the flange on the top of the FRP plies. The anchoring is divided into three groups, 50 mm, 100 mm and 100 mm double plate widths of FRP plates were fixed while the thickness of the plates was kept constant. The compression area was found to cover the entire flange. The application of the horizontal plate below the flange was shown to delay the debonding. A 14%, 19% and
2.6. ANCHORAGE OF THE FRP PLATES

A more simple and cost-effective way of anchoring the FRP at the web-flange junction is to bond the FRP to the underside of the flange as illustrated in Figure 2.19. This approach is less effective compared to the other anchorage type reported earlier. Deniaud and Cheng [2001a] investigated the shear behaviour of full-scale T-beams with U-strips and jacket anchored in this manner. All the beams failed in shear with the failure mode characterized by the debonding of FRPs.

Lee [2003] investigated the shear strengthening of a full-scale system using a novel anchoring system. The system involved embedding an L-shaped FRP plates into grooves at the flanges. The CFRP strips were bonded to the webs of the beams and extended until the top flange edge. This is accomplished through rectangular holes were drilled at the flange only at the locations of the strips. The holes that were filled with epoxy anchoring the FRP plates in place. Details of this anchorage scheme are illustrated in Figure 2.20. The L-shaped FRP strips separated at the overlap in the soffit of the beam. This method is difficult to use in the existing structures; drilling the slab requires knowledge of the position of slab reinforcement, which must not be damaged during the operation.

Figure 2.19: FRP bonded to the underside of the flange [Deniaud and Cheng, 2001a]

Figure 2.20: Details of the L-shaped anchorage [Lee, 2003]
2.7 FRP Shear-Strengthened Design Models

Design approaches proposed for regulatory standards should be safe and correct in concept, and simple to understand and apply. These design procedures are most effective if they are based on relatively simple conceptual models rather than complex empirical equations. Reinforced concrete beams strengthened in shear by means of externally bonded FRPs show complex behaviour. The complexity of the problem, combined with the limited experimental and numerical research available, makes it difficult to develop a robust predictive model suitable for practical design. Nevertheless, several attempts have been carried out to produce such models. Some of the models presented here have reached a sufficiently mature state to be implemented in code of practice. Some of them are relatively new and need further development.

In the case of beams strengthened with externally bonded FRP sheets, all of the existing models use a third term to account for the contribution of the FRP sheets to the shear strength. This expression is presented by:

\[ V_n = V_c + V_s + V_f \]  

(2.1)

In this equation the shear strength of the section, \( V_n \), is the sum of the shear strength of the concrete \( V_c \) (which is attributed to aggregate interlock, dowel action of the longitudinal steel, and diagonal tensile strength of concrete), of the strength contribution of the steel stirrups \( V_s \), and of \( V_f \), the shear strength contribution of FRP composites.

In order to determine the FRP shear contribution, the FRP effective strain is the most important and most effective term because it is the key factor expressing the FRP shear contribution. Some researchers considered the FRP effective strain to be the maximum strain of the FRP sheets assuming that there is a perfect bond between the fabric and concrete prior to failure [Chaallal et al., 1998a; Malek and Saadatmanesh, 1998a]. Others have considered it as a function of the bond between the stress between the concrete and the FRP sheets [Uji, 1992; Chajes et al., 1995], and many researchers have related it to the axial rigidity of the FRP sheets [Khalifa et al., 1998; Triantafillou, 1998]. Furthermore, for determining the effective strain, many equations have been proposed based on extended experimental results. The following section introduces the design of shear-strengthened reinforced concrete beams.
2.7. FRP SHEAR-STRENGTHENED DESIGN MODELS

2.7.1 Truss Design Model

The shear strength of a beam is traditionally evaluated using the simple truss analogy initially developed one century ago by Ritter and later by Morsch [ASCE, 1998]. This approach is still the basis of several current design codes. For conventional beams, this model assumes that the beam after cracking is replaced by a pin-connected, statically determinate truss, in which the concrete in compression zone is represented by the compression chord, the tensile steel reinforcement is represented by the tension chord, the steel stirrups corresponds to the tension web members, and the diagonal concrete between inclined cracks corresponds to the compression members. This approach assumes that the concrete cannot resist tension. Therefore, no diagonal tension members perpendicular to the concrete struts are considered. The predicted failure load is obtained when the vertical stirrups yield or when the applied stresses exceed the concrete compressive strength.

The truss model is usually used to estimate the shear contribution of FRP sheets. It is the most utilized model in shear strengthening for its ease to use and wide acceptance to researchers [Chaallal et al., 1998a; Khalifa et al., 1998; Triantafillou, 1998; Triantafillou and Antonopoulos, 2000; Chaallal et al., 2002; Chen and Teng, 2003a, b; Adhikary et al., 2004; Carolin and Taljsten, 2005; Zhang and Hsu, 2005; Pellegrino and Modena, 2006; Monti and Liotta, 2007]. The basic assumption of this model for shear-strengthened beams is that the externally bonded FRP composites contribute to the shear capacity in the same way as the internal steel stirrups. The shear design review here includes the design procedures found in [ACI, 2002; ISIS, 2001; BS, 2000; FIB, 2001; Taljsten, 2002] to predict the contribution of FRP sheets. The shear design model is used in all of the above mentioned references is the truss model.

2.7.1.1 ACI Model

The FRP shear strength after taking into account the crack angle equal to 45° degree can be written as:

$$V_f = A_f f_{te} (\sin \alpha + \cos \alpha) d_f / s_f$$  \hspace{1cm} (2.2)

$$f_{te} = \varepsilon_{te} E_f$$  \hspace{1cm} (2.3)
CHAPTER 2. LITERATURE REVIEW

where $\alpha$ is the stirrups inclination angle measured from the horizontal axis of the beam; $A_f$ is the area of the FRP sheets, $A_f = 2nt_w W_f$ (mm$^2$); $n$, $w_f$ and $t_f$ are the number of layers, width and the thickness of the FRP sheets, respectively; $f_{fe}$ is the effective stress of the FRP sheets; $E_f$ is the tensile modulus of elasticity of the FRP sheets (N/mm$^2$); $\varepsilon_{fe}$ is the effective strain level of the FRP sheets attained at section failure. ACI adopts equations based on a bond mechanism to obtain the effective FRP strain $\varepsilon_{fe}$ for side-bonded and U-wrap configurations. The bond capacity basically depends on the effective bond length $L_e$ (mm) and the concrete compressive strength $f_c$ (N/mm$^2$). The effective bond length is related to the stiffness of the FRP sheet by:

$$L_e = \frac{23300}{(nt_f E_f)^{0.58}} \quad (2.4)$$

Note that as the stiffness of the FRP increases the effective bond length decreases. Two modification factors, $k_1$ and $k_2$, which account for the concrete strength and the type of wrapping scheme used are expressed as follows:

$$k_1 = \left(\frac{f_c}{27}\right)^{2/3} \quad (2.5)$$

$$k_2 = \begin{cases} \frac{d_f - L_e}{d_f} & \text{for U-wraps} \\ \frac{d_f - 2L_e}{d_f} & \text{for side-bonded} \end{cases} \quad (2.6)$$

where $d_f$ is the depth of FRP sheets measured at the level of longitudinal steel reinforcement (mm). The bond-reduction coefficient $k_v$ and effective FRP strain $\varepsilon_{fe}$ are computed as:

$$k_v = \frac{k_1 k_2 L_e}{11900 \varepsilon_{fu}} \quad (2.7)$$

$$\varepsilon_{fe} = \min(k_v \varepsilon_{fu}, 0.75 \varepsilon_{fu}, 0.004) \quad (2.8)$$

$\varepsilon_{fu}$ is the ultimate tensile strength of FRP. For the fully wrapped configuration, the effective FRP strain is simply limited by the following expression:

$$\varepsilon_{fe} = \min(0.75 \varepsilon_{fu}, 0.004) \quad (2.9)$$
Finally, the maximum shear capacity of the section \((V_d)\) should be checked with the required shear capacity \((V_r)\) as:

\[
V_n = V_r
\]

(2.10)

if the shear capacity of the section does not satisfy this provision, the calculation has to start from the beginning with a new suggestion of the FRP dimensions.

### 2.7.1.2 ISIS Model

There is no major difference between the ACI and ISIS design methods. The formulae used to define the FRP shear strength is the same as in the ACI manual. Both methods limit the effective strain to 0.004.

### 2.7.1.3 FIB Model

FIB adopts the findings of Triantafillou and Antonopoulos [2000], which express the effective FRP strain in terms of the concrete compressive strength \(f_c\) and FRP axial rigidity \((E_f \rho_f)\), where \(\rho_f\) is the FRP reinforcement ratio defined as \((2t_f/b_w)\) for continuous FRP sheets, or \(((2t_f/b_w)/(b_f/s_f))\) for FRP strips. \(b_w, E_f\) and \(t_f\) are the the minimum beam's width cross section, elastic modulus of FRP sheets \((kN/mm^2)\), and thickness of the FRP composites. Two best-fit power-law expressions are derived from two separate sets of experimental data for debonding failure and fibre rupture. For the side-bonded and U-wrap configurations, both debonding failure and fibre rupture are considered as possible failure modes and hence the lower value obtained from the expressions is adopted, and it is given by:

\[
\varepsilon_{fe} = \min(0.65(\frac{f_c^{2/3}}{E_f \rho_f})^{0.56} \times 10^{-3}, 0.17(\frac{f_c^{2/3}}{E_f \rho_f})^{0.30} \varepsilon_{fu})
\]

(2.11)

For the fully wrapped ones, fibre rupture is taken to control the final failure and the expression corresponding to this failure mode applies. The effective FRP strain is determined as follows:

\[
\varepsilon_{fe} = 0.17(\frac{f_c^{2/3}}{E_f \rho_f})^{0.3} \varepsilon_{fu}
\]

(2.12)
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The FRP contribution to the shear capacity is obtained by the following equation after the effective strain is computed:

\[ V_f = 0.9 \frac{k \varepsilon_{fe}}{\gamma_f} E_f \rho_f b_w d (\cot \theta + \cot \alpha) \sin \alpha \]  

(2.13)

where \( d \), \( \theta \), \( \alpha \) are the effective depth, inclination of diagonal crack, and principal fibre orientation, respectively. \( k \) is the reduction factor to obtain the characteristic FRP strain from the mean value, \( k = 0.8 \). For fibre rupture, the partial safety factor is \( \gamma_f = 1.2 \). For the debonding failure mode, \( \gamma_f = 1.3 \). The threshold of the effective strain is increased, compared to the other codes, to a maximum value of 0.006.

2.7.1.4 BS Model

The ultimate strain of the FRP plies is limited to 0.004. The FRP effective strain is dependent on the failure mode. When the failure mode is rupture of the FRP sheets, the effective strain is expressed by:

\[ \varepsilon_{fe} = \varepsilon_{fu} (0.5622 (\rho_f E_f)^2 - 1.2188 (\rho_f E_f) + 0.778) \]  

(2.14)

and when the failure mode is debonding of the FRP sheets, the effective strain is computed from:

\[ \varepsilon_{fe} = 0.0042 ((0.835 f_c^{3/3} w_{fe}) / ((E_f t_f) 0.58 d_f)) \]  

(2.15)

where \( \varepsilon_{fu} \) is the design ultimate failure strain in the FRP, \( \rho_f \) is the FRP shear ratio, it is given by \( (2t_f/b_w)(w_{fe}/s_f) \) for beams strengthened with FRP strips and \( 2t_f/b_w \) for beams strengthened with FRP continuous sheets; \( E_f \) is the elastic modulus of FRP sheets \( (kN/mm^2) \); \( f_c \) is the concrete compressive strength \( (N/mm^2) \); \( w_{fe} \) is the effective width of FRP strips \( (mm) \) considered to be a function of the of the shear crack angle and FRP strengthening configuration, equal to \( (d_f - L_e) \) when the FRP sheets in the form of U-jacket and to \( (d_f - 2L_e) \) when the FRP in the form of side-bonded; \( L_e \) is the effective bond length equal to \( (461.3 / (t_f E_f)^{0.58}) \).

The FRP shear strength contribution can be calculated by using the same analogy as web steel reinforcement. Shear resistance of the FRP sheets is given by:

\[ V_f = \frac{1}{\gamma_{rmf}} A_f E_f \varepsilon_{fe} \sin \alpha (1 + \cot \alpha) (d_f/s_f) \]  

(2.16)
where $A_f$ is the area of the FRP sheets and it is equal to $2t_fw_f$ ($mm^2$); $\varepsilon_{fe}$ and $\alpha$ are the effective strain and the angle of FRP sheets with the horizontal axis of the beam, respectively; $d_f$ is the effective depth of FRP shear reinforcement (mm), usually equal to $d$ for the rectangular sections and $(d - \text{slab thickness})$ for T-sections; $s_f$ is the spacing between the centres of the FRP sheets (mm), for a continuous sheets $s_f = w_f$; $\gamma_mF$ is a partial safety factor for the FRP sheets and it is equal to 1.4 for CFRP, 1.5 for AFRP and 3.5 for GFRP.

2.7.1.5 Taljsten Model

In order to calculate the shear contribution of FRP sheets, Taljsten [2002] relates the effective stress of the FRP composites to its maximum strength. Taljsten also produces a new concept for the shear design by considering the angle of the FRPs' principal tensile strain instead of considering the angle of tensile strain coinciding with fibres' orientation angle.

By assuming the crack has occurred at an angle of 45°, the FRP shear contribution $V_f$ can be calculated from the following equation:

$$V_f = 1.2t_f b_f \varepsilon_{fu} E_f 0.9d \left(1 + \cot \alpha \frac{\sin \alpha \cos^2 \beta}{s_f}ight)$$  \hspace{1cm} (2.17)

where $b_f$ and $t_f$ are the width and the thickness of the FRP composites, respectively; $E_f$ is the tensile elastic modulus of the FRP composites; $s_f$ is the distance between the FRP composites parallel to the longitudinal direction of the member; $\alpha$ is the FRP composites inclination angle to the horizontal axis of the beam; $\beta$ is the angle between the principal tensile stress $(\theta)$ and the fibre direction $(\beta = \alpha + \theta - 90)$. Carolin [2003] showed that a reduction factor of the ultimate strain equal to 0.6 gives a good estimate for the effective strain of the FRP composites. Then the effective FRP strain is given by:

$$\varepsilon_{fe} = 0.6\varepsilon_{fu}$$  \hspace{1cm} (2.18)

where $\varepsilon_{fu}$ is the ultimate strain of the FRP composites. Taljsten's design approach is applicable only for those cases where failure occurs by rupture of FRP sheets.

There are similarities and differences among the various guidelines. All design guidelines have provisions to distinguish between FRP debonding and rupture. FIB and BS
employ the same equation for side-bonded and U-wrap strengthening schemes, while ACI and ISIS account for the difference in these two cases through the use of different factors. In the design of shear-strengthened beams, the author believe that it is more rational to use different equations for the different repair configurations.

It should be noted that the truss analogy is not more than a design tool conceptually convenient but practically it represents a conservative model that ignores the tensile strength of the concrete. Table 2.1 summarizes the equations used by various codes.

<table>
<thead>
<tr>
<th>Code</th>
<th>Equation</th>
<th>$\epsilon_p$</th>
<th>$L_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI</td>
<td>$V_f = A_f f_c (\sin \alpha + \cos \alpha) d_f / s_f$</td>
<td>$\epsilon_p = \min( k, \epsilon_{p,0.75}, 0.004)$</td>
<td>$L_s = \frac{23300}{(t, E_s)^{0.32}}$</td>
</tr>
<tr>
<td>ISIS</td>
<td>$V_f = A_f f_c (\sin \alpha + \cos \alpha) d_f / s_f$</td>
<td>$\epsilon_p = \min( k, \epsilon_{p,0.75}, 0.004)$</td>
<td>$L_s = \frac{23300}{(t, E_s)^{0.32}}$</td>
</tr>
<tr>
<td>FIB</td>
<td>$V_f = 0.9 \frac{k E_p}{(1 + A) E_s f_c} \sin \beta (\cot \theta + \cot \alpha) \sin \alpha$</td>
<td>$\epsilon_p = \min(0.65\left(\frac{E_f}{E_s}\right)^{0.17}, 0.17\left(\frac{E_f}{E_s}\right)^{0.17}, 0.006)$</td>
<td>-</td>
</tr>
<tr>
<td>BS</td>
<td>$V_f = (1 + \delta \sigma) A_f E_s f_c \sin \beta (1 + \cot \beta) (d_f / s_f)$</td>
<td>$\epsilon_p = \epsilon_{p,0.5622}(\rho, E_s)^2 - 1.2188(\rho, E_s) + 0.778$</td>
<td>$L_s = \frac{461.3}{(t, E_s)^{0.32}}$</td>
</tr>
<tr>
<td>Taljsten</td>
<td>$V_f = 1.2 t_i b_i f_c E_s 0.9 d (1 + \cot \beta) \sin \beta \cos^2 \theta$</td>
<td>$\epsilon_p = 0.6 \epsilon_{p,0.5622}$</td>
<td>-</td>
</tr>
</tbody>
</table>

2.7.2 Modified Compression Field Theory (MCFT)

Collins and Mitchell [1980] introduced a new approach called the compression field theory, which is similar to the tension field theory used for the design of thin webs steel plate girders. The compression field theory assumes that after cracking the concrete will not carry any tensile stresses and it acts as a series of struts. Furthermore, the inclination of the concrete struts initially coincides with the principal compressive strain. This theory uses the full strain compatibility between the concrete and steel stirrups.

The compression field theory has been modified by Vecchio and Collins [1986] to the modified compression field theory (MCFT). The modified compression field theory was proposed to account for the beneficial effects of small tensile stresses that still remain in diagonally cracked reinforced concrete members. This variable angle truss method requires iterations to converge to the appropriate solution.
2.7. FRP SHEAR-STRENGTHENED DESIGN MODELS

The compression field theory was used by Malek and Saadatmanesh [1998a,b] to estimate the shear contribution of the FRP sheets. This analytical model was based on a variable concrete crack angle ($\alpha$) and assumed a perfect bond between the concrete and FRP sheets. Classical laminate theory of composite materials was used to transform the FRP sheet stiffness from the local axis of the FRP to the principal axis. Prior to failure, it was assumed that the FRP sheets reach their full tensile capacity. Once the angle of the shear crack ($\theta$) is determined, the contribution of the FRP sheets to the shear capacity of the beam ($V_f$) can be written as:

$$V_f = \frac{f_{fu} t_f h}{\tan \theta}$$

where $t_f$ and $h$ are the thickness and the height of the FRP sheets, respectively; $f_{fu}$ is the ultimate resistance of the FRP sheets.

Gendron et al. [1999] presented a theoretical model to estimate the shear capacity of reinforced concrete beams bonded with FRP composites. The assumptions were the same as used in Malek and Saadatmanesh [1998a,b] and the analytical model consisted of two phases:

- Estimation of the shear crack angle ($\theta$) by the compression field theory
- Using the truss analogy to design the shear contribution of the FRP sheets.

Both models produced by Malek and Saadatmanesh [1998a,b], and Gendron et al. [1999] are limited to the CFRP sheets and fracture failure mechanism.

2.7.3 Shear Friction and Strip Model

The basic approach is to assume that the concrete may crack in an unfavourable manner, or the slip may occur along a predetermined plane of weakness. Reinforcement must be provided crossing the potential or actual crack to prevent direct shear failure. Recently, Loov [1998] applied the concept of shear friction to evaluate the shear strength of reinforced concrete beams.

Alexander and Cheng [1996] developed the strip method to evaluate the shear contribution of the FRP sheets. FRP sheets crossing the concrete web crack are described as
a series of strips and each strip is evaluated individually to find its maximum allowable strain from the geometry of the FRP sheets. The method assumes that the load is linearly distributed among the fibres. The load increases from zero at the lower edge of the crack to the maximum at the top edge of the crack. The stress distribution depends on the position of the strip. The strips at the top of the crack are subjected to high stresses and are proposed to resist high shear forces. After the rupture of the plate at the top end of the beam, the stresses are distributed to the remaining strips and the mechanism is continued until the complete failure of the beam.

The shear friction approach that was developed by Loov [1998] was used by Deniaud [2000] to express the shear contribution of the concrete and steel stirrups. The shear contribution of FRP plates ($V_f$) is calculated according to the strip method and given by:

$$V_f = \frac{d_f t_f E_f \varepsilon_{fu} R_L}{\tan \alpha}$$

where $d_f$ and $t_f$ are the depth and the thickness of the FRP strips, respectively; $E_f$ and $\varepsilon_{fu}$ are the modulus of elasticity and the maximum strain of the FRP sheets, respectively; $\alpha$ is the crack angle taken to be 26° from their experimental test results; $R_L$ is the ratio between the remaining bonded length and the initial bonded length.

The design approach of Deniaud [2000] is not sufficient to explain the rupture failure mechanism. In addition, in Deniaud's model the crack angle was taken to be 26° in order to match the experimental results.

2.8 Axial Strain Profile along the FRP Composites

Most of the conducted studies, experimental or numerical, have been mainly directed towards the predictions of global load-deflection relations. Less attention has been paid to investigate the strain profiles along the FRP sheets depth; most of the studies compared the applied load versus strain of FRP strip at the mid-depth of the beam for the section at the middle of the shear span. The proposed equations to estimate the effective FRP axial strain have also neglected this profile.

It was suggested by some researchers that the strain distribution in the FRP along the shear crack is not uniform [Teng et al., 2002]. This phenomenon is attributed to the fact
that the shear crack width varies along its length. As the strain in the FRP intersected by the crack is closely related to the width of the crack due to the linear elastic behaviour of the FRP, the strain in the FRP also varies along the shear crack. Different strain distributions were suggested according to experimental or theoretical research [Aedy et al., 1999; Chaallal et al., 1998a; Deniaud, 2000; Teng et al., 2002; Carolin and Taljsten, 2005].

The fibres crossing a concrete crack experienced the same level of strain along the path of a crack [Deniaud, 2000]. In other words, the load carried by the FRP sheet crossing the crack is uniformly distributed among the FRP strips. This observation differs from that stated by the others; the shear crack is not uniform. Chaallal et al. [1998a] had the same trend in his design proposal. Their basic assumption is that the bonded FRPs contribute to the shear resistance in the same way as that of internal steel stirrups. Clearly, all the FRPs intersected by the shear crack reaches its full tensile strength at beam failure.

Based on the observations measured of tested specimens carried out by Aedy et al. [1999], a strain distribution along a shear crack was suggested. The strain distribution showed that the maximum strain concentrates at the mid-way of a shear crack. The top of the bonded FRP sheets crossing a shear crack carry half of that observed on the middle, while those fibre on the tip experienced no strains. Teng et al. [2002] proposed a linear stress distribution along a shear crack. The stress in the FRPs increases linearly from zero at the crack tip to the ultimate tensile strength at the lower end of the shear crack. Therefore, in their design equation the effective strain is half of the ultimate strain in the FRPs.

The non-uniform strain field causes a non-uniform stress field in the composite over the cross section of the beam [Carolin and Taljsten, 2005]. The strain distribution implies that the most stressed fibre lies in the centre of the beam. For a normalized beam height, the axial strain profile for a shear-strengthened beam with inclined or vertical fibre alignment is described in Figure 2.21. Additionally, fibres experienced compression axial strains in the case of inclined sheets.

Monti and Liotta [2007] proposed three axial strain profiles depending on the type of FRP sheets' scheme (side-bonded, U-wrap and completely wrapped) for vertical and inclined FRP sheets. Figure 2.22 depicts the various axial strain profiles along the crack for the three different strengthening schemes. In the side-bonded configuration, the stress


Figure 2.21: Axial strain profile along the beam height [Carolin and Taljsten, 2005]

Figure 2.22: Axial strain profile the shear crack for: (a) side-bonded beams; (b) U-wrap beams; (c) completely wrapped [Monti and Liotta, 2007]

profile is truncated towards the top end of the crack, where the axial strain tends to zero (Figure 2.22a). In the U-wrap configuration, the stress profile remains constant where the available length allows the full debonding strength to be developed throughout the crack length (Figure 2.22b). As shown in Figure 2.22(c), the axial strain profile for the completely wrapped configuration increases towards the bottom end of the crack.

2.9 Numerical Modelling

2.9.1 Introduction

In comparison to analyses on FRP flexural strengthening, theoretical investigations concerning the behaviour of reinforced concrete beams strengthened in shear with FRP composites are rather limited. As laboratory tests are expensive and time-consuming, there is
2.9. NUMERICAL MODELLING

an obvious advantage to having accurate numerical procedures that are capable of accurately simulating the complex behaviour of such structures. Good numerical models have the added value that they can further be exploited to lead to improved understanding of the various failure mechanisms and the influence of the important governing parameters. Realistic and reliable models must be able to account for the numerous complexities of concrete behaviour such as the nonlinear response in compression, post-cracking behaviour in tension, and the bond-slip relationships associated with both the steel/concrete and FRP/concrete interfaces.

In early work, linear elastic analyses were implemented, while the recent trend has been towards the use of nonlinear finite element models. A review of past studies on the application of finite element analysis for the modelling of FRP shear-strengthened reinforced concrete beams is presented in this chapter. A description of concrete constitutive laws, concrete cracking behaviour, and various techniques for modelling FRP strengthening materials is given. A special emphasis is placed on the different bond-slip models that have been incorporated in finite element analyses to characterize the FRP/concrete interfacial behaviour and to simulate the various debonding phenomena. A summary of the numerical modelling of shear-strengthened beams in terms of structural modelling and material modelling are presented in Table 2.2 and Table 2.3, respectively.

2.9.2 Finite Element Packages

To date, various commercial finite element packages have been used for the analysis of FRP shear-strengthened beams. These include ABAQUS [Kaliakin et al., 1996; Malek and Saadatmanesh, 1998a,b], ANSYS [Kachlakev and McCurry, 2000; Santhakumar et al., 2004; Elyasian et al., 2006] and DIANA [Al-Mahaidi et al., 2001; Lee et al., 2001; Lee, 2003]. In other studies, in-house numerical codes have been developed based on various approaches [Arduini et al., 1997; Wong and Vecchio, 2003]. An obvious advantage of commercial finite element packages is their versatility; they generally offer a wide range of element types, and they are capable of dealing with a large variety of complex structural problems.

Numerical models simulating the behaviour of FRP shear-strengthened beams have been developed using two-dimensional models [Malek and Saadatmanesh, 1998a,b; Al-
CHAPTER 2. LITERATURE REVIEW

Mahaidi et al., 2001; Lee et al., 2001; Wong and Vecchio, 2003] and three-dimensional models [Kachlakev and McCurry, 2000; Lee, 2003; Santhakumar et al., 2004; Elyasian et al., 2006]. In the two-dimensional models, the out-of-plane stresses are neglected as plane stress elements are employed to simulate the concrete; brick elements are used in the three-dimensional analyses. All the numerical models, since no debonding had been reported between the concrete and steel (flexure and shear) in the experimental phase [Kaliakin et al., 1996], assumed perfect bonding between the concrete and steel. Additionally, the steel reinforcement elements embedded in the concrete were represented using truss elements.

2.9.3 Modelling of Concrete

2.9.3.1 Concrete in Compression

Various constitutive models have been adopted in simulations of FRP shear-strengthened beams to describe the behaviour of concrete under a wide range of complex stress and strain histories. These models included nonlinear elastic models [Arduini et al., 1997; Al-Mahaidi et al., 2001; Lee et al., 2001; Lee, 2003], plasticity-based models whether perfect plasticity models [Kachlakev and McCurry, 2000; Santhakumar et al., 2004; Elyasian et al., 2006], or elastic-plastic models [Kaliakin et al., 1996; Malek and Saadatmanesh, 1998a,b]. Failure of concrete under general stress states has been represented using theories such as the Drucker-Prager [Malek and Saadatmanesh, 1998a,b] and Willam-Warnke criteria [Kachlakev and McCurry, 2000]. Figure 2.23 shows the typical uniaxial stress-strain relationship for concrete.

In general, the existing non-linear elasticity or plasticity-based concrete models have been relatively successful in predicting the load-deflection behaviour of shear-strengthened beams with FRP. This is because of the fact that the behaviour depends mainly on the tensile and cracking behaviour of the concrete while the compressive behaviour plays a secondary role.
2.9. NUMERICAL MODELLING

2.9.3.2 Crack Modelling

A primary factor governing the nonlinear response of FRP shear-strengthened beams is progressive cracking and the influence of these cracks on the bond mechanisms between the FRP and concrete. Two approaches have been proposed to simulate crack initiation and propagation in FRP-strengthened elements; namely, the discrete crack approach [Lee et al., 2001], and the smeared crack approach [Kabiakin et al., 1996; Arduini et al., 1997; Malek and Saadatmanesh, 1998a,a; Al-Mahaidi et al., 2001; Kachlavev and McCurry, 2000; Lee et al., 2001; Lee, 2003; Santhakumar et al., 2004; Elyasian et al., 2006]. The discrete and smeared crack models are depicted in Figure 2.24.

With a discrete crack model, crack discontinuities are simulated using separated points...
in a finite element mesh, as shown in Figure 2.24. This approach requires continuous re-meshing of the elements in the vicinities of the cracks as they propagate. Consequently, this approach requires a relatively high computational capacity, along with care to avoid numerical errors. To overcome inherent difficulties associated with this approach, the technique of implementing interface elements at predefined locations and paths of crack growth has sometimes been incorporated in the model. An example of this is shown in Figure 2.25. Some studies have considered only a critical shear crack [Lee et al., 2001]. Using a predefined discrete crack model has generally been incorporated with a smeared crack model to describe the tensile behaviour between cracks. Clearly, the accuracy of this approach is linked to properly defining the locations for crack initiation identifying the paths for crack propagation.

The material response in a smeared crack model is described by continuum constitutive relations, in terms of stress and strain, for the material including cracks. At crack initiation, the initially isotropic stress-strain relation is replaced by an orthotropic stress-strain relation. This means that the individual cracks generally cannot be traced. Instead, regions with distributed cracking are detected and the crack orientations are obtained as a part of the solution. Compared with the discrete crack approach, the smeared crack approach is computationally more convenient, since no special elements are needed to model the cracks. This also means that the locations of the cracks need not to be known and specified by the analytical beforehand.

In the smeared crack approach, two procedures have been reported in the literature for FRP shear-strengthened beams: (i) the fixed crack model [Kaliakin et al., 1996; Kach-
2.9. NUMERICAL MODELLING

(2.9) (2.26) Typical tension stiffening model for concrete

Figure 2.26: Typical tension stiffening model for concrete

Lakev and McCurry, 2000], and (ii) the rotating crack model [Malek and Saadatmanesh, 1998a,b]. A third procedure, the multidirectional crack model has apparently not yet been implemented for FRP shear-strengthened beams.

Another approach for simulating the behaviour of FRP shear-strengthened beams has been based on the modified compression field theory (MCFT) [Wong and Vecchio, 2003]. Wong and Vecchio have incorporated the MCFT in a two-directional in-house finite element code to investigate the behaviour of FRP shear-strengthened beams. Their analysis was based on a smeared, rotating crack model for the concrete.

2.9.3.3 Tension Stiffening Model

As explained by Floegl and Mang [1981], tension stiffening refers to the capacity of the intact concrete between neighboring cracks to carry a limited amount of tensile forces. The shear and tensile behaviour of the cracked concrete and the degree of tension stiffening depend mainly on the crack width. Since the characteristics of the FRP laminates in a strengthened structure influence the cracking behaviour, they might be expected to play a role in determining the appropriate tension stiffening model for FRP-strengthened beams and slabs. Figure 2.26 shows the typical tension stiffening model for concrete. Sato and Vecchio [2003] have concluded that the tension stiffening model is independent on the stiffness of FRP laminates. This was attributed to the negligible cross sectional area of the FRP laminates compared to that of the concrete section.
2.9.3.4 Shear Retention Factor

The so-called "shear retention factor" is a parameter that is generally employed to account for the friction between the two surfaces of a crack. A higher shear retention factor implies that the cracked concrete is capable of carrying a higher shear load and that the growth of shear cracks is impeded [Lee, 2003].

To date, only one study has focused on the appropriate shear retention factor to be used for FRP shear-strengthened beams [Lee, 2003]. This investigation has addressed the issue of the shear retention factor of concrete for the case of a conventional reinforced concrete beam. Various values of shear retention factor ranging from 0.025 to 0.2 were adopted in his numerical analysis (Figure 2.27). In order to fit the numerical predictions of the load-deflection relationships with those observed experimentally, the optimal value of the shear retention factor was found to be 0.05 [Lee, 2003].

2.9.3.5 Convergence of Results

An important aspect of finite element modeling is mesh size and convergence of the numerical predictions. Some studies have found that a convergence of the results was obtained when an adequate number of elements was used and that any refinement of the mesh had negligible effect on the results, as shown in Figure 2.28 [Kachlakov and McCurry, 2000; Santhakumar et al., 2004]. Kaliakin et al. [1996] investigated the influence of the number
of nodes per element on the convergence of the results. For an unstrengthened beam, they stated that no appreciable difference in the numerical results was found when 8-node or 20-node per concrete element was used. To date, apparently no similar convergence studies have been carried out for the case of FRP shear-strengthened beams.

### 2.9.4 Modelling of Bonded FRP Composites

Generally, two approaches have been adopted to represent the behaviour of the bonded FRP composites in shear-strengthened concrete beams. The first approach involves converting the FRP sheets to equivalent truss elements. It is attractive because of its simplicity [Wong and Vecchio, 2003]; however, this method does not explicitly account for the widths of the bonded FRPs. In the second approach, shell elements are employed. With these elements the out-of-plane stresses are considered to be negligible. A failure criterion also needs to be defined to describe FRP rupture under general plane stress conditions. Such criteria must account for the orthotropic nature of the FRP composites [Kaliakin et al., 1996; Arduini et al., 1997; Malek and Saadatmanesh, 1998a,b; Kachlakov and McCurry, 2000; Santhakumar et al., 2004; Elyasian et al., 2006].

### 2.9.5 Modelling of FRP/Concrete Interfacial Behaviour

Debonding failures often govern the behaviour of FRP shear-strengthened concrete beams and prevent such beams from attaining their full load capacity. Consequently, the proper
modelling of the FRP/concrete interfacial behaviour is essential for developing accurate numerical simulations. For this purpose, appropriate interface elements are required that must be able to capture the interfacial nonlinearities, including slip, and account for all possible failure modes. To date, some key studies that have considered the FRP/concrete interfacial behaviour are those of Lee et al. [2001]; Wong and Vecchio [2003]; Lee [2003]. Theses studies lead to good predictions of the overall load-deflection response and load capacity enhancements. However, they do not address the complete details of FRP debonding (i.e., slip profiles along the FRP/concrete interface).

In finite element analysis, two approaches can be adopted to simulate the debonding. In the first approach, debonding is simulated by modelling the cracking and failure of the concrete elements adjacent to the adhesive layer. This approach, which is referred to as the mesoscale model, utilized a very fine mesh with element sizes (0.2 – 0.5 mm) being one order smaller than the thickness of the fracture layer of the concrete [Lu et al., 2005]. This method generally requires large computational resources. In the second approach, interface elements are utilized to predict the nonlinear behaviour between the FRP and concrete [Lee et al., 2001; Wong and Vecchio, 2003; Lee, 2003].

In the work of Lee et al. [2001], predefined discrete shear cracks based on the actual major shear crack pattern observed from experiments were proposed to simulate the debonding failure of the FRP composites (Figure 2.29). To simplify the modelling, the crack pattern was idealized into several straight lines. The CFRP strips were bonded to the web over the shear crack. This was performed by an additional layer of elements over the concrete elements with both the CFRP and concrete elements sharing typical nodes; i.e., perfect bond was assumed.
2.9. NUMERICAL MODELLING

Figure 2.30: Constitutive relationships for bond interface: (a) elastic-plastic; and (b) linear elastic [Wong and Vecchio, 2003]

Figure 2.31: Load-deflection curves for RWOA specimens [Wong and Vecchio, 2003]

The Wong and Vecchio [2003] model is perhaps considered to be the pioneering model in implementing FRP/concrete interface elements for shear strengthened beams. Two bond-slip models were proposed to simulate the interfacial behaviour: elastic, and elastic-plastic models (Figure 2.30). These models were based on the characteristics of the adhesive layer and neglected the characteristics of the FRP laminates and concrete. After doubling the concrete compressive strength while keeping the strain at peak stress unchanged [Wong, 2001], the numerical model was shown to give reasonable results in terms of ultimate load capacity. The numerical model was more accurate when the elastic-plastic bond relationship was applied. The linear elastic bond relation led to a premature shear failure. The predicted load-deflection curves, along with the comparisons against the experimental results, are plotted in Figure 2.31 and show very good agreement. The authors conclude that more clearly defined constitutive relations must be developed for the FRP/concrete interface elements to further improve the modeling capabilities.
A three-dimensional finite element model was developed by Lee [2003] in order to obtain overall strength improvement predictions for shear-strengthened beams. The slip between the concrete and FRP strips was modelled using structural interface elements. Furthermore, since the CFRP strips were well anchored in the flange and soffit of the beam, perfect bond between the two surfaces was assumed at these locations, as shown in Figure 2.32. In that study, the bond-slip behaviour between the concrete and CFRP plates was established based on experimental results of shear-lap specimens. Furthermore, to correctly and accurately model CFRP debonding failure due to shearing failure of the concrete layer, a fine mesh was employed for the concrete layer lying between the outer surface and the steel stirrups. A linear elastic behaviour was used for the steel reinforcements since failure was observed to occur without any steel yielding. Overall, the numerical predictions were very close to the experimental loading capacities.

2.10 Summary

The latest advancements in using FRP composites to increase a beam’s shear carrying capacity was reviewed and the behaviour of shear-strengthened beams was studied. In this manuscript, the published factors affecting the shear-strengthened beams were addressed and the contradiction of certain parameters was observed. Among the parameters influencing such beams are beam dimensions (i.e., steel stirrups, concrete strength, size
Table 2.2: Review of structural modelling of shear-strengthened beams

<table>
<thead>
<tr>
<th>Program type</th>
<th>Concrete elements</th>
<th>Steel elements</th>
<th>FRPs elements</th>
<th>Interface elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kaliakin et al. (1996)</td>
<td>ABAQUS software</td>
<td>8-node brick</td>
<td>3-D bar</td>
<td>4-node shell</td>
</tr>
<tr>
<td>Arduini et al. (1996)</td>
<td>In-house code</td>
<td>8-node brick</td>
<td>NA*</td>
<td>NA</td>
</tr>
<tr>
<td>Malek and Saadatmanesh (1998)</td>
<td>ABAQUS software</td>
<td>4-node plane stress</td>
<td>2-node bar</td>
<td>4-node shell</td>
</tr>
<tr>
<td>Kachlakov et al. (2001)</td>
<td>ANSYS software</td>
<td>8-node brick (SOLID65)</td>
<td>2-node (LINK8)</td>
<td>4-node (SOLID46)</td>
</tr>
<tr>
<td>Al-Mahaidi et al. (2001)</td>
<td>DIANA software</td>
<td>8-node plane stress</td>
<td>3-node truss</td>
<td>8-node plane stress</td>
</tr>
<tr>
<td>Lee et al. (2001)</td>
<td>DIANA software</td>
<td>8-node plane stress</td>
<td>3-node truss</td>
<td>8-node plane stress</td>
</tr>
<tr>
<td>Lee (2003)</td>
<td>DIANA software</td>
<td>8-node brick (HX24L)</td>
<td>3-node truss</td>
<td>4-node plane stress (Q6MEM)</td>
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<tr>
<td>Wong and Vecchio (2003)</td>
<td>In-house code</td>
<td>4-node plane stress</td>
<td>truss</td>
<td>truss elements</td>
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<tr>
<td>Santhakumar and Chandrasekaran (2004)</td>
<td>ANSYS software</td>
<td>brick (SOLID65)</td>
<td>truss (LINK8-3D)</td>
<td>SOLID46</td>
</tr>
<tr>
<td>Elyasian et al. (2006)</td>
<td>ANSYS software</td>
<td>8-node cubic (SOLID65)</td>
<td>3-D spar (LINK8)</td>
<td>shell (SHELL43)</td>
</tr>
</tbody>
</table>

*It is not available at the reference*
Table 2.3: Review of material modelling of shear-strengthened beams

<table>
<thead>
<tr>
<th>Material modelling</th>
<th>Concrete model</th>
<th>Steel model</th>
<th>FRPs model</th>
<th>Interface model</th>
<th>Mesh sensitivity</th>
<th>Parameters studied</th>
</tr>
</thead>
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<tr>
<td>Kaluskin et al. (1996)</td>
<td>ABAQUS concrete</td>
<td>elastic-perfectly plastic</td>
<td>linear elastic isotropic</td>
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<td>Yes</td>
<td>Concrete strength, FRP elastic modulus of elasticity and plate thickness</td>
</tr>
<tr>
<td>Arduini et al. (1996)</td>
<td>nonlinear behaviour</td>
<td>elastic-perfectly plastic</td>
<td>linear elastic isotropic</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Malek and Saadatmanesh (1998)</td>
<td>ABAQUS concrete</td>
<td>NA</td>
<td>NA</td>
<td>No</td>
<td>No</td>
<td>Plate thickness, fibre orientation angle, steel stirrups spacing</td>
</tr>
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<td>Kachlakov et al. (2001)</td>
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<td>linear elastic orthotropic</td>
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<td>Yes</td>
<td>No</td>
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<td>No</td>
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<td>linear elastic orthotropic</td>
<td>Mod. shear-lap specimen</td>
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<td>No</td>
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<tr>
<td>Wong and Vecchio (2003)</td>
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<td>linear elastic isotropic</td>
<td>elastic-plastic</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Sarnbatmatar and Chandrashekaran (2004)</td>
<td>ANSYS concrete</td>
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<td>NA</td>
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<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Elyasian et al. (2006)</td>
<td>ANSYS concrete</td>
<td>uniaxial tension-compression</td>
<td>elastic</td>
<td>No</td>
<td>No</td>
<td>Fibre orientation angle, concrete strength, tensile steel and steel stirrups spacing</td>
</tr>
</tbody>
</table>

The shear-lap test was modified for shear-strengthened beams.
2.10. SUMMARY

Numerous experiments were carried out pointing the way to comprehensive guidelines that can be used for design; design procedures were reported. While there has been a considerable amount of research conducted on this area, a satisfactory shear design method has not been developed so far.

Due to the many advantages of using finite element analyses, experimental tests alone cannot verify the behaviour and all the parameters of shear-strengthened beams. Therefore, numerical and experimental investigations are required to link the behaviour of such beams. Analyses based on a perfect bond between the FRP and concrete substrate were seen to generally lead to over-predictions of the ultimate load capacities and stress levels in the bonded FRPs. Such analyses are inherently incapable of accounting for debonding, thus underscoring the need for developing a numerical tool that properly simulate the behaviour at the FRP/concrete interfaces for cases where failure is governed by FRP debonding. Numerical tools having good predictive capabilities can be very valuable for optimizing FRP strengthening schemes and for gaining insight into the role of various factors on the performance of FRP shear-strengthened beams.

While there has been a considerable amount of research carried out to develop a reliable numerical model to simulate the behaviour of FRP shear-strengthened beams, a satisfactory model has not been developed so far. As a contribution to fill this need, a versatile numerical model is developed in this study to predict the response of reinforced concrete beams strengthened in shear with bonded FRP composites, with particular emphasis on the interfacial behaviour and debonding phenomena.
Chapter 3

Development of a Reliable Numerical Model

3.1 Introduction

A numerical model is only as accurate as the assumptions used in creating it. This chapter consists of two main parts. In the first part, numerical aspects concerning the material modelling of FRP shear-strengthened beams are discussed. These aspects are the hypoelastic constitutive law with a fixed smeared crack approach to simulate the concrete behaviour. Appropriate material models are used to represent the behaviour of steel reinforcement and FRP sheets. Interface elements between the FRP and concrete are used to simulate the FRP/concrete interfacial behaviour. The constitutive law of these elements is represented using a bond–slip relationship.

In the second part, a comparison between the applicability of using truss elements and shell elements to capture the behaviour of FRP composites is presented. The truss modelling approach involves converting the FRP sheets to equivalent truss elements. However, this method does not explicitly account for the width of the FRP sheets. This issue is considered in the second approach. With shell elements, the out-of-plane stresses are considered to be negligible. This study constitutes a pioneering attempt to simulate the FRP/concrete behaviour with various interface elements. In addition to identify the best element representing the FRP composites, three appropriate interface elements are in-
vestigated for the purpose of comparison. They are the spring elements, discrete truss elements and continuous truss elements. The best interface element is identified based on the accuracy of the simulation. In addition to the vertical interface elements, this study also investigates the influence of including horizontal interface elements. The interfacial response is provided in two arrangements. In the first arrangement, the interface elements are implemented parallel and perpendicular to the beam axis, while in the second arrangement, interface elements are oriented only perpendicular to the beam axis. Different mesh sizes are used to investigate the effect of mesh sensitivity on the accuracy of the results.

The numerical model presented here aims to capture the three-dimensional and nonlinear behaviour of the concrete, as well as to accurately model the bond-slip behaviour at the FRP/concrete interface. The proposed three-dimensional analysis is applied to various cases having different FRP strengthening configurations. This investigation involves the beams tested by Lee and Al-Mahaidi [2008]; Adhikary and Mutsuyoshi [2004]; Pellegrino and Modena [2002]; Khalifa and Nanni [2000]; Chaallal et al. [1998a]. The accuracy of the model will be evaluated by comparing the numerical predictions to the experimental results. Then, having verified the accuracy of the numerical models, information on the slip of debonding along the FRP/concrete interface are performed in order to gain better insight into the debonding mechanism of shear-strengthened beams. The significance of the present findings with respect to potential retrofits of existing concrete beams will be discussed in the next chapter.

3.2 ADINA Finite Element Model

In this study, an appropriate three-dimensional finite element model is developed to accurately simulate the behaviour of FRP shear-strengthened beams. The initial phase of the numerical study involves the development of an accurate finite element model of a control beam. The control beam has no external FRP composites for shear strengthening. The quality of this model is assessed by comparing the numerical results with experimental measurements. In the second phase of the numerical study, the control beam model is modified to include the presence of the FRP composites for shear strengthening. The accuracy of this model is again measured by comparing with experimental data. The goal of these phases is to ensure that the numerical models are representing very well the actual
tested beams. The data are analyzed in a sequence of increasing complexity. The most simple consists of a side-bonded FRP strengthening, and the most complex consists of a U-wrap strengthening scheme. All computations reported herein are performed using the ADINA [2004a] computer software.

3.2.1 Material Modelling

3.2.1.1 Concrete

The constitutive model used for the concrete behaviour is provided in the ADINA [2004b] software. It is characterized by a nonlinear stress–strain relation to allow for the strain softening behaviour of the material under increasing compressive stresses. In addition, it utilizes failure envelopes that define both failure in either tension or compression by crushing. It also features a strategy to model the post-cracking and post-crushing behaviour of the material. The failure envelopes also account for multiaxial stress conditions and identify whether tensile or crushing failure of the material has occurred. For the stress–strain relationship of the concrete in compression, the elastic limit is taken as 30% of the maximum concrete compressive strength, $f'_{c}$. This is followed by a nonlinear behaviour until the maximum concrete strength is reached, beyond which the behaviour softens until concrete crushing occurs.

The tensile behaviour of the concrete takes into account cracking, shear modulus degradation, fracture energy and tension stiffening. Tension stiffening is modelled as a linearly descending branch in the stress–strain relationship after the peak point at which the concrete has cracked. The slope of the ascending branch is equal to the concrete modulus of elasticity. In the descending part of the stress-strain curve, the fixed smeared crack model is used, in which the plane of failure occurs perpendicular to the corresponding principal stress direction. Figure 3.1 shows the uniaxial stress–strain curve for the concrete. For the finite element implementation, the values of the compressive strength $f'_{c}$ (MPa), tensile strength $f_{t}$ (MPa) and elastic modulus $E_{c}$ (MPa) are taken from the corresponding experimental set of data. When $E_{c}$ and $f_{t}$ are not given, they can be approximated based on the following CSA-A23.3-04 [2004] equations:

$$f_{t} = 0.6\sqrt{f'_{c}}$$

(3.1)
3.2. ADINA FINITE ELEMENT MODEL

$E_c = 3,300 \sqrt{f'_c} + 6,900 \quad (3.2)$

A detailed description of the concrete constitutive model of the finite element package ADINA [2004b] is presented in Appendix A.

3.2.1.2 Steel Reinforcement and FRP Composites

The steel is represented by an elastic-plastic constitutive relation with linear strain hardening, as shown in Figure 3.2(a). To indirectly include the effect of steel dowel action, an increased value for the shear retention factor (0.5) is used. The ratio between the slopes in the elastic range to the plastic range is taken from 100 to 200. A linear elastic tensile model until failure is assumed to represent the FRP composites. A rupture point on the stress–strain relationship defines the maximum stress and strain of the FRP composites, as shown in Figure 3.2(b).

3.2.1.3 FRP/Concrete Interface

Since the bond-slip models developed by Lu et al. [2005] has received wide acceptance, this model is one of the most accurate bond stress–slip models that can be incorporated into a finite element analysis. The behaviour of the FRP/concrete interface is simulated by
a relationship between the local shear stress, $\tau$, and the relative displacement, $s$. Three different bond-slip relations have been suggested by these authors; they are classified according to their level of sophistication and are referred to as the precise, the simplified, and the bilinear models. In the current study, the bilinear model, as shown in Figure 3.3, is adopted for its simplicity. It should be noted that the fracture energy values with the bilinear model and the two other models are essentially identical. The other two models were highlighted in detail in Lu et al. [2005].

Since the debonding is observed to occur at a few centimeters inside the concrete cover, no parameter considering the adhesive properties or thickness is taken into account. One of the parameters that governed the FRP/concrete interfacial behaviour is the tensile strength of the concrete ($f_t$). Considering $\tau_{max}$ to be the maximum bond stress (MPa) and $s_0$ the corresponding slip (mm), then for the ascending part ($s \leq s_0$)

$$\tau = \frac{\tau_{max}}{s_0} s \tag{3.3}$$

$$\tau_{max} = 1.5\beta_w f_t \tag{3.4}$$

$$s_0 = 0.0195\beta_w f_t \tag{3.5}$$

$$\beta_w = \sqrt{(2.25 - b_f/b_c)/(1.25 + b_f/b_c)} \tag{3.6}$$
3.2. ADINA FINITE ELEMENT MODEL

For the descending part \((s_0 < s \leq s_{\text{max}})\)

\[
\tau = \tau_{\text{max}} \frac{(s_{\text{max}} - s)}{(s_{\text{max}} - s_0)}
\]

\[s_{\text{max}} = 2G_f/\tau_{\text{max}}\]  

where

\[G_f = 0.308\beta^2_w \sqrt{f_t}\]

Here \(b_f, b_c\) and \(G_f\) are width of FRP composites, the central spacing between the strips (in the case of continuous sheets \(b_f/b_c = 1\)), and the interfacial fracture energy, respectively.

### 3.2.2 Structural Modelling

Three-dimensional brick elements with three degrees of freedom per node are employed to discretize the concrete; the number of nodes is selected based on the accuracy of the results. Using such elements satisfies shear and bending deformations due to their quadratic interpolation functions. The steel reinforcement embedded in the concrete is represented by truss elements. Ideally, the bond strength between the concrete and steel reinforcement should be considered. However, in the current application, the truss elements representing the longitudinal steel and steel stirrups are directly connected to the concrete elements.
since no debonding was observed between the two components in the experimental phase. Clearly, possible steel debonding did not influence the global behaviour of FRP shear-strengthened beam.

3.2.2.1 Modelling of FRP Composites

In order to determine the best strategy to simulate the behaviour of the FRP composites, two approaches are compared. In the first approach, the FRP is modelled as truss elements aligned in the direction corresponding to the fibre orientation. Each FRP strip is thus converted to an equivalent truss element. When there is a continuous FRP sheet, it is subdivided into several truss elements along the FRP sheet length; the number of elements were aligned corresponding to the meshing of the concrete beam. The area of each truss element is equivalent to that of the FRP composites. In the second approach, shell elements are utilized and the orthotropic nature of the FRP composites is accounted for in the constitutive relation for the material.

3.2.2.2 Modelling of FRP Concrete Interface

Special attention was given to simulate the bond-slip between the concrete and FRP laminates. Bilinear interface elements are employed for this purpose. These elements allow relative movements between the two adjacent surfaces, as shown in Figure 3.4. The difference in displacement between the concrete and FRP represents the slip at the interface. The interface elements are arranged parallel to the fibre orientations and full strain compatibility is assumed between the two adjacent surfaces in the other directions. Three types of interface elements are investigated to represent FRP/concrete interfacial behaviour. These are spring elements, discrete truss elements and continuous truss elements. The most appropriate of these three interface elements will be identified on the basis of comparison with experimental data.

The uniaxial tensile-compressive spring elements are illustrated schematically in Figure 3.5a. These elements are employed with a bilinear force-slip relationship to simulate the interfacial behaviour along the FRP/concrete bonding layer. In order to predict the delamination between the concrete and FRP, the bilinear force-slip curve ($F - s$) of the
3.2. ADINA FINITE ELEMENT MODEL

Figure 3.4: Finite element model

Spring elements is based on the relation proposed by Lu et al. [2005] (Equations 3.3 to 3.9). The area of each spring element represents the shear area for each concrete node. Debonding is assumed to occur if the slip, \( s \), in spring element reaches the maximum allowed slip, \( s_{\text{max}} \).

In the second type of interface element, the slippage between the concrete and FRP is simulated via two-node discrete truss elements aligned in a discrete manner. Each interface element connects the FRP nodes and the corresponding concrete nodes; the interface elements are totally independent from each other (Figure 3.5b). The discontinuities of the discrete truss elements allow each truss to fluctuate from negative to positive stresses depending on their location from the shear crack. The constitutive law of these elements \( (\tau - \varepsilon) \) depends on their length. The difference in displacement between the concrete and FRP represents the slip at the interface elements, while the axial stress in these elements represents the interfacial shear stress. Additionally, these elements allow both the tensile and compressive movements to occur parallel to the fibre orientations between the two materials.
CHAPTER 3. DEVELOPMENT OF A RELIABLE NUMERICAL MODEL

Figure 3.5: Various arrangements of interface elements

The continuous truss elements, the third type of interface element, are similar to the discrete truss elements in most respects. The only difference is the continuous truss elements are introduced all over the contact layer of FRP/concrete interface to describe continuous connection between the two adherents, as shown in Figure 3.5c. These elements are discretized according to the concrete nodes and the corresponding FRP nodes. Continuous truss elements type are the most simple to implement with ADINA.

It should be emphasized that interface elements do not directly represent the adhesive. They represent the overall concrete/FRP interfacial response, which depends on the concrete, the FRP, as well as the adhesive. The interface element connects one concrete node with a corresponding FRP node. The stiffness of these bond elements is associated with the difference in displacement between the two adherent materials. The interface elements are considered to act only in the directions parallel to the main fibre orientations. The constraint equations are enforced in the other directions between the interface and the concrete nodes, and between the interface and the FRP nodes. Thus, the total displacement between Point 1 (slave) and Point 2 (master) of the interface element (Figure 3.6) represents the interfacial slip.
3.2. ADINA FINITE ELEMENT MODEL

3.2.3 Horizontal Interface Elements

When the principal diagonal-tension stresses reach the tensile strength of the concrete, a diagonal crack will develop [Kong and Evans, 1990]. In all of the previous studies, the interfacial behaviour is considered only perpendicular to the beam axis, while full strain compatibility was assumed in the other directions [Wong and Vecchio, 2003; Lee and Al-Mahaidi, 2008]. Therefore, the influence of the horizontal component of the principal diagonal-tension stresses to the general response of the beam was neglected. Additionally, there is no published information on the effectiveness of utilizing horizontal interface elements beside the vertical ones.

In addition to the vertical interface elements, the current study investigates the influence of including horizontal interface elements. The interfacial response is provided in directions parallel and perpendicular to the fibre orientations to investigate the horizontal component of the principal diagonal-tension stresses. It should be noted that the interaction between the vertical and horizontal interface elements is disregarded. The shear stress–slip curve for the discrete truss elements acting perpendicular to the fibre orientations is a function of the shear stress–slip curve of the vertical interface elements. As such, analyses using shear stress–slip curves for horizontal interface elements ranging from 5%, 10% to 20%, of the vertical interface elements are used. In order to account for this investigation, the beam with steel stirrups strengthened with single continuous sheets tested by Pellegrino and Modena [2002] is considered. The comparison between the results is given in terms of load–deflection relationships. For this phase of the study, discrete truss elements and shell elements are employed for the FRP/concrete interfacial behaviour and FRP composites, respectively.
3.2.4 Finite Element Discretization

An important step in the finite element modelling is the selection of the mesh density. A convergence of results is obtained when an adequate number of elements is used in a model. This is achieved practically when an increase in the mesh density has a negligible effect on the results. Therefore, in the current research, a convergence study is carried out to specify an appropriate mesh density and to ensure that the spatial discretization used did not introduce excessive approximations into the simulations. Another purpose of this study is to verify the fluctuation of the slip profiles along the sheet depth and the locations of the maximum interfacial slip values. For these purposes, only the beam strengthened with side-bonded FRP strips tested by Chaallal et al. [1998b] is investigated. The number of elements along the beam depth are increased, while the number of elements in the other dimensions are kept identical. That is five elements, ten elements and twenty elements are performed along the beam depth of 200 mm. Note that shell elements and discrete truss elements are employed to simulate the FRP composites and FRP/concrete interfacial behaviour, respectively. A number of response parameters are compared, including the load-deflection relationship and the interfacial behaviour between the concrete and FRP composites.

3.3 DIANA Finite Element Model

Intensive research at Monash University (Australia) in collaboration with the Université de Sherbrooke examined the mechanism of shear-strengthened beams. The ultimate objective of this investigation is to evaluate the capability of the numerical models to simulate the behaviour of shear strengthened beams. This study presents non-linear numerical analyses of two various finite element packages, DIANA [2000], and ADINA [2004b], in order to simulate the response of shear-strengthened T-beams tested by Lee [2003]. The DIANA finite element simulations were conducted at Monash University. The accuracy of these models is evaluated by comparing the numerical predictions to the experimental results.

For the DIANA numerical model, a three-dimensional finite element model capable of predicting the ultimate load and modes of failure was conducted. Furthermore, as the beams were symmetrical, only a quarter of the beam was simulated with appropriate
boundary conditions. The plain concrete, longitudinal and transverse steel, FRP strips, and bond between the concrete and FRP composites were simulated using a variety of elements. Additionally, the nonlinear load-deformation behaviour was simulated under displacement-controlled loading conditions. The details of the material and geometrical modelling are summarized below.

3.3.1 Concrete

Three-dimensional 8-node solid brick elements (HX24L) with three degrees of freedom per node were employed to simulate the concrete. The DIANA concrete model follows the implementation of the modified compression field theory of Vecchio and Collins [1986]. The model represents concrete cracking using a smeared fixed-crack approach and can simulate the post-peak response of concrete. The concrete constitutive model is fully defined in compression and in tension. Both the hardening and softening behaviour of the concrete in compression are described. Concrete tension stiffening in the analysis and the nonlinear descending portion of the tensile stress-strain curve is based on the mode-I fracture energy. Concrete elements account for the degradation in the modulus of rigidity after crack initiation using a constant reduced shear modulus. The values of concrete density, \( \rho \), and Poisson's ratio, \( \nu \), was taken equal to 2400 kg/m\(^3\) and 0.2, respectively. The elastic modulus, \( E_c \), and the tensile strength of concrete, \( f_t \), were identified from the following SAI [2001] equations:

\[
\begin{align*}
    f_t &= 0.4\sqrt{f_c'} \\
    E_c &= 0.043\sqrt{f_c'(\rho)^{1.5}}
\end{align*}
\] 

3.3.2 Steel Reinforcement and FRP Composites

Steel reinforcing bars were simulated by three node truss elements. Slip between the steel reinforcement and surrounding concrete was assumed to be negligibly small. Hence the steel elements are rigidly attached to the concrete nodes. The CFRP plates were modelled using four-node quadrilateral isoparametric plane stress elements (Q8MEM). This was attributed to the fact that the forces carried by the CFRP strips are mainly in the in-plane direction; out of plane stresses were considered to be negligible. For
CHAPTER 3. DEVELOPMENT OF A RELIABLE NUMERICAL MODEL

(a) (b) (c)

Figure 3.7: Interface element (L8IF): (a) topology, (b) displacements, (c) traction [Lee, 2003]

the flexural and shear reinforcement, a linear elastic material until the yield point was employed since failure was observed to occur without any steel yielding. An orthotropic linear elastic material with properties described experimentally was adopted for the CFRP plates. This assumption is reasonable since the failure is governed by concrete crushing in the experiments.

3.3.3 FRP/Concrete Interface

In order to create a reliable numerical model to simulate the actual response of externally shear-strengthened beams, the presence of bond between the concrete and FRP composites should be considered. In the DIANA models, slip between the concrete and FRP strips was modelled using structural interface elements (L8IF), as shown in Figure 3.7. The CFRP strip was superimposed on the top of the concrete connected via interface elements. Furthermore, since the CFRP strips were well anchored in the flange and soffit of the beam, perfect bond between the two adherents was assumed in these locations. In the web, there were two sets of nodes separating the concrete and CFRP composites. In that study, the bond–slip behaviour between the concrete and CFRP plates was established based on experimental results of shear-lap specimens. The bond–slip behaviour of such experiments was reproduced herein to fit with shear-strengthened beams (Figure 3.8). Furthermore, to correctly and accurately model the debonding failure mode due to shearing failure of concrete layer, a fine mesh was performed for the concrete layer lying between the outer surface and the steel stirrups.
3.4 Specimens Investigated

The primary objective of the numerical investigations is to develop and validate a reliable numerical model to simulate the behaviour of FRP shear-strengthened beams. The proposed three-dimensional analysis is applied to various cases having different FRP strengthening configurations. These are selected for numerical analysis so as to cover the widest possible range of FRP strengthening schemes, and to include both rectangular and T-shaped beam sections. This investigation involves the beams experimented by Lee and Al-Mahaidi [2008]; Adhikary and Mutsuyoshi [2004]; Pellegrino and Modena [2002]; Khalifa and Nanni [2000]; and Chaallal et al. [1998b]. The various shear-strengthening configurations considered include strengthening with vertical and inclined FRPs attached to the sides of beams, and strengthening with U-shaped wraps for rectangular sections. Furthermore, the validity of the numerical model is extended to examine the strengthening of T-sections with the U-wrap FRP configuration. It is also used to investigate whether the use of a top-end rod anchoring for a continuous U-shaped wrapping can prevent debonding. The specimens investigated are presented in a sequence of increasing complexity. The most simple consists of a side-bonded FRP strengthening, and the most complex consists of a U-wrap strengthening scheme.

Table 3.1 and Table 3.2 list the geometrical characteristics, strengthening details and material properties of the various specimens. In Table 3.1, the geometric properties of the
Table 3.1: Geometrical characteristics and FRP shear-strengthening configurations of tested beams

<table>
<thead>
<tr>
<th>No.</th>
<th>Beam set</th>
<th>Section type</th>
<th>Specimen</th>
<th>Beam dimensions (mm)</th>
<th>Steel stirrups</th>
<th>Strength type</th>
<th>Strength details</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Lee and Al-Mahaidi (2008)</td>
<td>T-sec.</td>
<td>R1</td>
<td>0.75D</td>
<td>No 10@365mm</td>
<td>U-wrap</td>
<td>Vertical strips</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>0.6D</td>
<td></td>
<td>U-wrap</td>
<td>Vertical strips</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>0.5D</td>
<td></td>
<td>U-wrap</td>
<td>Vertical strips</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>B-8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Pellegrino and Modena (2002)</td>
<td>Rect.</td>
<td>TR30D1</td>
<td>2700 150 300 250 3</td>
<td>No 8@200mm</td>
<td>Side-bonded</td>
<td>1 Cont. ply</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td>TR30D3</td>
<td></td>
<td></td>
<td>Side-bonded</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td>TR30D4</td>
<td></td>
<td></td>
<td>Side-bonded</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>TR30D2</td>
<td></td>
<td></td>
<td>Side-bonded</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td>BT1</td>
<td></td>
<td></td>
<td>U-wrap</td>
<td>1 Cont. ply</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td>BT2</td>
<td></td>
<td></td>
<td>U-wrap</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Khalifa and Nanni (2000)</td>
<td>T-sec.</td>
<td>BT3</td>
<td>3050 150 405 382 2.8</td>
<td></td>
<td>U-wrap+H</td>
<td>2 Cont. plies</td>
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<td>14</td>
<td></td>
<td></td>
<td>BT4</td>
<td></td>
<td></td>
<td>U-wrap</td>
<td>Vertical strips</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td>BT5</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td>BT6</td>
<td></td>
<td></td>
<td>U-wrap</td>
<td>Cont. sheet</td>
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<tr>
<td>17</td>
<td>Chaallal et al. (1998)</td>
<td>Rect.</td>
<td>US</td>
<td>1300 150 250 170 2.5</td>
<td>No 6@200mm</td>
<td>Side-bonded</td>
<td>Vertical strips</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td>RS90</td>
<td></td>
<td></td>
<td>Side-bonded</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td>RS135</td>
<td></td>
<td></td>
<td>Side-bonded</td>
<td>Inclined strips</td>
</tr>
</tbody>
</table>

*Web width is considered.

Table 3.2: Material properties of tested beams

<table>
<thead>
<tr>
<th>No.</th>
<th>Beam set</th>
<th>Concrete ( f_c ) (MPa)</th>
<th>Long. steel ( f_y ) (MPa)</th>
<th>Steel stirrups ( f_y ) (MPa)</th>
<th>Steel stirrups ( t ) (mm)</th>
<th>Ultimate stress ( f_u ) (MPa)</th>
<th>Ultimate strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lee and Al-Mahaidi (2008)</td>
<td>31.1</td>
<td>450</td>
<td>250</td>
<td>1.2</td>
<td>3150</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>Adhikary and Mutsuyoshi (2004)</td>
<td>34.0</td>
<td>391</td>
<td>386</td>
<td>0.167</td>
<td>3400</td>
<td>1.50</td>
</tr>
<tr>
<td>3</td>
<td>Pellegrino and Modena (2002)</td>
<td>31.4</td>
<td>548</td>
<td>548</td>
<td>0.165</td>
<td>3550</td>
<td>1.50</td>
</tr>
<tr>
<td>4</td>
<td>Khalifa and Nanni (2000)</td>
<td>35.0</td>
<td>470</td>
<td>350</td>
<td>0.165</td>
<td>3790</td>
<td>1.66</td>
</tr>
<tr>
<td>5</td>
<td>Chaallal et al. (1998)</td>
<td>35.0</td>
<td>400</td>
<td>400</td>
<td>1.000</td>
<td>2400</td>
<td>1.40</td>
</tr>
</tbody>
</table>
3.4. SPECIMENS INVESTIGATED

beam are span (L), depth of the beam (H), and the width of the beam (B_w). The value a/d is the ratio of the shear span to the effective depth of the longitudinal reinforcement. The specimen designations correspond to those used in the original references. It should be emphasized that, for a given set of specimens (with and without FRP strengthening), the same values of the various material parameters were used. In view of the geometrical and loading symmetries, only one quarter of the beams is simulated. The nonlinear load–deformation behaviour of the structure is simulated under displacement-controlled loading conditions.

3.4.1 Pellegrino and Modena Specimens

The beams tested by Pellegrino and Modena [2002] investigated the side-bonded FRP shear-strengthening scheme with single, double and triple plies. In our analyses, only those beams having internal steel stirrups have been considered. Three beams were examined in order to study the influence of increasing the number of FRP layers. TR30D1 refers to the control specimen (i.e., without FRP strengthening), while TR30D3 is the specimen strengthened with a single layer of FRP sheet. TR30D4 and TR30D2 represent the specimens strengthened with two and three plies, respectively.

Four beam specimens with a total span length of 2000 mm and a rectangular cross-section of 300 mm deep and 150 mm wide were tested. All the beams have identical longitudinal steel reinforcement (two 22 mm and 20 mm diameter bars at the tension face
and two 22 mm diameter bars at the compression face). 8 mm diameter steel stirrups with a 300 mm spacing were provided in the shear span to ensure shear failure. All beams were tested under four-point loading. The loading scheme and strengthening configuration are presented in Figure 3.9. The ratio of shear span to effective depth was constant (3) in all the tests. The principal data of the beams are summarized in Table 3.1 and Table 3.2.

### 3.4.2 Chaallal et al. Specimens

Chaallal et al. [1998b] examined FRP shear strengthening with vertical and inclined side-bonded plates. These specimens were designed to compare the efficiency of vertical and 45° inclined side-bonded strengthening schemes with FRP strips. US, RS90 and RS135 are the control beam, the beam with vertical FRP strips and the beam with inclined FRP strips, respectively. The three specimens of 1200 mm-long RC beams having cross-sectional dimensions of 150 mm x 250 mm (Figure 3.10) were considered. All specimens had the same flexural and shear steel reinforcement; the steel reinforcement details are shown in Figure 3.10. The mechanical properties and geometrical data of the beams are summarized in Table 3.1 and Table 3.2. Unidirectional carbon fibre strips, 50 mm wide and 1 mm thick, were used in that study. The two specimens strengthened with FRP strips had the same FRP reinforcement ratio. All specimens were tested in four-point bending. The schematic loading arrangement and strengthening details are shown in Figure 3.10.

### 3.4.3 Adhikary and Mutsuyoshi Specimens

For the beams tested by Adhikary and Mutsuyoshi [2004], one was a control specimen while the other seven were strengthened using FRP sheets. Only two specimens are simulated with the proposed numerical model: the control specimen and one strengthened specimen with a U-wrap up to the top edge of the beam, as shown in Figure 3.11. Specimen B-1 (the control specimen) is analyzed to serve as a reference, while only Specimen B-8 of the remaining FRP-strengthened beams is considered as this represented the strengthening configuration of practical interest; specimen B-8 is the only specimen strengthened with a U-wrap strengthening scheme up to the top edge of the concrete section. Table 3.1 and Table 3.2 show the details of the test variables in the experiments. The dimensions of the
Figure 3.10: Shear strengthening configurations and loading arrangement [Chaallal et al., 1998b]
tested beams were 150 mm (width) x 200 mm (depth) x 2,600 mm (length), as shown in Figure 3.11. No internal shear reinforcement was provided in the shear span to ensure shear failure even after the application of FRP sheets. Additionally, this is to investigate the effect of FRP sheets in shear strengthening when there is no steel stirrups provided. All beams were tested under four-point loading.

### 3.4.4 Khalifa and Nanni Specimens

The proposed model has also been verified against the experimental results reported by Khalifa and Nanni [2000]. The control specimen is BT1. The specimens were reinforced with longitudinal steel bars with no shear reinforcement to favour shear failure (Figure 3.12). The material and mechanical properties are listed in Table 3.1 and Table 3.2. That study examined the behaviour of a T-section strengthened with side-bonded strips (BT5), a continuous U-wrap (BT2), U-wrap strips (BT4), and a continuous U-wrap with the superposition of horizontal sheets along the sides of the beam (BT3). Another experimented configuration was the rod anchoring of a continuous sheet at the top of the FRP sheet (BT6). The strengthening details are shown in Figure 3.13.
For specimen BT6, the ends of the U-wraps were anchored to the flanges on both sides of the beam using an FRP rod, as given in Figure 3.13a. The anchoring was attained by grooving the concrete flanges at the corners. The groove dimensions were about 15 x 15 mm and extended throughout the strengthened part. A 10 mm diameter GFRP rod was placed into the groove and then the groove was filled with paste.

3.4.5 Lee and Al-Mahaidi Specimens

In order to simulate the behaviour of a T-section strengthened with a U-wrap scheme, the beams tested by Lee and Al-Mahaidi [2008] are considered (Figure 3.14). The experimental program consisted of four full-scale RC beams with a T-shaped cross-section. Three of these beams were shear strengthened with CFRP strips at various spacing. The control beam is denoted by R1. Three beams were shear-strengthened with CFRP L-shaped laminate strips, spaced at 0.75D, 0.6D and 0.5D, respectively, where D is the overall depth of the beam. The designation of these specimens corresponds to their spacing. The details of the beams are given in Figure 3.15. Additionally, the CFRP strips were placed in the same manner in both sides of the shear span. The CFRP strips were bonded to the webs of the beams and extended until the top flange edge. This was accomplished through rectangular holes drilled at the flange only at the locations of the strips. The specimens were loaded under four-point loading, and the shear span to depth ratio was 3.0.

3.5 Summary

Numerical models were created with the ADINA finite element package to predict the behaviour of externally shear-strengthened beams using fibre reinforced polymers (FRPs). Nonlinear material behaviours, as they relates to plain concrete, steel reinforcing bars, FRP composites and FRP/concrete interfaces were simulated with appropriate constitutive models. In the finite element analysis, two approaches were employed to simulate the behaviour of the FRP composites. These were truss elements and shell elements. To gain better insight into the behaviour of shear-strengthened beams, the influence of three types of interface element to represent the FRP/concrete interfacial behaviour was exam-
Figure 3.13: Shear strengthening configurations [Khalifa and Nanni, 2000]
Figure 3.14: Beam’s dimensions and reinforcement details [Lee and Al-Mahaidi, 2008]

Figure 3.15: Shear strengthening configurations [Lee and Al-Mahaidi, 2008]
CHAPTER 3. DEVELOPMENT OF A RELIABLE NUMERICAL MODEL

These elements were spring elements, discrete truss elements and continuous truss elements. The appropriate interface elements are selected on the basis of the best fit of numerical predictions obtained for a predetermined set of experimental data.

In addition to the vertical interface elements which is normally presented, the interfacial response was provided parallel and perpendicular to the fibre orientations to examine the horizontal component of the principal diagonal-tension stresses. By increasing the number of elements along the depth of the FRP sheet, a mesh sensitivity study was performed. The goal of this study was to ensure that the spatial discretization used did not introduce excessive approximations into the simulations.

Numerical models were developed by Lee and Al-Mahaidi [2008] using the DIANA finite element package to simulate the behaviour of shear-strengthened beams. The plain concrete, longitudinal and transverse steel, FRP strips, and bond between the concrete and FRP composites were simulated using a variety of elements. The accuracy of the DIANA and ADINA models is evaluated by validating the numerical predictions against published experimental data.
Chapter 4

Validation of Numerical Results

4.1 Introduction

A number of investigations were conducted in this research with the objective of establishing the ability of the proposed model to simulate the response of FRP shear-strengthened beams. The results presented in this chapter are given, firstly, for the comparison between the truss and shell modelling of FRP composites in terms of ultimate load carrying capacities, load-deflection relationships, and failure modes. In addition, special concerns are placed on the results for the FRP/concrete interfacial response for various types of interface elements. The strain profiles along the sheet depth are also examined in this study.

4.2 Comparison Between Shell and Truss Modelling of FRP Composites

An examination of the shell and truss modelling will include a comparison between numerical predictions and numerous experimental data on ultimate load carrying capacities and load-central deflection relationships. All subsequent comparisons of this section employed discrete truss elements to simulate the FRP/concrete interfacial behaviour. The beams tested by Pellegrino and Modena [2002] investigated the side-bonded FRP shear-
strengthening scheme with single, double and triple plies. Only the results of the control beam (specimen TR30D1), and beam with steel stirrups and shear-strengthened with single continuous sheet (specimen TR30D3) are considered in this analysis. The second set of specimens modelled numerically was those tested by Chaallal et al. [1998b]. In order to simulate the behaviour of a rectangular section strengthened with a U-wrap scheme, the beams tested by Adhikary and Mutsuyoshi [2004] are considered.

4.2.1 Load–Deflection Relationships and Failure Modes

The numerical predictions and experimental results of the various specimens are summarized in Table 4.1, while the failure modes are presented in Table 4.1. Note that the failure mode with both types of FRP composite elements are identical. For the Pellegrino and Modena shear-strengthened beam with a single CFRP sheet (specimen TR30D3), Figure 4.1 represents the result of the FE analysis by using the shell and truss element modelling of the FRP sheet, and compares them with the experimental results. As can be seen from Figure 4.1, the curves for the experimental and numerical results of the control beam, TR30D1, are virtually identical. Similarly, numerical results for the beam with one layer of FRP sheet, TR30D3, are similar to the experimental results when the simulation is done with shell elements. In this case, the load carrying capacity of the model with shell elements overestimated the failure load by 4%, over the experimental data, and the corresponding mid-span deflection was approximately identical to the experimental value. With truss elements, the ultimate load carrying capacity is lower than the experimental results by 6%. The corresponding mid-span deflection is higher than the test value by 21%. The experimental failure mode of this beam is debonding of the FRP sheets, which was detected by the two modelling approaches. The load–deflection curves predicted from the numerical analysis is almost identical to the experimental curve prior to and up to the initial cracking phase. For both FRP models, at the stage of excessive stress transformation from the concrete to the adherent FRP composites, the numerical behaviour of the model using the shell elements was identical to the tested beam. In the case of the adopted truss elements, the load–deflection response was lower and more ductile than the experiment.

The second set of specimens modelled numerically was that tested by Chaallal et al.
4.2. COMPARISON BETWEEN SHELL AND TRUSS MODELLING OF FRP COMPOSITES

The load–deflection curves for the control beam (US) and for beam strengthened with vertical FRP strips (RS90) are shown in Figure 4.2. The numerical predictions for the control beam are essentially identical to the experimental results. For the vertical FRP side bonded strips, the shell modelling of the FRP strips predicted a failure load 7% higher than the experimental values, while the truss simulation underestimated the failure load by 9%. The mid-span deflection corresponding to the failure load was identical for the shell modelling and higher by 9% for the truss modelling results.

The comparison between the experimental and the numerical results of the inclined side-bonded strengthened beam (RS135) of Chaallal et al. [1998b] is plotted in Figure 4.3. It should be emphasized that the shear stress–slip curve used for the inclined FRP strips was similar to that used for the vertical FRPs. The model successfully estimated the load carrying capacity of the inclined strips when utilizing the shell elements; 4% higher than the test value. As shown in Table 4.1, with the truss modelling of the FRP composites, the ultimate load is 11% lower than the test results. In addition, the experimental mid-span deflection at the maximum load was identical to the shell modelling; it is 15% higher than the experimental values for the shell and truss modelling, respectively. Failure with both simulation types of FRPs was dominated by debonding of the FRP which, in fact, is the same as what was observed in the experiments.

In order to simulate the behaviour of a rectangular section shear-strengthened with a
CHAPTER 4. VALIDATION OF NUMERICAL RESULTS

Figure 4.2: Applied load–central deflection relationships for US and RS90 beam of Chaallal et al. [1998b]

Figure 4.3: Applied load–central deflection relationships for US and RS135 beam of Chaallal et al. [1998b]
4.2. COMPARISON BETWEEN SHELL AND TRUSS MODELLING OF FRP COMPOSITES

U-wrap scheme, two beams tested by Adhikary and Mutsuyoshi [2004] were simulated; the control specimen (B-1) and one specimen strengthened with a U-wrap up to the top edge of the beam (B-8). The behaviour of the control beam and U-wrap strengthened beam are shown in Figure 4.4. It can be seen that a very satisfactory agreement is observed between the experimental values and the numerical analysis with the shell modelling of the FRP sheets. The simulation of the FRP composites by the truss elements shows the same behaviour as the test results before the failure load. In addition, discrepancies in the slope of the curves are observed near the ultimate load. The U-wrap modelled beam (B-8) with shell elements overestimated the maximum load by 5% and the corresponding mid-span deflection was 4% lower than the test value. The truss modelling of the FRP composites attached to the sides and the bottom of the beam (B-8) predicted a lower failure load by 2%. At the maximum load, the reported mid-span deflection was 5% higher than the test value. The observed failure criterion of the numerical analysis (debonding of FRP sheets) was identical to the corresponding experimental observation.

The proposed finite-element model proved its capability to accurately predict the load–deflection relationships of the FRP shear strengthened beams, when shell elements are employed to simulate the FRP sheets. Figure 4.5 shows the comparison between the experimental and numerical results in terms of load–deflection relationships for the rest of the strengthened specimens tested by Pellegrino and Modena [2002] (specimens TR30D3 and TR30D2). The results for the Khalifa and Nanni [2000] beams are shown in Figure 4.6.
Table 4.1: Comparisons between shell and truss modelling of FRP composites

<table>
<thead>
<tr>
<th>No.</th>
<th>Beam set</th>
<th>Spec.</th>
<th>Experimental results</th>
<th>Numerical results (shell)</th>
<th>Numerical results (truss)</th>
<th>$\varepsilon_{\text{num}}$</th>
<th>$\varepsilon_{\text{exp}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Max. load (kN)</td>
<td>Max. def. (mm)</td>
<td>Max. load (kN)</td>
<td>Max. def. (mm)</td>
<td>Max. load (kN)</td>
</tr>
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<td>Adhikary and</td>
<td>B-1</td>
<td>80</td>
<td>6.2</td>
<td>82</td>
<td>6.3</td>
<td>82</td>
</tr>
<tr>
<td>2</td>
<td>Mutsyoshi (2004)</td>
<td>B-8</td>
<td>172</td>
<td>16.0</td>
<td>180</td>
<td>15.4</td>
<td>168</td>
</tr>
<tr>
<td>3</td>
<td>Pellegrino and</td>
<td>TR30D1</td>
<td>322</td>
<td>9.8</td>
<td>323</td>
<td>9.6</td>
<td>323</td>
</tr>
<tr>
<td>4</td>
<td>Modena (2002)</td>
<td>TR30D3</td>
<td>323</td>
<td>14.5</td>
<td>336</td>
<td>13.1</td>
<td>303</td>
</tr>
<tr>
<td>5</td>
<td>Chaallal</td>
<td>US</td>
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<td>3.0</td>
<td>119</td>
<td>2.9</td>
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</tr>
<tr>
<td>6</td>
<td>et al. (1998)</td>
<td>RS90</td>
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<td>4.0</td>
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<td>RS135</td>
<td>194</td>
<td>6.0</td>
<td>202</td>
<td>5.9</td>
<td>173</td>
</tr>
</tbody>
</table>

*Deflection measured at maximum load.*

Table 4.2: Comparisons between the experimental and numerical failure modes

| No. | Beam set         | Spec. | Failure modes       | Experimental | Numerical | |
|-----|------------------|-------|---------------------|--------------|----------||
| 1   | Adhikary and     | B-1   | Concrete crushing  | Concrete crushing |         |         |
| 2   | Mutsyoshi (2004)| B-8   | Debonding           | Debonding    |          |         |
| 3   | Pellegrino and  | TR30D1| Concrete crushing  | Concrete crushing |         |         |
| 4   | Modena (2002)   | TR30D3| Debonding           | Debonding    |          |         |
| 5   | Khalifa and     | TR30D4| debonding           | debonding    |          |         |
| 6   |                  | TR30D2| debonding           | debonding    |          |         |
| 7   |                  | BT1   | diagonal shear      | diagonal shear |         |         |
| 8   |                  | BT2   | debonding           | debonding    |          |         |
| 9   |                  | BT3   | debonding           | debonding    |          |         |
| 10  |                  | BT4   | debonding           | debonding    |          |         |
| 11  |                  | BT5   | debonding           | debonding    |          |         |
| 12  |                  | BT6   | flexural failure    | concrete crushing |         |         |
| 13  | Chaallal        | US    | Concrete crushing  | Concrete crushing |         |         |
| 14  | et al. (1998)   | RS90  | Debonding           | Debonding    |          |         |
| 15  |                  | RS135 | Debonding           | Debonding    |          |         |
4.2. COMPARISON BETWEEN SHELL AND TRUSS MODELLING OF FRP COMPOSITES

4.2.2 Axial Strain Profiles along the FRP Composites

Based on the conclusion that the proposed numerical model successfully predicted the response of the experimental beams in terms of the load–deflection relationships and ultimate capacities, the numerical model is considered to give realistic descriptions for the distributions of the strains in the bonded FRPs with both truss and shell modelling of FRP composites.

Strain profiles along the FRP sheet depth at different load levels for the single-ply FRP side-bonded configuration (TR30D3) of Pellegrino and Modena [2002] are presented in Figure 4.8 and Figure 4.9 for the truss and shell modelling of the FRP composites, respectively. These figures are for the section at the middle of the shear span. Generally, both modelling approaches showed similar axial strain profiles along the FRP sheet depth. The predicted strain values along the FRP sheet depth for the 115 kN load level show small amounts of strain. These low strain values appear at the initial cracking stage of the concrete. With the increase of crack width and the successive transfer of stresses from the concrete to the FRP sheet, higher strain values are obtained. The maximum strain value along the FRP sheet depth occurs around the mid-depth.
Figure 4.6: Applied load–central deflection relationships for Ref., BT2 and BT3 beams of Khalifa and Nanni [2000]

Figure 4.7: Applied load–central deflection relationships for BT4, BT5 and BT6 beams of Khalifa and Nanni [2000]
4.2. COMPARISON BETWEEN SHELL AND TRUSS MODELLING OF FRP COMPOSITES

Figure 4.8: Axial strain distribution of the FRP along the sheet depth for specimen TR30D3 of Pellegrino and Modena [2002] with truss elements

Figure 4.9: Axial strain distribution of the FRP along the sheet depth for specimen TR30D3 of Pellegrino and Modena [2002] with shell elements
A comparison between the axial strain profiles along the sheet depth of the two modelling approaches of FRPs (shell and truss) for the single-ply strengthened beam plotted in Figure 4.8 and Figure 4.9 show that the modelling approach of FRPs has a significant influence on the axial strain profiles. For example, the predicted axial strain values for the truss elements modelling approach is higher than those with shell elements. This results in a dependence of the width of the bonded FRPs.

The influence of the various simulation techniques of the FRP strips to the load-deflection response showed that modelling FRPs with shell elements gives the best performance. The author suggests that an increase of the number of truss elements to represent the behaviour of FRP composites might lead to better results.

### 4.3 Comparison Between Various Interface Elements

The objective of this comparison is to identify the appropriate interface elements to predict the FRP/concrete interfacial behaviour accurately. Three interface elements are utilized to represent such behaviour: spring elements, discrete truss elements and continuous truss elements. The comparison between the various types of interface elements is discussed in terms of load-deflection relationships, and the interfacial slip profiles. For the detailed results, the experimental results by Pellegrino and Modena [2002] are selected for the sake of comparison with the finite element predictions. In the current phase of the investigation, the specimen having internal steel stirrups and strengthened with a single side-bonded continuous CFRP sheet (TR30D3) is considered. Based on the conclusion that the numerical model with the shell simulation of the FRP composites can successfully predict the response of the experimental beams in terms of ultimate capacities and load-deflection relationships, the FRP composites in the following analysis phases are simulated by shell elements.

#### 4.3.1 Load–Deflection Relationships

Figure 4.10 compares the experimental load–deflection behaviour of the selected beam with models using interface elements. From the figure, it can be observed that the responses of
4.3. COMPARISON BETWEEN VARIOUS INTERFACE ELEMENTS

![Graph showing applied load vs central deflection relationships for specimen TR30D3 with various interface elements.](image)

**Figure 4.10:** Applied load–central deflection relationships for specimen TR30D3 [Pellegrino and Modena, 2002] with various interface elements

The numerical predictions are comparable to the experimental results at low load levels. At higher load levels, the numerical models exhibited stiffer responses compared to the experiments. For each interface element type, the predicted load carrying capacities are fairly similar to each other. The numerically predicted ultimate load for beams modelled with spring elements is 335 kN, with discrete truss elements is 336 kN and with continuous truss elements is 338 kN. These values are within 4%, 4% and 5%, respectively, over the experimental results. Consequently, the predicted maximum deflection value is virtually identical for the three types of interfacial elements. In conclusion, it can be stated that the type of interface element has no influence on the global structural behaviour.

### 4.3.2 Slip Profiles along the FRP/Concrete Interface

The slip profiles along the sheet depth with different interface elements models are also investigated to determine the appropriate interface element. These quantities are valuable and difficult to measure directly in the laboratory. The sections where the slip profiles obtained are described in Figure 4.11.

The slip profiles obtained with spring elements are shown in Figure 4.12(a)–(c), which describe the slip values at different locations corresponding to 150 mm, 400 mm and 600 mm from the point of applied load, respectively. The shear span of the beam considered is 750 mm. On each figure, the slip values are illustrated at various load values up to
failure. Due to the small differences between the maximum and minimum slip values at the 115 kN load level, the interfacial slip values appear as a straight line through the FRP laminate depth. With a load increase, the slip values presented are significantly higher at the bottom edge of the beam and low towards the top end. This indicates that, if debonding of the FRP sheet were to occur, it would first be observed at the bottom edge of the specimen, at all locations. Also, the slip values increased beyond the maximum slip value and the failure is characterized either by crushing of concrete or rupture of the FRPs. In fact, there is no difference between the general responses of slip profiles in all locations along the shear span. Also, the failure mode reported experimentally was debonding of the CFRP sheets, while concrete crushing failure was obtained numerically. Therefore, spring interface elements are apparently not appropriate to represent the FRP/concrete interfacial behaviour.

For the discrete truss interface elements, the slip profiles along the sheet depth are plotted in Figure 4.13(a)–(c) at the same locations, along the beam tested by Pellegrino and Modena [2002], as for the spring elements. With this type of interface element, the presence of a shear crack may be identified by fluctuating from negative to positive values in the slip profiles. The slip profiles dramatically increase at the regions near the vicinity of a shear crack. Furthermore, the locations of the maximum slip values correspond to
4.3. COMPARISON BETWEEN VARIOUS INTERFACE ELEMENTS

Figure 4.12: Interfacial slip profiles along the FRP sheet depth at different locations for specimen TR30D3 [Pellegrino and Modena, 2002] for spring interface elements.
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Figure 4.13: Interfacial slip profiles along the FRP sheet depth at different locations for specimen TR30D3 [Pellegrino and Modena, 2002] for discrete truss interface elements

the shear crack intersections with the FRP composites. For the slip profiles drawn at a distance 150 mm from the load, shown in Figure 4.13a, the maximum slip value is obtained near the top end of the sheet. For the slip profiles at a distance 400 mm from the load, as shown in Figure 4.13b, the maximum slip value is found around the mid-depth of the beam. This is attributed to a local effect caused by a shear crack. For the slip profiles at a distance 600 mm from the load plotted in Figure 4.13c, the maximum slip value is found near the bottom end of the sheet. Generally, shear cracks are expected to propagate as the load increases and the failure occurs due to debonding of FRP sheets over the main diagonal shear crack. The numerical model is able to capture the failure process identical to that observed experimentally.

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4.3. COMPARISON BETWEEN VARIOUS INTERFACE ELEMENTS

Figure 4.14: Interfacial slip profiles along the FRP sheet depth at different locations for specimen TR30D3 [Pellegrino and Modena, 2002] for continuous truss interface elements

For the strengthened beam modelled with continuous truss interface elements, Figure 4.14a–Figure 4.14c show the interfacial behaviour along the sheet depth at the same locations as for the previous types of interface elements. The prediction of the interfacial slip behaviour is very similar to that with the discrete truss interface elements. However, the continuous truss elements are not able to characterize the failure process as smoothly as that observed experimentally. This is attributed to the redundant elements found in the continuous interface truss elements representing the FRP/concrete interfacial behaviour.
4.3.3 Bond–Slip Model

The above comparison between the various types of interface element is extended to study the trend of shear stress versus slip. The investigation is conducted for an interface element corresponding to the location that exhibited debonding failure. This element is taken at the bottom edge near the loading point for the spring elements, while it is selected to be at the top edge next to the point of applied load for the discrete and continuous truss elements. For the strengthened specimen TR30D3 of Pellegrino and Modena [2002], the following values are calculated for the shear stress-slip bilinear curve using [Lu et al., 2005] model: $s_0 = 0.05$, $s_{max} = 0.2$ and $T_{max} = 4MPa$. It should be noted that the total amount of interfacial slip within the interface element should be equal to or less than the maximum slip value. Therefore, debonding is deemed to have occurred if the total slip extends beyond the maximum slip value $s_{max}$.

Figure 4.15(a)–(c) show the shear stress–slip relations for the spring elements, discrete truss elements and continuous truss elements, associated with the provided shear stress–slip curve, respectively. From Figure 4.15(a), the slip of the spring elements reached the maximum slip value and increased progressively with zero shear stress. This process is continued until crushing of the concrete. The continuous increase of slip with zero shear stress specifies that at a certain load level no further load is transferred to the FRP sheets without indication of delamination. Furthermore, based on this conclusion, it is obvious that, with this type of interface element, no debonding failure is obtained in the element determined before. The impossibility for the spring element to capture the debonding failure mode is attributed to the deficiency of these elements in the software.

The shear stress–slip curves for the discrete and continuous truss elements are given in Figure 4.15(b) and (c), respectively. The results obtained herein exhibited similar trends to the original bilinear curve. It can be observed from Figure 4.15(a) and Figure 4.15(c) that the shear stress increases linearly up to the maximum shear stress, $T_{max}$, beyond that the shear stresses level off progressively up to debonding of the sheets. Additionally, for both types of interface elements, the debonding occurred beyond the peak value of shear stress and during the descending part of the curve. From the graphs, the interfacial slip response of the discrete truss interface elements and continuous truss elements follows the original bond stress–slip curve up to debonding. These observations explain that the response
4.3. COMPARISON BETWEEN VARIOUS INTERFACE ELEMENTS

Figure 4.15: Comparison of shear stress–slip curves for the various interface elements for specimen TR30D3 [Pellegrino and Modena, 2002]
CHAPTER 4. VALIDATION OF NUMERICAL RESULTS

of the interfacial behaviour with discrete truss elements is smoother than that with the continuous truss elements. This is attributed to the redundant elements associated with the continuous truss elements representing the FRP/concrete interfacial behaviour.

In summary, the finite element model that best captured the interfacial FRP/concrete response was that using discrete truss elements. The numerical predictions are less successful when spring elements are implemented.

4.4 Influence of Horizontal Interface Elements

In Figure 4.16, the numerical results corresponding to the load–deflection relation for the various ranges of horizontal interface elements are compared with the curve with only vertical interface elements. The experimental results are shown for reference. It can be observed that the load–deflection behaviour with bidirectional interface elements exhibited behaviour comparable to that for the beam without horizontal interface elements. Debonding of the vertical interface elements governed the failure criteria. In light of the above findings, it can be concluded that horizontal interface elements are not necessary to be included when modelling the behaviour of shear-strengthened beams. This is due to the fact that the horizontal components of the principal diagonal-tension stresses are low and resisted by concrete horizontal stresses. Moreover, this can be attributed to the direction of the fibres that most influence the transfer of stresses from the concrete to the FRP sheets.

4.5 Results of Finite Element Discretization

Apparently no similar convergence studies have been carried out for the case of bonded FRPs attached to the concrete surface. This is because the previous studies considered only the reference beam (i.e., without FRP strengthening). In Figure 4.17, the numerical predictions of the load–deflection relationship are compared for each of the three element sizes associated with the experimental results. The element sizes along the beam depth are 40 mm, 20 mm or 10 mm. The experimental ultimate load carrying capacity was 184 kN.
4.5. RESULTS OF FINITE ELEMENT DISCRETIZATION

The model successfully estimated the load carrying capacity when using a 40 mm element size (197 kN); 7% higher than the test value, as well as implementing a 20 mm element size (192 kN); 4% higher than the test result). In addition, the predicted maximum load was 190 kN (3% higher than the experimental value) for a 10 mm element size. The influence of the various element sizes along the sheet depth to the load-deflection response showed that there is no significant difference between the 20 mm and 10 mm element size. A convergence of results is obtained when using a 20 mm element size. Furthermore, in order to reduce the computational efforts, modelling with a 20 mm element size along the beam depth gives a good performance.

Another investigation for the mesh size can be performed by using the results of the slip profiles along the sheet depth. Figure 4.18(a)–(c) illustrate the interfacial slip profiles for a 40 mm, 20 mm and 10 mm element size in the vertical direction, respectively. The slip profiles are only plotted at the centre of the shear span. It is observed that small discrepancies between the locations of the ultimate slip values occur between the 40 mm and 20 mm element size. The maximum slip value for the 40 mm element size is obtained at 125 mm from the bottom edge, while it is at 137.5 mm for the 20 mm element size. The locations of the maximum slip values along the beam depth with the 20 mm and 10 mm element size are identical. Thus, it can be stated that the decrease of element size along the sheet depth did not yield appreciable differences in the numerical results.

Figure 4.16: Applied load–central deflection relationships for specimen TR30D3 [Pellegrino and Modena, 2002] with consideration of horizontal interface elements

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Figure 4.17: Applied load–central deflection relationships for specimen RS90 [Chaallal et al., 1998b] for different mesh sizes

Figure 4.18: Interfacial slip profiles along the FRP sheet depth for specimen RS90 [Chaallal et al., 1998b] for different mesh sizes
4.6 Comparison between DIANA and ADINA Results

The shear capacities of the four T-beams tested by Lee and Al-Mahaidi [2008] along with the results obtained from their finite element models are tabulated in Table 4.3. Failure was characterized experimentally by the formation of large shear cracks and separation of the CFRP strips, as shown in Table 4.4. For the control beam, as illustrated in Figure 4.19, both finite element packages (DIANA and ADINA) showed satisfactory agreement with the experimental curves. At early stages of loading, the numerical models exhibited a slightly higher stiffness compared to the experimental beam. The DIANA finite element package showed 10% and 28% lower results for the ultimate loading capacity and maximum central deflection, respectively. The numerical predictions of the ADINA finite element package are comparable to the experimental data. The ADINA model predicts a load carrying capacity that corresponds to 4% greater than the experimental data, and the corresponding mid-span deflection is lower than the test results by 7%.

<table>
<thead>
<tr>
<th>No.</th>
<th>Spec.</th>
<th>Concrete $f_c$ (MPa)</th>
<th>Experimental results</th>
<th>DIANA results</th>
<th>ADINA results</th>
<th>DIANA P_exp.</th>
<th>ADINA P_exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Max. load (kN)</td>
<td>Max. def. (mm)</td>
<td>Max. load (kN)</td>
<td>Max. def. (mm)</td>
<td>P_num.</td>
</tr>
<tr>
<td>1</td>
<td>Control</td>
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<td>495</td>
<td>14</td>
<td>440</td>
<td>10</td>
<td>515</td>
</tr>
<tr>
<td>2</td>
<td>0.75D</td>
<td>31.1</td>
<td>762</td>
<td>21</td>
<td>840</td>
<td>21</td>
<td>804</td>
</tr>
<tr>
<td>3</td>
<td>0.6D</td>
<td>30.9</td>
<td>797</td>
<td>20</td>
<td>850</td>
<td>20</td>
<td>816</td>
</tr>
<tr>
<td>4</td>
<td>0.5D</td>
<td>31.6</td>
<td>892</td>
<td>24</td>
<td>900</td>
<td>20</td>
<td>940</td>
</tr>
</tbody>
</table>

Table 4.4: Comparison between experimental and numerical failure modes

<table>
<thead>
<tr>
<th>No.</th>
<th>Spec.</th>
<th>Experimental</th>
<th>DIANA</th>
<th>ADINA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Control</td>
<td>Denoding</td>
<td>Concrete crushing</td>
<td>Debonding</td>
</tr>
<tr>
<td>2</td>
<td>0.75D</td>
<td>Denoding</td>
<td>Concrete crushing</td>
<td>Debonding</td>
</tr>
<tr>
<td>3</td>
<td>0.6D</td>
<td>Denoding</td>
<td>Concrete crushing</td>
<td>Debonding</td>
</tr>
<tr>
<td>4</td>
<td>0.5D</td>
<td>Denoding</td>
<td>Concrete crushing</td>
<td>Debonding</td>
</tr>
</tbody>
</table>

For specimen 0.75D, the comparison between the experimental and numerical results of the applied load–central deflection curves are plotted in Figure 4.19(b). From the
figure, it can be observed that the responses of the numerical models are similar to their respective experimental curve. At higher load levels, the numerical models enhance the stiffness response compared to the experiments. The DIANA model overestimated the failure load and the corresponding mid-span deflection by 10%. Using the ADINA model, the ultimate load and the central deflection are higher than the experimental values by 6% and 5%, respectively. Figure 4.20 and Figure 4.21 compare the applied load–central deflection relationships with the both numerical packages to the experimental results.

In summary, we can conclude that the DIANA model carried out by Lee [2003] could not capture the failure modes that were reported experimentally. In fact, the failure mode reported experimentally was debonding of the CFRP strips, while shear compression failure was obtained with DIANA model. By contrast, the ADINA model is able to predict typical failures as what were observed in the experiments. The difference between the failure modes predicted by the two numerical models is attributed to the shear stress–bond slip model used.
Figure 4.20: Applied load–central deflection relationships for 0.6D specimen of [Lee, 2003]

Figure 4.21: Applied load–central deflection relationships for 0.5D specimen of [Lee, 2003]
CHAPTER 4. VALIDATION OF NUMERICAL RESULTS

4.7 Summary

In this study, a general methodology for the modelling of reinforced beams with web-bonded FRP composites was presented. A key feature of the numerical modelling is the implementation of nonlinear transitional link elements to simulate the interfacial behaviour between the externally bonded FRPs and the concrete. The finite element model utilized to analyze the experimental beams proved to be capable of properly modelling the general trends and behaviour, and with an excellent accuracy. An investigation of varying approaches to simulate the behaviour of FRP composites was carried out. By comparing the measured and computed load-deflection responses of shear-strengthened beams, it has been shown that the numerical model can effectively simulate the global response when shell elements are employed for the FRP composites.

The simulation of the FRP/concrete interfacial behaviour is affected by the type of interface element used to represent this behaviour. While the global behaviour was well predicted, meaning accurate load-deflection behaviour was achieved, the nonlinear spring interface elements were unable to accurately capture the debonding failure mode. It would be desirable to accurately predict the interfacial slip response and determine the regions of severe cracking with implementing discrete truss elements. Considering such elements, a consistency was observed between the shear crack region and the maximum values of the interfacial slips; thus indicating that the delamination occurs over the main diagonal shear crack.

By conducting a numerical investigation to examine the effect of including horizontal interface elements in the numerical model, it was found that including horizontal interface elements is not essential. The discretization study conducted here is apparently a pioneering study on shear-strengthened beams. The study considered the increase of the number of elements along the depth of the FRP sheet while keeping the mesh size in the other dimensions identical. It can be stated that the decrease of element size along the sheet depth did not yield appreciable differences in the numerical results.

Numerical models have been conducted using the ADINA and DIANA software to simulate the behaviour of the shear-strengthened beams tested by Lee and Al-Mahaidi [2008]. In summary, we can conclude that the DIANA model carried out by Lee [2003]
could not capture failure modes similar to those reported experimentally. In fact, the failure mode reported experimentally was debonding of the CFRP strips, while shear compression failure was predicted. By contrast, the ADINA model was able to predict the typical failure modes observed in the experiments.
Chapter 5

Size Effects for RC Beams Strengthened with FRP Composites

5.1 Introduction

The behaviour of reinforced concrete members strengthened in shear with external FRP stirrups is influenced by several parameters. The size of the beam is undoubtedly one of the most important parameters affecting the ultimate capacity of a shear-strengthened beam. However, few researchers have investigated this effect, particularly for specimens that have an effective depth greater than 300 mm [Bousselham and Chaallal, 2004a]. In fact, the importance of this parameter has emerged from the relation established between the required effective bond length and the effective FRP axial strain [Deniaud and Cheng, 2001b; Khalifa and Nanni, 2002]. Furthermore, the size has only effect on the shear strength of reinforced concrete beams [ASCE, 1998; Kong and Evans, 1990]. On the other hand, from a review of the published experimental and numerical results, it appears that to date there has been no extensive evaluation of the distribution of axial strains in the FRP composites along their depths. The principal motivation of the experimental program of the present study is for a better understanding of the interaction between the beam size and the axial strains in the FRP sheets, and the corresponding shear capacity.

In this study, various sizes of CFRP shear-strengthened beams are considered. This investigation includes an experimental program, carried out within a collaboration ef-
5.2 EXPERIMENTAL PROGRAM

The experimental program was carried out at the Department of Civil Engineering of Tsinghua University, China [Qu et al., 2005]. It involved three series of rectangular reinforced concrete beams with the configurations shown in Figure 5.1. Each series included a reference beam and a CFRP U-jacket strengthened beam. The third series includes an additional beam strengthened with completely enclosed CFRP jackets. The reference beams are labelled as RC, followed by a number that denotes the series. The notations U4, U5 and U6 refer to the U-jacket strengthened beams for the first, second and third series, respectively, while W7 identifies the only specimen with CFRP completely wrapped of the third series.

In order to verify the size effect of the shear-strengthened beams, the span \( L \) and the width \( b \) were increased proportionally with the height \( h \) of the beams. In addition, the width \( w_f \), the spacing \( s_f \) and the thickness \( t_f \) were also increased proportionally with the height \( h_f \) of the CFRP U-jackets. The dimensions of the specimens along with the amount of longitudinal steel for each beam are summarized in Table 5.1. In this table, \( a \) and \( a_0 \) indicated the outer and the inner shear span, as illustrated in Figure 5.1. The nominal shear span-to-depth ratio (\( = a/d \)) was 2.0 for all the beams; it is defined as the distance from the loading point to the centre of the support. However, due to the relative large stiffness of the loading plates at the loading point and supports, it is believed that the real shear span-to-depth ratio (\( = a_0/d \)) is 1.51.
CHAPTER 5. SIZE EFFECTS FOR RC BEAMS STRENGTHENED WITH FRP COMPOSITES

Figure 5.1: Specimens configurations details

Table 5.1: Geometrical dimensions of tested specimens

<table>
<thead>
<tr>
<th>No.</th>
<th>Beams sets</th>
<th>Spec.</th>
<th>Beam dimensions (mm)</th>
<th>Beam dimensions (mm)</th>
<th>Number of long. steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>L</td>
<td>b</td>
<td>h</td>
</tr>
<tr>
<td>1</td>
<td>First set</td>
<td>RC1</td>
<td>900</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>U4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Second set</td>
<td>RC2</td>
<td>1800</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>U5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Third set</td>
<td>RC3</td>
<td>2700</td>
<td>300</td>
<td>600</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>U6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>W7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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5.2. EXPERIMENTAL PROGRAM

All beams were cast in a plywood mould from a single batch of concrete and were cured under the same condition for six days before the mould was stripped. The rectangular cross sections were then smoothed at the bottom edges to avoid any stress concentrations in the FRP wraps at these locations. The maximum aggregate size was 25 mm. The ages at which the beams were tested varied from 40 days to 90 days. The concrete compressive strengths $f'_c$ of all the specimens, together with their tensile strengths $f_t$, and the ultimate properties of the CFRP sheets are listed in Table 5.2. All specimens had the same flexural reinforcement ratio ($A_s/bd$). Furthermore, all beams were heavily reinforced in bending to ensure that they would fail in shear. The flexural reinforcement bars were cold drawn deformed steel with yield strength of 400 MPa. No steel stirrups were installed in the shear span of interest (right shear span); however, sufficient steel stirrups were placed in the other span (left shear span) to ensure that the failure would occur in the shear span of interest.

Before bonding the composite material to the concrete surface, special care was given to the surface preparation. Sandblasting was used to roughen the concrete surface until the aggregates were exposed. This was followed by compressed air cleaning to remove dust and loose particles. Once the surface was prepared to the required standard, the epoxy resin was mixed in accordance with the manufacturer's instructions. The epoxy resin was set to the concrete surface. Then, the FRP strips were placed against the epoxy resin coating and the resin was squeezed through the roving of the strips with a plastic roller. Large entrapped air bubbles at the epoxy/concrete or epoxy/FRP strips was avoided. During the hardening of the epoxy, a constant uniform pressure (about 0.22N/mm$^2$ was

<table>
<thead>
<tr>
<th>No.</th>
<th>Beam set</th>
<th>Specimen</th>
<th>Concrete (MPa)</th>
<th>Composites (MPa)</th>
<th>Ultimate stress</th>
<th>Ultimate strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>First</td>
<td>RC1</td>
<td>51.2</td>
<td>2.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>set</td>
<td>U4</td>
<td>51.2</td>
<td>2.55</td>
<td>3550</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>Second</td>
<td>RC2</td>
<td>49.7</td>
<td>2.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>set</td>
<td>U5</td>
<td>51.2</td>
<td>2.93</td>
<td>3550</td>
<td>0.15</td>
</tr>
<tr>
<td>5</td>
<td>Third</td>
<td>RC3</td>
<td>50.5</td>
<td>3.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>set</td>
<td>U6</td>
<td>51.0</td>
<td>3.16</td>
<td>3550</td>
<td>0.15</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>W7</td>
<td>50.7</td>
<td>3.16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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applied on the FRP surface to ensure a good contact. The operation was carried out at room temperature.

The strengthening material was a unidirectional carbon fibre sheet. It was applied in evenly spaced U-wrap strips. The number of CFRP layers were varied from a single ply \((t_f = 0.111 \text{ mm})\) in the first series, to double plies \((t_f = 0.222 \text{ mm})\) in the second series, to three layers \((t_f = 0.333 \text{ mm})\) in the third series. All strengthened beams had the same CFRP reinforced ratio \((\rho_f = A_f/bd)\). The CFRP sheets were applied perpendicular to the beam axis. The geometrical dimensions of the CFRP sheets are listed in Table 5.3. For the U-wrap strengthened beams (U4, U5 and U6), two strengthening schemes were used for each beam specimen. The shear span of interest (right shear span) was strengthened with CFRP strips in the form of U-shape strips, while completely wrapped CFRP strips were applied on the other span (left shear span). This was done to induce failure at the shear span of interest. For the additional beam specimen of the third series (W7), it was strengthened with completely wrapped CFRP strips on both shear spans.

### Table 5.3: CFRP shear-strengthening dimensions and configuration

<table>
<thead>
<tr>
<th>No.</th>
<th>Spec.</th>
<th>Composites dimensions</th>
<th>Strength. type</th>
<th>Strength. details</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>U4</td>
<td>0.111 30 20</td>
<td>U-wrap</td>
<td>Vertical strips</td>
</tr>
<tr>
<td>2</td>
<td>U5</td>
<td>0.222 60 40</td>
<td>U-wrap</td>
<td>Vertical strips</td>
</tr>
<tr>
<td>3</td>
<td>U6</td>
<td>0.333 90 60</td>
<td>U-wrap</td>
<td>Vertical strips</td>
</tr>
<tr>
<td>4</td>
<td>W7</td>
<td>0.333 90 60</td>
<td>Completely wrapped</td>
<td>Vertical strips</td>
</tr>
</tbody>
</table>

All beams were simply supported and subjected to a monotonic static loading applied at the centre of the beam. Vertical displacements were measured at the centre of the span using linear variable differential transformers (LVDTs). The strains in the CFRP strips were also measured, on the vertical face of the strengthened specimens, in order to estimate the shear strength contribution of each CFRP stirrup. Strain gauges were located based on the final crack pattern observed in the control specimen. As illustrated in Figure 5.2, the CFRP strips were numbered sequentially along the tested shear span beginning from the support.
5.3 Numerical Analysis

The proposed three-dimensional finite element program, ADINA [2004a], was used for the numerical analysis of the seven tested beams. In the analysis, appropriate material models were employed to represent the behaviour of the concrete, the steel reinforcement and the CFRP sheets. These are described in detail in the ADINA theory and modelling guide ADINA [2004b]. In addition, to model the bond behaviour at the FRP/concrete interfacial behaviour, discrete truss elements that properly represent the local bond stress slip characteristics and failure were assumed. The details of the constitutive models and their implementation into the numerical analysis are described in detail in Chapter 3.

The nonlinear load deformation behaviour of the beams was simulated under displacement-controlled loading conditions, as was the case for the laboratory experiments. In view of the geometrical and loading symmetries, only one quarter of the beam is simulated. Additionally, using the dimensions of the actual strengthened beam of the first series (U4), a precision study was performed. The goal of this study is to ensure that the one-quarter model of the beam did not introduce excessive approximation in the simulations. The complete CFRP wrapping scheme installed in the left shear span was simulated by three different approaches. In the first approach, CFRP sheets completely wrapped the section was carried out. U-wrap strengthening scheme was assumed in the second approach, while the side-bonded scheme was used in the third approach. For these three approaches, the CFRP sheets on the left shear span were fully connected to the concrete. However, the U-jacket strengthening scheme and the interface elements lying in the right shear span (shear span of interest) were kept the same for each of these approaches. The accuracy of the numerical model was evaluated by comparing the numerical predictions to experimental
results.

5.4 Experimental and Numerical Results

The results presented in the following sections are in terms of ultimate load carrying capacities, load–deflection relationships, and failure modes. Comparisons are also provided in terms of the CFRP axial strain profiles along the sheet depth. Once the accuracy of the numerical analysis was established, numerical studies were carried out to investigate the interfacial slip profiles and shear crack angles.

5.4.1 Ultimate Load Carrying Capacities and Failure Modes

For the beams of the first series, the smallest among the three series with a span of 900 mm and a depth of 200 mm, the flexural cracks were observed experimentally in the control specimen near the mid-span at the bottom of the beam, at a load level of 60 kN. Then shear cracks began to appear at a load of approximately 75 kN. The main shear crack was located close to the mid-depth and it extended towards the bottom and the top edge of the beam. As the load increased, additional shear cracks formed, widened and propagated up to the final failure at a load level of 159 kN. The mode of failure was shear crushing of the concrete under the concentrated load. This agrees with the findings by Pellegrino and Modena (2002), which state that RC beams with transverse steel fail with a diagonal cracked area, whereas those without web reinforcement fail with one principal diagonal crack.

The failure progress of the control beams at different load levels are summarized in Table 5.4. This table shows the loads, and their values as a percentage to the failure loads, at the stages of flexural cracking, shear cracking, and failure. In terms of these percentages, we observe that the beams RC1 and RC2 had similar values at the stage of flexural cracking. For the control beam of the third series (RC3), the flexural cracks occurred at a lower load percentage than the previous two beams. Similarly, the shear cracks formed at approximately the same load percentage for the specimens RC1 and RC2, but at a significantly lower level for the control beam of the third series. In summary, the
5.4. EXPERIMENTAL AND NUMERICAL RESULTS

general trend is that, as the beam size decreases, the ratio of the load at shear cracking to that at failure increases.

Table 5.4: Failure progress of the control specimens at different load levels

<table>
<thead>
<tr>
<th>No.</th>
<th>Spec.</th>
<th>Applied load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Flexural cracks</td>
</tr>
<tr>
<td>1</td>
<td>RC1</td>
<td>60 kN (38%)</td>
</tr>
<tr>
<td>2</td>
<td>RC2</td>
<td>280 kN (40%)</td>
</tr>
<tr>
<td>3</td>
<td>RC3</td>
<td>433 kN (27%)</td>
</tr>
</tbody>
</table>

With regard to the ultimate load carrying capacities of the three control specimens, we observe that the failure load of the control beam of the second series (RC2) was 346% higher than that of the first series, while the ultimate capacity of the control beam of the third series occurred at loads 923% and 129% higher than those of the first and second series, respectively. The failure modes of the three control beams were similar.

The crack patterns at different load levels for the three control beams are illustrated in Figure 5.3. For the control beams RC1 and RC2, prior to the failure, one major shear crack formed in the web of the specimens. The shear cracks propagated from the mid-depth of the beam towards the point of the applied load and the support. In the specimen RC1, minor shear cracks formed close to the support. For the specimen RC3, two major shear cracks were observed in the web prior to failure. The locations at which the major shear cracks formed were similar to those reported for the two previous beams. As the load increased, minor shear cracks occurred along the major shear cracks and support. In fact, more distributed shear cracks were seen in the control beam of the third series (RC3) compared to what was observed for the other control specimens. This can be attributed to the depth increase, which results in a relative decrease of the aggregate interlock.

For the strengthened specimen of the first series, U4, it was not possible to observe cracks on the sides of the beam because of the presence of the bonded CFRP sheets. However, during loading a clicking sound occasionally emitted from the beam. The sound increased in frequency as the beam was loaded closer to the maximum load bearing ca-
Other than this, no significant warning signals preceded the sudden failure of the specimens. The governing failure mode was delamination of the CFRP sheets from the sides of the specimens. The debonding initiated at the strip closest to the applied load and propagated to the support while peeling off a thin layer of the concrete. After the debonding of the first three strips nearest to the loading point, the splitting failure propagated towards the support as the load descended progressively until crushing of the concrete. Opening of a shear crack along the depth of the beam induces tension in the CFRP strips bridging the crack. The resistance forces in these strips tend to decrease the crack opening, making it more difficult for the shear crack to grow. The shear capacity of the member is hence improved. The maximum recorded load level was 202 kN. This represents an increase of 28% in the ultimate capacity over that of the control specimen. Table 5.5 shows the progress of the debonding failure mode in the bonded strips with the applied load for the first three strips from the point of applied load. It can be observed from this table that the loading percentages relative to the failure load for specimen U4 (strengthened specimen of the first series) at the various stages of failure are higher than those of the other strengthened specimens.

In Table 5.6 comparisons between the experimental results and numerical predictions
5.4. EXPERIMENTAL AND NUMERICAL RESULTS

Table 5.5: Failure progress of the strengthened specimens at different load levels

<table>
<thead>
<tr>
<th>No.</th>
<th>Spec.</th>
<th>Applied load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>U4</td>
<td>123 kN (61%)</td>
</tr>
<tr>
<td>2</td>
<td>U5</td>
<td>420 kN (52%)</td>
</tr>
<tr>
<td>3</td>
<td>U6</td>
<td>1080 kN (54%)</td>
</tr>
<tr>
<td>4</td>
<td>W7</td>
<td>840 kN (38%)</td>
</tr>
</tbody>
</table>

are presented. These show that the numerical model is quite capable of predicting the ultimate load carrying capacities with a high degree of accuracy. Furthermore, the analysis is able to properly simulate the failure modes that were observed experimentally. For the control specimen of the first series (RCl), the predicted ultimate carrying capacity is 4% greater than the test value. The simulated failure mode is identical to the experimental observation. For the strengthened beam of the same series (U4), the numerical prediction of the ultimate load carrying capacity is 105% of the experimental value; in addition, the proposed model successfully simulates the debonding failure of the CFRP U-strips which was the dominant failure mode. At the stage of significant stress transfer from the concrete to the bonded CFRP sheets, the predicted debonding initiated at the interface elements at the top edge of the strip beside the load. After removing the debonded interface elements, the analyses were continued and the load increased. As the load was further increased, the interface elements of the second strip delaminated at a few centimeters below the top edge of the CFRP sheet. Delamination then continued towards the top edge. The position of the first delamination of the interface elements corresponds to the experimental observations. The delamination progressed until the crushing of the concrete. The maximum load occurred after a few strips had already failed, which is identical to the experimental observations.
Table 5.6: Comparison between experimental and numerical results

<table>
<thead>
<tr>
<th>No.</th>
<th>Beams sets Spec.</th>
<th>Experimental results</th>
<th>Numerical results</th>
<th>Failure modes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Max. load (kN)</td>
<td>Max. def. (mm)</td>
<td>Pexp,</td>
</tr>
<tr>
<td>1</td>
<td>First set RC1 U4</td>
<td>160</td>
<td>1.9</td>
<td>166</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>203</td>
<td>2.2</td>
<td>213</td>
</tr>
<tr>
<td>3</td>
<td>Second set RC2 U5</td>
<td>709</td>
<td>3.6</td>
<td>745</td>
</tr>
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<td>809</td>
<td>4.2</td>
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<tr>
<td>5</td>
<td>Third set U6 W7</td>
<td>1626</td>
<td>6.6</td>
<td>1659</td>
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<td>6</td>
<td></td>
<td>2018</td>
<td>6.5</td>
<td>2053</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>2221</td>
<td>8.4</td>
<td>2203</td>
</tr>
</tbody>
</table>

Failure modes: Experimental | Numerical
- concrete crushing
- debonding
- CFRP rupture

*a deflection measured at maximum load.

5.4.2 Load–Deflection Relationships

Comparisons between the numerical and experimental results in terms of load–deflection relationships are obviously important to assess the accuracy of the numerical simulations. Figure 5.4, Figure 5.6 and Figure 5.7 present such comparisons for the tested beams of the first, second and third series, respectively. As displacement-controlled solutions were adopted in the analyses, the numerical models are able to simulate the post-peak behaviour.

5.4.2.1 First Series

The experimental and the numerical load–deflection curves of the first series are plotted in Figure 5.4. For the control specimen RC1, both curves for the numerical and tested results are essentially similar before cracking occurs. Small discrepancies in the load–deflection curves are observed in the cracking zone. The measured deflection values corresponding to the maximum load carrying capacity are 1.9 mm and 1.7 mm for the experimental and numerical specimens, respectively. It can be seen that the numerical analysis slightly overestimated the stiffness in the cracking region. For the strengthened specimen U4, the numerical results predict load–deflection trends similar to those of the experiments. The numerical predictions underestimate the experimental maximum deflection by 5%.
For the case of considering the actual dimensions of the strengthened beam of the first series (U4), the numerical predictions of the load–deflection relationships are compared for the three different approaches assumed for the CFRP sheets in the left shear span. In Figure 5.5, the experimental results are plotted for the seek of comparison. Small discrepancies between the load–deflection behaviours of those approaches are observed (Figure 5.5). This in particular is attributed to the fact that debonding of the CFRP sheets on the shear span of interest governed the failure. For the completely wrapped, U-jacket and side-bonded strengthening schemes implemented in the left shear span, the analytical ultimate loads are higher than the experimental values by 6%, 5% and 5% respectively, while the actual deflections are approximately typical.

5.4.2.2 Second Series

Overall, the numerical load–deflection curve of the control specimen is in good agreement with the experimental data. The numerical analysis of the control specimen, depicted in Figure 5.6, conservatively overestimates such behaviour. The predicted deflection corresponding to the maximum load is lower by 8%. For the strengthened specimen, the predicted load–deflection curve descends abruptly at about 97% of the maximum deflection of the corresponding tested specimen (Figure 5.6). The strengthened beam shows a higher load (for the same deflection) than the control beam. This is attributed to the fact that the CFRP strips provide forces to resist the growth of the shear cracks.
There is no significant difference in the response between the experimental and numerical load-deflection curves.

5.4.2.3 Third Series

The numerical predictions of the load-deflection curve of the control specimen show identical behaviour to the experimental curve prior to the initial cracking phase, as shown in Figure 5.7. This is because the CFRP strips carry little stress before the beam initiates to crack. At the stage of excessive cracking and excessive stress transfer between the adherent materials, the numerical curve is stiffer than the experimental trends. Consequently, the maximum numerical mid-span deflection is lower than the test value by 7%. For both strengthening schemes, an excellent agreement is obtained in terms of the load-deflection behaviour between the numerical predictions and the experimental data, as seen in Figure 5.7. Overall, for the beam with a U-jacket configuration, U6, the numerical analyses underestimate the maximum deflection by 2%. In the case of the beam with completely wrapped scheme, W7, the maximum load capacity was reached when the first strip rupture occurred. The numerical predictions of the maximum deflection corresponding to the ultimate load carrying capacity is underestimated by 10%.

It appears that applying a complete FRP wrapping for shear strengthening converts
5.4. EXPERIMENTAL AND NUMERICAL RESULTS

**Figure 5.6:** Load–deflection relationships for the specimens of the second set

**Figure 5.7:** Load–deflection relationships for the specimens of the third set
the mode of failure from debonding to rupture of the FRPs. This is observed both in the numerical predictions and experimental tests. Furthermore, this particular strengthening approach is effective in enhancing the load carrying capacity. The ultimate failure occurred at a large deflection. The author assumes that this explains the absence of the size effect on the shear capacity of beams strengthened with fully wrapped CFRP strips.

5.4.3 Strain Distribution along the FRP Sheet Depth

In order to verify the influence of the size effect in the contribution of the CFRP sheets to ultimate shear capacity, the CFRP axial strains of the strengthened beams were measured. The numerical model is also successful to assess such results through a comparison with the experimental data. The plots of the axial strain in the CFRP strips are presented in Figure 5.8–Figure 5.11. In addition, to avoid crowded graphs and to be able to draw suitable conclusions, the CFRP axial strains are plotted at only two load levels; one at a low load value and the other corresponds to the maximum load.

5.4.3.1 First Series

The presented experimental strain profiles shown in Figure 5.8(a)–Figure 5.8(c) are for F1, F2 and F3 strips, respectively. For the strip near the support F1 (Figure 5.8a), little axial straining in the CFRP strip was observed prior to cracking (not shown). With the load increased, the maximum axial strain is measured at the bottom edge of the CFRP strip. The profile of the axial strain along the CFRP strip depth is similar to the behaviour of a direct shear test. The axial strain descends progressively towards the top end of the CFRP sheet. On the other hand, the axial strain profile along CFRP strip F2 (Figure 5.8b) showed the peak strain value beside the bottom end of the CFRP sheet. The numerical and experimental results are similar with approximately zero and a small value at the top edge and bottom edge of the CFRP strip, respectively. The experimental axial strain profiles were not smooth and this is attributed to the crack presence.

The experimental and numerical axial strain profiles along the depth of strip F3 are presented in Figure 5.8(c). At a low stress level and a small crack size, we observe a profile similar to the classical strain distribution along the strip depth. As the load is further
Figure 5.8: Axial strain profiles along the CFRP depth for F1, F2 and F3 bonded strips of specimen U4
increased, the maximum axial strain is found near the mid-depth of the sheet. These results show that the maximum axial strain and the interfacial slip along the sheet depth are observed at the same location. It can be concluded that the maximum axial strain in this CFRP strip at failure was 0.0038 mm/mm, which is 24% of the ultimate strain of the CFRP sheet. Furthermore, the numerical results show that when debonding occurs, the axial strain degrades to small values in the debonded regions.

We could conclude that in this investigation, the maximum axial strain readings were found along the shear crack.

### 5.4.3.2 Second Series

For specimen U5, similar trends as for specimen U4 were observed for strip F1 and F2. Figure 5.9 shows the axial strain in CFRP strip F3; the strip located at midway of the shear span. The experimental axial strain profile along the depth is non-uniform, as was the corresponding strip of specimen U4. This is due to numerous shear cracks. Both the numerical and the experimental results showed the location of the maximum axial strain to be consistent with the maximum interfacial slip location. This is explained by the fact that the shear crack intercepted the CFRP strip in this region and, consequently, the maximum axial strain was recorded. The maximum experimental axial strain was 0.0036 mm/mm which is 24% of the ultimate CFRP strain.

---

**Figure 5.9:** Axial strain profiles along the CFRP depth for F3 bonded strip of specimens U5
5.4. EXPERIMENTAL AND NUMERICAL RESULTS

5.4.3.3 Third Series

The axial strain profile of strip F3 for specimens U6 and W7 are presented in Figure 5.10 and Figure 5.11, respectively. Similarly, the same behaviour for the aforementioned strengthened specimens, U4 and U5, was observed for specimen U6. The numerical predictions produce similar results as the experimental data. In this case, the evaluated axial strain is 20% of the ultimate axial CFRP strain. As can be seen from Fig. 12, the completely wrapped CFRP sheets showed its superiority over the U-jacket strengthening scheme. The maximum axial strain was 0.012 mm/mm which is 80% of the ultimate strain. This explains the absence of the size effect on the shear capacity of beams strengthened with fully wrapped CFRP scheme. The numerical predictions are in very good agreement with the experimental results.

In summary, as the dimensions of the beams as well as the dimensions of the CFRP strips are increased proportionally, the contribution of the CFRP strips is higher in the smaller specimens. Therefore, with a larger beam size, one can expect less improvement of the shear capacity with FRPs. One interesting point concerning the maximum axial strain in the CFRP strips should be highlighted. CFRP strips near the middle of the shear span intersected by a wide shear crack, and hence more stretched the other strips. As a result, the maximum axial strain occurs at this location.
CHAPTER 5. SIZE EFFECTS FOR RC BEAMS STRENGTHENED WITH FRP COMPOSITES

5.4.4 Slip Profiles along the FRP/Concrete Interface and Shear Crack Angles

In view of the good agreement between the numerical and experimental results in terms of the load-deflection behaviour, the numerical model is expected to provide valuable insight into aspects of the interfacial behaviour that are very difficult to assess experimentally. The interfacial slip profiles are thus used to predict the crack formation angle along the shear span. It can generally be stated that in the experimental program shear cracks propagated as the load increased and failure occurred due to debonding of the FRP strips over the main diagonal shear crack. This debonding leads to an instantaneous increase in the vertical strains of the bonded CFRP strips. As far as the interfacial slip behaviour of the strengthened specimens is concerned, it can be concluded that the crack inclinations resemble the measured crack angles.

5.4.4.1 First Series

Figure 5.12(a)–Figure 5.12(e) show the slip profiles along the interface for the U-jacket strengthened specimen U4. The predicted slip profiles for the CFRP strip closest to the applied load, F5, is presented in Figure 5.12(a). At a low load level (52 kN), the interfacial slip is concentrated around the middle of the beam depth and decreases towards the ends of the beam. As the applied load is increased up to the cracking stress, the interfacial slip
increases with maximum values moving to the top end of the strip. The presence of the cracks influences the behaviour of the slip profile by fluctuating the values from negative to positive. With an increase of the applied load, the shear cracks propagate and the slip values increase correspondingly. As a result, the slip profile increases dramatically in the region near the vicinity of the applied loads, thus causing debonding of that strip, F5.

The overall evolution of the interfacial slip profiles predicted for the next strip from the applied load, F4, is presented in Figure 5.12(b). These are consistent with the behaviour of the previous strip, F5. The slip distributions show that the maximum slip values occur at 50 mm below the top edge of the CFRP strips. The debonding initiates at the point of the maximum slip value and propagates towards the top end of the CFRP strip. At the same load levels, the strip F5 shows higher slip values than does the strip F4. This indicates that the delamination initiates in the strip closest to the applied load and propagates towards the support.

Figure 5.12(c) shows the slip profiles along the strip depth for the third strip, F3, placed midway along the shear span. Clearly, before crack initiation, small slip values are predicted around the mid-depth of the CFRP strip. The abrupt increase of the slip values around the mid-depth was due to a local effect caused by the shear crack. In this case, delamination initiates at the position of the maximum interfacial slip, leading to eventual failure towards the top edge. For the last two CFRP strips, F2 and F1, the slip profiles are plotted in Figure 5.12(d) and Figure 5.12(e), respectively. It can be observed that the maximum values of these slip profiles concentrate at a distance 50 mm and 25 mm from the bottom end of the strip, respectively. Furthermore, the interfacial slip values decrease towards the top ends of the CFRP strips.

The shear crack angle is a key parameter in the calculation of the FRP shear resistance capacity. In Figure 5.13(a)–Figure 5.13(d), the formations of the shear cracks in the experimental tests for the strengthened beams are depicted, and the crack angle is labelled correspondingly. The maximum slip value of the first strip adjacent to the applied load was observed at the top edge; however, the higher slip values of the CFRP near the support were measured at the bottom edge of the CFRP strip. Consequently, this means that the connection of the points of the ultimate interfacial slip values can be considered as an indication of the crack inclination angle. As indicated in Figure 5.13(a), the crack angle of the strengthened specimen U4 was 32°. In the numerical model, the inclination angle
Figure 5.12: Interfacial slip profiles along the FRP depth of the bonded strips for specimen U4
between the maximum slip values varied. Overall, the average angle of the maximum slip values, which represents the crack angle, was 28°. This result shows that the predicted crack angle of the numerical model is in very good agreement with the experimental data.

5.4.4.2 Second Series

The previous analysis for the first series was repeated for the strengthened specimen of the second series. The slip profiles for strip beside the applied load, F5, is presented in Figure 5.14(a). Figure 5.14(b) is for the strip adjacent to the support, F1. The interfacial slip behaviours of these strips are selected to show the relation between the position of the strip along the shear span and the location of the maximum interfacial slip. Typical interfacial behaviour to that described for specimen U4 is obtained also for specimen U5, although, the slip trends for this case are not smooth as for the previous specimen. This is attributed to the fact that the increase of depth size leads to more shear cracks, which is indeed similar to the response observed experimentally.
Figure 5.14: Interfacial slip profiles along the FRP depth of bonded strips F5 and F1 for specimen U5

According to the aforementioned interfacial debonding behaviour, the maximum slip values appeared at various locations along the bonded FRP strips. It should be noted that the location of the maximum slip values depends on the inclination of the shear crack and the position of the strip along the shear span. The experimental reported shear crack, as shown in Figure 5.13(b), was 33°. The numerical model can successfully simulate the angle of the shear crack with a value of 36°. The predicted results support the conclusion that the numerical model is excellent in representing the experimental behaviour.

5.4.4.3 Third Series

The predicted slip profiles for specimen U6 follow the same trend as obtained for the previous experimented series. The entire interfacial slip profile of the first and last strips, F1 and F5, are plotted in Figure 5.15(a) and Figure 5.15(b), respectively. For specimen W7, full strain compatibility is employed between the two adherent materials. Consequently, the interfacial slip is not measured. The experimental crack angle orientation in specimen U6 is 34° (Figure 5.13(c)), while the numerical model predicted a crack angle of 26°. Obviously, the accuracy of the numerical model to assess the shear crack angle for this specimen is less than the previous specimens. Moreover, during the experiments numerous cracks were observed near the support. This is particularly attributed to the higher depth, which leads to uncertainty in measuring the crack angle accurately.
5.5 Summary

Figure 5.15: Interfacial slip profiles along the FRP depth of bonded strips (F5 and F1) for specimen U6

Generally, the predicted results support the conclusion that the numerical model is representative of the experimental behaviour.

5.5 Summary

The experimental program reported here investigated the shear performance of rectangular reinforced concrete beams strengthened with CFRP U-strips as well as completely wrapped. The size effect was examined for these beams with varying depths. The experimental program was conducted to obtain a better understanding the behaviour and to improve the database of the influence of the depth size to the ultimate load carrying capacity. Within the indicated scope of investigation, the particular conclusions emerged from this study are summarized as follows:

- For the control beams, it is observed that the smaller beam dimensions, the higher loading percentage is attained before the cracking phase.

- The results confirm that there is an optimum FRP quantity beyond which the strengthening effectiveness is not beneficial.

- The experiment also supplied valuable information on the effect of the depth increase. An increase in depth size is consistent with excessive shear cracks prior to
The axial strain in the CFRP sheets is not uniformly distributed along their heights. The strip near the support showed a response similar to the direct shear test.

Numerical modelling was carried out to investigate the behaviour of FRP shear-strengthened beams. A nonlinear constitutive model was incorporated to represent the interfacial behaviour between the concrete and CFRP strips. The comparison between the numerical predictions and experimental results presents excellent agreement in terms of the ultimate load carrying capacities, load-deflection relationships, and failure modes. Based on the numerical results, the following conclusions are drawn:

- Accounting for the slip profiles along the CFRP depth is necessary to assess the debonding phenomena.

- The maximum axial strain in the CFRP sheets is concentrated along the shear crack. The strip lies on the middle of the shear span has the highest axial strain value. This finding herein is different to those of the other researchers.

- A consistency is observed between the shear crack locations and the maximum values of the interfacial slips; the delamination occurs over the main diagonal shear crack.

- The numerical model is able to capture the shear crack angle along the shear span.
Chapter 6

Numerical Predictions for Various Configurations of FRP Composites

6.1 Introduction

In addition to the experimental test results considered in the previous chapter, another batch of tested specimens are simulated to verify the accuracy of the proposed numerical model. Comprehensive research was carried out at Tsinghua University (China) to enhance the database of FRP shear-strengthened beams. These studies showed that the shear-strengthening of a reinforced concrete beam is strongly influenced by the strengthening scheme. The additional attraction of that study is that the shear span to depth ratio \((a/d)\) and steel stirrups are found to play an important role on the maximum load carrying capacity.

The investigation of the numerical model was extended to another batch of shear-strengthened beams with different strengthening schemes and with different shear span to effective depth ratios. The various shear-strengthened beams considered included strengthening with strips and continuous U-wrap, as well as completely wrapped strengthening schemes. These beams were also used to investigate whether the use of a top horizontal band anchoring for a U-shaped wrapping can prevent debonding. The specimens were strengthened with either GFRP or CFRP sheets to examine the effectiveness of various strengthening materials.
CHAPTER 6. NUMERICAL PREDICTIONS FOR VARIOUS CONFIGURATIONS OF FRP COMPOSITES

The basic motivation of the present numerical study is to produce a satisfactory numerical tool that is capable of accurately predicting the performance of shear-strengthened beams. The experimental results of the ten simply supported shear-strengthened beams with various shear-strengthening configurations and values of shear span to depth ratio \( (a/d) \) are used to assess the validity of the finite element model. The proposed numerical model is validated against the experimental results in terms of ultimate load carrying capacities and applied load-central deflection relationships. Additionally, the capability of the model to predict the FRP axial strain distributions along the shear crack for the various strengthening configurations are illustrated.

6.2 Experimental Program

Ten beam specimens with a total span length of 1500 mm and a rectangular cross-section of 260 mm deep and 150 mm wide were tested. The specimens were categorized into three main series according to the shear span to effective depth ratio \( (a/d) \). All the beams have identical longitudinal steel reinforcement. 8 mm diameter of steel stirrups with 60 mm of spacing were provided in the right half to ensure shear failure in the span of interest (left shear span). The steel reinforcement details and the dimensions of the specimens are shown in Figure 6.1. All the test specimens were strengthened within their shear span of interest.

The first series consisted of one specimen, SGU-1-1, with a ratio of shear span to effective depth of 1.5. In the span of interest (left shear span), the specimen was reinforced with 6 mm steel stirrups with a spacing of 200 mm, as shown in Figure 6.1a, to induce failure in this span. The second series included eight beams with the same cross-section dimension, longitudinal steel reinforcement and steel stirrups as for the first series. The beams of the second series were tested at a shear span to effective depth ratio of 2.155. One beam of this series, SGU-2-1b, was different in the design of internal transverse steel. It was provided with five 8 mm-diameter at 160 mm centre-to-centre along the shear span of interest, as shown in Figure 6.1(b). The third series included one specimen (SGU-3-1), with 2.8 ratio between the shear span to effective depth.

The concrete of the manufactured beams was designed for a 28-day compressive strength.
of 35 MPa. In reality, the compressive strength varied between 31.8 MPa and 40.1 MPa. For the longitudinal steel reinforcement, the average measured yield strength was 395 MPa and the mean ultimate tensile strength was 576 MPa. The transverse steel stirrups of 6 mm diameter were made with a yield strength of 576 MPa and ultimate tensile strength of 555 MPa. For the 8 mm diameter reinforcement, 307 MPa and 435 MPa were measured for yield and ultimate strength, respectively. The relevant strengthening configuration details with their FRP width, \( w_f \), and spacing, \( s_f \), and the concrete compressive strength, \( f'_c \), as well as their tensile strength, \( f_t \), for all tested specimens are provided in Table 6.1.

One beam of the tested specimens, belonging to the second series, was kept without strengthening as a reference specimen (specimen S0-2-0). Eight specimens were strengthened with GFRP sheets following different strengthening schemes. In the second series, specimen SCU-2-1 was strengthened with CFRP sheets to examine the influence of using different FRP materials. It may be noted that the thickness for the GFRP sheets was 0.169 mm, whilst for the CFRP sheets it was 0.111 mm. The GFRP sheets had an ultimate strength of 2777 MPa and modulus of elasticity of 97 GPa. For the CFRP strips, 235 GPa and 3550 MPa were measured for the Young's modulus and tensile strength,
Figure 6.2: Shear strengthening configurations
### Table 6.1: Concrete properties and shear-strengthening details of the tested specimens

<table>
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<tr>
<th>No.</th>
<th>Beams series</th>
<th>Spec.</th>
<th>(a/d) ratio</th>
<th>Concrete (MPa)</th>
<th>Strengthening type</th>
<th>Strengthening details</th>
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<th>sheet depth s_f (mm)</th>
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<td>50</td>
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<td>U-wrap</td>
<td>Vertical strips</td>
<td>50</td>
<td>50</td>
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</table>

* Horizontal 62 mm band superimposed on the top of U-wrap strips.

The details of the strengthening configurations are illustrated in Figure 6.2. The figure shows only the strengthened part of the beams. Specimen SGU-1-1 (first series) was strengthened with four GFRP U-wrap strips along the shear span of interest (left shear span) with the fibre direction oriented perpendicular to the longitudinal axis of the beam. The strip width was 50 mm as well as the clear spacing between the strips. Specimen SGU-2-1a (second series) was strengthened in a manner similar to that of the previous specimen, but with five U-wrap strips throughout the shear span. The identical strengthening scheme was provided for specimen SGU-2-2 with a wider strip width (80 mm) and smaller clear spacing between the strips (25 mm). CFRP sheets were used to repair specimen SCU-2-1 (not shown) with a similar strengthening scheme of specimen SGU-2-1a. Continuous GFRP sheets in a U-wrap strengthening scheme were used to strengthen specimen SGU-2-3. The layout of the GFRP U-wrap strips of beam SGUB-2-1 was similar to beam SGU-2-1a; however, anchorage for the GFRP strips was improved by means of an additional 65 mm-wide longitudinal band along the shear span. The amount of internal steel stirrups of specimen SGU-2-1b was varied from the previous specimens (not shown). 8-mm stirrup diameter with a 160 mm spacing was provided, while the strengthening configuration was kept identical to specimen SGU-2-1a. Specimen SGO-2-1 was strengthened with closed GFRP strips. The strip width and the clear spacing between strips were 50 mm. For the specimen of the third series, SGU-3-1, seven GFRP
U-wrap strips were installed along the shear span \((a/d=2.8)\). The cross-section dimension and steel stirrups were similar to those in the previous series. A 50 mm strip width and spacing between the strips were used for this specimen.

All beams were simply supported and subjected to a monotonic four-point static loading. Vertical displacements were measured at the centre of the span using linear variable differential transformers (LVDTs). The strain in the FRP sheets was also measured on the vertical face of the specimens to estimate the FRP axial strain. Strain gauges were placed based on the final failure crack pattern observed in the control specimen. The placement of the strain gauges is illustrated in Figure 6.3. The notation of the FRP strips along the test span (Figure 6.3) was arranged according to the citation of the strip from the applied load.

### 6.3 Numerical Program

The proposed three-dimensional finite element program, ADINA [2004a], was used for the numerical analysis of the ten tested specimens. In the analysis, appropriate material models were employed to represent the behaviour of the concrete, the steel reinforcement and the CFRP sheets. These are described in detail in the ADINA theory and modelling guide ADINA [2004b]. In addition, to model the bond behaviour at the FRP/concrete interfacial behaviour, discrete truss elements that properly represent the local bond stress-slip characteristics and failure were used. The details of the constitutive models and their implementation into the numerical analysis are described in detail in Chapter 3. Furthermore, to investigate the proposed model for the completely wrapped FRP strengthening
scheme, perfect bond was imposed between the two adherent materials. This implies that complete displacement continuity was assumed and rupture of the FRP plies governs failure.

The anchorage of the GFRP strips of specimen SGUB-2-1 was improved with a horizontal sheet installed at the top edge of the U-wrap strips. Two approaches were used to model the contact interface between the additional horizontal sheet at the top band of specimen SGUB-2-1 and the adjacent surfaces (concrete and FRP strips). In the first approach, full strain compatibility in all directions was assumed between the horizontal sheet and the U-wrap strips, while in the second approach the horizontal sheet was connected to the adjacent surfaces via horizontal interface elements. Additionally, these interface elements had the same constitutive relation as those for the vertical interface elements and full contact bond was assumed in the other directions.

The nonlinear load deformation behaviour of the beams was simulated under displacement-controlled loading conditions, as was the case for the laboratory experiments. In view of the geometrical and loading symmetries, only one quarter of the beam was simulated, as shown in Figure 6.4. Comparisons between the experimental data and numerical predictions are carried out for the various ratios of shear span to effective depth of the beams, with the objective of establishing the validity of the numerical model, with consideration of the interfacial slip behaviour as the chief parameter.
CHAPTER 6. NUMERICAL PREDICTIONS FOR VARIOUS CONFIGURATIONS OF FRP COMPOSITES

6.4 Numerical Results and Discussion

The results presented in the following sections are in terms of ultimate load carrying capacities, load-deflection relationships, and failure modes. The axial strain profiles along the shear crack were compared with the experimental results. Special emphasis is placed on the slip profiles at the interface between the concrete and FRP strips.

6.4.1 Ultimate Carrying Capacities

Table 6.2 shows the comparison between the numerical and experimental results. As can be seen from the results given in Figure 6.5, the finite element results show a very good agreement with the experimental results with an average accuracy of 103%.

For the beam of the first series, SGU-1-1, the finite element model predicted load carrying capacity corresponds to 104% of the experimental results. With regard to the beams of the second series, the theoretical predictions estimate typical values of load carrying capacity for specimens SGU-2-1a and SGU-2-1b. The numerical predictions obtained for specimens SGU-2-2, SGU-2-3 and SCU-2-1 are 106%, 106% and 98% of the test values, respectively. On the other hand, the numerical result of the control specimen, S0-2-0, shows a 4% higher load carrying capacity than the tested specimen. Compared to the experimental results, the numerical predictions assess 3% greater load carrying capacity for specimen SGUB-2-1. Successfully, the proposed model simulates the behaviour of closed GFRP strips. The ultimate load carrying capacity from the numerical model is 2% greater than the experimental value. In addition, the model accurately simulates the behaviour of the third series specimen, SGU-3-1; in this case, the numerical prediction of the load capacity is overestimated by 108% of the test value.

Based on the above discussions, it is obvious that the proposed numerical model reveals satisfactory load capacities predictions compare to the test data. In addition the simulation of the debonding criterion is the key factor to obtain accurate results.
6.4. NUMERICAL RESULTS AND DISCUSSION

Table 6.2: Comparison between numerical and experimental results

<table>
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<th>No.</th>
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<th>Numerical results</th>
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<td></td>
<td></td>
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<td>Max. def. (mm)</td>
<td>Max. load (kN)</td>
<td>Max. def. (mm)</td>
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<td>4.5</td>
<td>239</td>
<td>4.3</td>
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</tbody>
</table>

* Deflection measured at maximum load

Figure 6.5: Ratios of the numerical to experimental results for the various specimens
6.4.2 Load–Deflection Relationships and Failure Modes

Figure 6.6–Figure 6.10 compare the applied load–central deflection relations recorded during the experiments and those obtained from the analysis. Overall, the applied load–central deflection curves of the analytical results are in good agreement with the experimental data. Besides, minor discrepancies in the stiffness of the applied load–central deflection relationships are slightly overestimated by the theoretical results. Deflections are measured at midspan at the centre of the bottom face of the beams.

6.4.2.1 First Series

The applied load–central deflection relationship of the specimen of the first series (SGU-1-1) agrees quite well with the experimental data, as illustrated in Figure 6.6. The numerical applied load–central deflection relationship plotted in the linear range is identical to the test data. After first cracking, the stiffness of the finite element model is greater than that of the experimental beams. In addition, the maximum deflection of the specimen considered is underestimated by 10% compared to the corresponding experimental value. A comparison of the failure behaviour shows that the dominant failure mode from the analysis is the delamination of the GFRP strips, which is in agreement with the experimental observations. Numerically, the model is capable of predicting the typical failure scenario of the tested specimen. At the stage of excessive stress transformation from the concrete to the adjacent FRP composites, the delamination is initiated at the intersection between the shear crack and the bonded strips. Debonding of the interface elements at the top edge of the strip beside the load is the first indication of the delamination. After removing the debonded interface elements with the load increase, the interface elements of the adjacent strip are delaminated at the intersection between the shear crack and the bonded strips and continue towards the top edge. The delamination occurred at the corresponding points of maximum slip values as will be shown in the subsequent section. This behaviour progressed until concrete crushing.
6.4. NUMERICAL RESULTS AND DISCUSSION

Figure 6.6: Applied load-central deflection relationships for the specimen of the first series

6.4.2.2 Second Series

For the beams of the second series (a/d=2.155), the numerical results of the control beam, S0-2-0, are similar to the experimental results (Figure 6.7); however, small discrepancies are observed. In addition, the maximum central deflection of the control beam is underestimated by 8%. As can be seen in Figure 6.7, for strengthened specimens SGU-2-la and SGU-2-1b (the strengthening configuration was identical, while the amount of internal steel stirrups was increased in specimen SGU-2-1b) the numerical model showed good agreement with the experimental results. Obviously, the use of steel stirrups enhances the load carrying capacity of specimen SGU-2-1b. The stiffness of the numerical model is slightly overestimated in the cracking phase. Overall, the numerical analysis underestimated the maximum deflection by about 11% and 6% for specimens SGU-2-la and SGU-2-1b, respectively. Failure of those specimens is dominated by debonding of the FRP, which indeed is the same as that observed in the experiments. Successfully, the numerical model is able to capture the same failure process as that observed experimentally.

Similar stiffness is observed for specimens SGU-2-2, SGU-2-3 and SCU-2-1, as provided in Figure 6.8. The results indicate that the installation of continuous sheet (specimen SGU-2-3) is effective to increase the shear capacity. With regard to the deflection, 4%, 8% and 8% lower deflection values are obtained from the numerical predictions for specimens SGU-2-2, SGU-2-3 and SCU-2-1, respectively, compared to their experimental data. From Figure 6.9 we can see that the numerical and experimental results of specimen SGO-2-1 are
in a very good agreement; however, the numerical model is stiffer than the experimental plot at the same load levels.

Figure 6.9 shows the experimental results of specimen SGUB-2-1 as well as the numerical results with both simulating approaches; full constraint and horizontal interface elements. For the approach of full constraint, we observe that the load carrying capacity is higher by 3% compared to the actual measurements, while the use of horizontal interface elements overestimates that value by 2%. Besides, there is no difference in the stiffness between either approach of simulation. This is due to the fact that the vertical interface elements govern the delamination of the strips. The maximum central deflection values
6.4. NUMERICAL RESULTS AND DISCUSSION

Figure 6.9: Applied load-central deflection relationships for the specimens SGO-2-1 and SGUB-2-1 of the second series

are greater when using the full strain compatibility approach than those using horizontal interface elements by 3%. From the analysis we conclude that the addition of the horizontal sheets at the top band of the GFRP strips is not effective in enhancing the load carrying capacity and delaying the debonding of the strips. Close agreement between the numerical and experimental failure state is reported.

6.4.2.3 Third Series

As can be seen from Figure 6.10, the curves for the experimental and numerical results of specimen SGU-3-1 are similar. There is agreement between the failure progress observed in the experimental and numerical results, where the debonding of the bonded strips is governing the failure criterion.

In general, the predicted applied load–central deflection plots showed an excellent agreement with the test data. However, the numerical model sometimes exhibited a slightly stiffer behaviour than that observed in the experiments.
CHAPTER 6. NUMERICAL PREDICTIONS FOR VARIOUS CONFIGURATIONS OF FRP COMPOSITES

6.4.3 Strain Distribution Along the FRP Sheet Depth

In order to verify the accuracy of the numerical results, the FRP axial strain distributions are validated against the experimental data. The strain of the FRP sheets was measured along the crack pattern observed in the control specimen. The arrangement of the strain gauges along the shear crack was shown in Figure 6.3. The numerical results of the FRP vertical strain values are lower compared to the experimental data; this is explained by using the interface elements with a particular stiffness. To avoid crowded graphs and to be able to draw a sufficient conclusion of the accuracy of the numerical predictions, the numerical results of the longitudinal strain are compared to the experimental data at three load levels.

Figure 6.11 shows the experimental results of the vertical strain profiles of the GFRP strips compared with the numerical results along the shear crack of the specimen of the first series (SGU-1-1). At relatively low load levels, the numerical strain distribution agrees well with the experimental results with fairly smooth curves along the shear crack. The small amounts of the vertical strain values appear at the initial cracking stage of the concrete. The strain distribution obtained varies considerably along the shear crack. In particular, the shear cracking results in a localized maximum strain value around the mid-depth. With the successive transfer of stresses from the concrete to the FRP sheet, higher strain values are obtained. The strain profiles along the shear crack are lower at the top edge of the crack, while higher towards the bottom end. This is attributed to
the presence of the compressive stresses at the top end, while tensile stresses occur at the bottom.

The experimental and numerical strain distributions along the proposed shear crack at selected load levels are compared in Figure 6.12 for specimen SGU-2-1a (second series). The strain trends indicate that at low load levels the GFRP strips experience small strain values. As a result of the load increase and continuous transfer of stresses, the strain is observed to increase rapidly around the mid length of the shear crack. For the CFRP shear strengthened beam (specimen SCU-2-1), the vertical strain distribution predicted for the CFRP sheets is plotted in Figure 6.13. The general trends are similar to those for the GFRP shear-strengthened beam; both specimens have the same strengthening scheme. The CFRP strains are higher on the strip located in the mid-way along the shear span. A comparison between the strains for specimen SGU-2-1a and specimen SCU-2-1 clearly demonstrates the influence of the type of FRP sheets. The magnitudes of the FRP strains are higher for the GFRP shear-strengthened specimen than that repaired with the CFRP strips. This can be explained by the lower elastic modulus for the GFRP strips as compared to the CFRP sheets (almost three times lower).

The predicted vertical strain and the corresponding test data are graphed in Figure 6.14 for the specimen of the third series (specimen SGU-3-1). It was strengthened with seven GFRP strips installed along the shear span of interest. Similar to the previous trends, the formation of the shear crack in the beam affects the peak value of the vertical strain.
CHAPTER 6. NUMERICAL PREDICTIONS FOR VARIOUS CONFIGURATIONS OF FRP COMPOSITES

Figure 6.12: Axial strain profiles along the shear crack for specimen SGU-2-la of the second series

![Figure 6.12: Axial strain profiles along the shear crack for specimen SGU-2-la of the second series](image)

Figure 6.13: Axial strain profiles along the shear crack for specimen SCU-2-1 of the second series

![Figure 6.13: Axial strain profiles along the shear crack for specimen SCU-2-1 of the second series](image)
6.4. NUMERICAL RESULTS AND DISCUSSION

As expected, the vertical strain has a small value beside the support and progressively increases around the mid-way of the shear crack. The predicted results are in good accordance with the experimental observation, as shown in Figure 6.14.

6.4.4 Slip Profiles along the FRP/Concrete Interface and Shear Crack Angles

Knowing the values of the interfacial slips between the FRP sheets and concrete can be very helpful for a better understanding of the FRP/concrete interfacial behaviour and bond performance. These observations are virtually impossible to detect in experiments. One of the advantages of this numerical analysis is its capability to estimate the shear crack angles along the shear span of the shear-strengthened beams. The shear crack angle is calculated according to the location of the maximum slip values along the sheet depth. Generally, the maximum slip values appear at virtual positions of the intersection between the shear crack and bonded strips. In the current analysis, the average angle of the connection between the points of the maximum slip values is considered to be the shear crack angles. Successfully, according to the following discussion, the theoretical model proved its ability to predict shear crack angles similar to those observed experimentally.
CHAPTER 6. NUMERICAL PREDICTIONS FOR VARIOUS CONFIGURATIONS OF FRP COMPOSITES

Figure 6.15: Interfacial slip profiles along the FRP depth for specimen SGU-1-1 of the first series

6.4.4.1 First Series

Figure 6.15(a)–(d) show the slip profiles of specimen SGU-1-1 along the depth of the GFRP strips along the shear span at different load levels. The slip profile for the strip beside the applied load, U1, is illustrated in Figure 6.15(a). At a load level (60 kN) slightly above the corresponding cracking load, the slip values are relatively small. With a load increase (275 kN), the maximum slip value appears below the top edge. Finally, debonding initiated at the top edge of the bonded strip and this is mainly due to the small length between the shear crack to the top edge of the strip. Additionally, the presence of a shear crack influences the behaviour of the slip profile by fluctuating the results from negative to positive.
6.4. NUMERICAL RESULTS AND DISCUSSION

For the second strip labelled as U2, the slip profile along the sheet depth is presented in Figure 6.15(b). At the same load levels, the maximum slip profiles for this strip are smaller than that of the previous strip. This indicates the fact that the debonding initiated at the strips adjacent to the applied load and then propagated towards the support, which is indeed similar to what was observed in the experiment. The slip values plotted in Figure 6.15(b) are significantly higher at 100 mm below the top end of the bonded strip (U2). This is attributed to the shear crack intersects the bonded strip at this location. Delamination thus initiated at this location and propagated towards the top edge. The negative values in the Figure 6.15(b) indicate that the FRP strip moves downwards relative to the concrete.

The bonded strips beside the support, U3 and U4, are shown in Figure 6.15(c) and Figure 6.15(d), respectively. It can be concluded that, the slip profiles are consistent with the value of the applied load. The maximum slip distributions along the sheet depth concentrated at 100 mm and 25 mm from the bottom edge for strip U3 and U4, respectively. It is noticed that the interfacial slip values decrease towards the top ends of the FRP strips. Furthermore, zero value of the slip at the bottom edge of the beam is due to the different arrangement of the interface elements.

A comparison between the slip profiles along the sheet depth plotted in Figure 6.15 indicated that the presence of the shear crack has a crucial influence on the slip profiles along the sheet depth. Consequently, the location of the higher slip values definitely depends on the intersection of the shear crack with bonded strips. The same method used in the previous chapter to estimate the shear crack angle is considered. Overall, the average angle of the maximum slip values, which represent the shear crack angle, is estimated to be 40°. No value was provided from the experimental measurements.

6.4.4.2 Second Series

With regard to specimen SGU-2-la (second series), the slip profiles along the sheet depth is plotted in Figure 6.16(a)–(e). Similar to that observed from the previous specimen, the interfacial slip profiles varied significantly from one strip to another along the shear span. This is due to the influence of the shear crack. The interfacial slip curves of the specimen beside the applied load, U1, are described in Figure 6.16(a). The maximum interfacial
CHAPTER 6. NUMERICAL PREDICTIONS FOR VARIOUS CONFIGURATIONS OF FRP COMPOSITES

slip values for this strip corresponding to load level at 295 kN appeared at top vicinity of the bonded strip. In addition, the interfacial slip profiles along the FRP sheet are higher near the top end and descend towards the bottom edge.

The predicted slip profiles for strip U2 are presented in Figure 6.16(b). As shown in the figure, as the load increases above the cracking load, the interfacial slips profiles increases and its maximum values concentrated at 75 mm from the top edge of the beam. The increase in the shear crack width causes delamination of the bonded strip at the intersection and progressed rapidly towards the top end. For the other bonded strips, U3, U4 and U5, the slip profiles are presented in Figure 6.16(c)–(e), respectively. From the figure, the maximum slip values occur at 155 mm, 105 mm and 52 mm from the bottom edge of the beam for strip U3, U4 and U5, respectively.

According to the aforementioned interfacial behaviour, the maximum slip values appeared at various locations along the bonded FRP strips. The maximum interfacial slip value is a function of the presence of the shear crack and the strip citation along the shear span. The numerical model can successfully simulate the angle of the shear crack with a value of 32°.

The numerical results of the interfacial slip profiles presented in Figure 6.17 are for specimen SCU-2-1. Only the results of the bonded strips near the applied load, U1, and adjacent to the support, U5, are illustrated in Figure 6.17(a) and Figure 6.17(b), respectively. At debonding, the location of the peak value of the interfacial slip occurs at the top end of the bonded strip. On the other hand, the interfacial slip profiles of U5 bonded strip (Figure 6.17b) showed a peak value at 50 mm from the bottom edge of the beam. Similar to the previous specimens, the numerical model can successfully predict the shear crack angle. The shear crack angle reported from the experiment is 30°, while the numerical model estimated the shear crack angle with a value of 36°. The predicted results support the conclusion that the numerical model is excellent in presenting the experimental behaviour.

A direct comparison of the shear crack angle between specimens SGU-2-la and SCU-2-1 might imply the influence of the stiffness of the FRP material. It is difficult to verify the difference since the concrete strength is not the same for the two specimens. The concrete strength for specimen SGU-2-la was 39.2 MPa, while 37.6 MPa was reported for
Figure 6.16: Interfacial slip profiles along the FRP depth for specimen SGU-2-1a of the second series
specimen SCU-2-1. It can be observed that the predicted shear crack angle of the CFRP shear-strengthened beam (36°) is higher than that strengthened with GFRP strips (30°), although the beam strengthened with CFRP strip has less concrete strength. This is the result of the increase in the modulus of elasticity of the CFRP sheets, which increases the stiffness of the beam. Particularly, the increase of the shear crack angle is related to the increase of the stiffness of the FRP strips.

### 6.4.4.3 Third Series

For the third series, specimen SGU-3-1, the development of the interfacial slip profiles along the sheet depth of the bonded strips beside the applied load, U₁, and adjacent to the support, U₇, are presented in Figure 6.18(a) and Figure 6.18(b), respectively. Typical to the technique developed to account for the shear crack angle, the numerical model predicted a crack angle of 25°. A comparison for the shear crack angle for the various ratios of shear span to effective depth shows that an increase of this ratio has a significant influence on the shear crack angle. Clearly, the predicted shear crack angle for a low shear span to effective depth ratio (a/d=1.5) is relatively steeper than that with a higher ratio.
6.5 Summary

The primary objective of this research was to investigate the computational efficiency of the proposed numerical model for the analysis of reinforced concrete beams strengthened in shear with externally bonded FRPs. A key feature of the numerical modelling was the implementation of nonlinear transitional link elements to simulate the interfacial behaviour between the externally bonded FRPs and the concrete.

It has been shown that the proposed model is capable of accurately predicting the load carrying capacities and load-deflection relationships for a wide variety of FRP shear-strengthening schemes. However, the numerical model sometimes exhibited a slightly stiffer behaviour than that observed in the experiments. In addition, it can provide valuable insight into quantities that are difficult to measure in the laboratory, such as the interfacial slip profiles and shear crack angles. The analysis is also capable of predicting the axial strains along the shear crack.

Comparisons between the numerical predictions and experimental trends lead to the following conclusions:

- The numerical slip profiles are consistent with the debonding behaviours reported in the corresponding experimental studies.

Figure 6.18: Interfacial slip profiles along the FRP depth for specimen SGU-3-1 of the second series
• A comparison for the shear crack angle for the various ratios of shear span to effective depth showed that increase of that ratio has a significant influence on the shear crack angle. Clearly, the predicted shear crack angles for low shear span to effective depth ratio (a/d=1.5) are relatively steeper than those with higher ratio.

• The existence of cracks affects the interfacial slip profiles along the beam depth. Consequently, the presence of cracks may cause shifts in the signs of the slip values, from negative to positive.

• There is a significant difference in the delamination behaviour for the FRP bonded strips along the shear span. The strips lying beside the applied load have higher slip values around the top end of the laminate. By contrast, for the strips installed adjacent to the support the maximum slips occur near the bottom edge.
Chapter 7

Parametric Studies and Design Equations

7.1 Introduction

Experimental studies on shear strengthening with FRPs carried out in recent years have provided interesting findings and conclusions, particularly with regard to the effect of the stiffness of the composites on the shear strength enhancement (Triantafillou and Antonopoulos 2000). Many aspects are still not fully verified. The relatively large scatter observed in the research studies indicates that other parameters not yet captured may influence the shear resistance. For example, the effect of the steel stirrups has been shown to have an important influence on the shear behaviour of shear-strengthened beams.

In current codes of practice, the shear resistance of a beam \( V_n \) is the sum of the shear resisted by the concrete \( V_c \) and that resisted by the transverse steel \( V_s \). If the beam is strengthened with externally FRPs, then a third term representing the contribution of the FRP \( V_f \) is added to that sum, maintaining unchanged the contributions of the concrete and the internal steel. The contribution of the FRP to the shear resistance is often idealized as analogous to that of the internal steel stirrups. However, in practice the FRP never reaches its full strength, and the effective strain \( \varepsilon_{fe} \) value is a fraction of the ultimate tensile strain \( \varepsilon_{fu} \). Improved formulae were proposed as more data become available. These formulae, which were derived from calibrated experimental test measure-
ments, are generally expressed in terms of the FRP stiffness, the concrete strength and mode of failure. The truss analogy, on the other hand, is used to estimate the effective strain as a function of the shear force obtained experimentally. Steel stirrups are internal, whereas FRP is external. Therefore, the resistance mechanism of the first case differs from the second. There is a lack of data on experimental strains in the FRP. When made available, this data will be invaluable to the understanding of the mechanisms involved and to the calibration of the developed models.

In most research studies, the relationship between the effective strains ($\varepsilon_{fe}$) estimated using the truss analogy and the stiffness of the FRP with respect to the concrete's resistance for a given mode of failure and scheme of strengthening was particularly emphasized. However, the experimental studies indicated that other parameters also influence the shear resistance. This chapter proposes to examine and evaluate the effect of important parameters; namely, the FRP modulus of elasticity, FRP thickness, concrete compressive strength, shear steel stirrups, and width ratio between the FRP to the concrete beam, on basis of data collected from the proposed numerical model. The output of this parametric study is used into a statistical analysis to produce a new design equation describing the influence of these parameters on the behaviour of the effective FRP axial strain. Details of the statistical analysis will be presented in the following sections. This study was taken for both side-bonded and U-wrap schemes. The author favours the gain in the FRP axial strain instead of shear force as a criterion for discussion because it is the utmost important value influence of the behaviour a shear-strengthened beam. As discussed earlier, debonding of the FRPs is the predominant failure mode for FRPs bonded on sides of beams. This mode also controls the strength for most beams strengthened with U-wrap schemes. The design equation developed in this section is concerned with shear-strengthened beams whose failure is controlled by FRP debonding. Additionally, the effective FRP strain predicted by the model proposed in this study will be compared with that measured from the test data and strains predicted by other modelling approaches. In addition, a detailed parametric study is performed to investigate the effect of FRP/concrete interfacial bond properties in terms of stiffness, local bond strength, and fracture energy. We hope this research study clarify the main parameters affecting the behaviour of shear-strengthened beams.
7.2 Parametric Studies

7.2.1 Parameters of Bond–Slip Model

As far as the FRP/concrete interfacial behaviour of shear-strengthened beams is concerned, it is generally accepted that debonding propagation is governed by mode II fracture behaviour (in-plane shear/sliding). Bond-slip curves were identified experimentally or theoretically by an ascending branch before reaching the bond strength ($\tau_{\text{max}}$), followed by softening behaviour. As shown earlier, the bilinear bond-slip model may be used to characterize the FRP/concrete interfacial behaviour. In this model, initiation and propagation of debonding can be represented with three parameters: interfacial stiffness ($k_s$), local bond strength ($\tau_{\text{max}}$), and interfacial fracture energy ($G_f$), as shown in Figure 7.1. In this section, the numerical predictions of a Pellegrino and Modena beam having steel stirrups and strengthened with a single layer of FRP sheets (TR30D3) is used to gain a clear understanding of FRP/concrete interfacial properties on the performance of shear-strengthened beams. The numerical aspect for the interfacial behaviour is investigated by modifying the order of the bond–slip model of Lu et al. [2005].

7.2.1.1 Effect of Interfacial Stiffness

Interfacial slip suppose to have a direct effect on the stress transfer rate from the concrete to the FRP sheets. After concrete cracking, low interfacial stiffness may result in a slow stress flow into FRP sheets causing debonding. Three values for the corresponding interfacial slip to the maximum interfacial shear stress ($s_0$) were investigated: 0.02 mm, 0.04 mm and 0.05 mm. The analyses were stopped when the first delamination occurred. The reason for this is to investigate the effect of such parameters on the strengthening performance. The comparison is presented in terms of load–deflection relationships. As shown in Figure 7.2, the interfacial stiffness ($k_s$) has no effect on the overall structural performance and also, the debonding point remains almost the same. In the figure, the same ultimate load carrying capacity and the corresponding midspan deflection are obtained despite the variation in the interfacial stiffness, where the debonding initiated at the same load value.
CHAPTER 7. PARAMETRIC STUDIES AND DESIGN EQUATIONS

Figure 7.1: Typical bond-slip relationship

Figure 7.2: Effect of interfacial stiffness on the applied load-central deflection relationship
7.2. PARAMETRIC STUDIES

7.2.1.2 Effect of Interfacial Bond Strength

To clearly demonstrate the effect of FRP/concrete interfacial bond strength ($\tau_{\text{max}}$), local bond strengths ranged between 4 MPa, 6 MPa and 8 MPa were considered. As seen in Figure 7.3, increase of bond strength has a low effect on the global structural stiffness. If keeping other parameters constant, high bond strength presents more stress transfer, which resulted in small increase of ductility of the beam. Generally, this means that interfacial bond strength does not have a significant influence on the load carrying capacity and also the stress in the FRPs. This may be due to the fact that the debonding occurs at the softening branch of the interfacial bond curve.

![Figure 7.3: Effect of interfacial bond strength on the applied load–central deflection relationship](image)

7.2.1.3 Effect of Interfacial Fracture Energy

The larger interfacial fracture energy ($G_f$) the harder debonding becomes. The influence of the interfacial fracture energy ($G_f$) is investigated through various values of maximum interfacial slip ($s_{\text{max}}$). The following values were considered for the comparison of interfacial fracture energy: $s_{\text{max}} = 0.2$ mm, $s_{\text{max}} = 0.4$ mm and $s_{\text{max}} = 0.8$ mm. With the increase of interfacial fracture energy, the ultimate load carrying capacity and the structural ductility can be enhanced, as shown in Figure 7.4. Low interfacial fracture energy resulted in early debonding and limited the transferable stresses to the FRPs leading to
crushing of the concrete.

To have insight into the trend of the FRP/concrete interfacial behaviour, the above comparison values is extended to investigate bond-slip interfacial relationships for the various fracture energy values. The investigation is conducted for an interface element corresponds to the location of first debonding occurrence. This element is taken at the top edge near the loading point. The influence of the interfacial fracture energy on the interfacial bond-slip relation is depicted in Figure 7.5(a)–(c). For a maximum interfacial slip \( s_{\text{max}} \) is 0.2 mm (Figure 7.5a), the debonding occurred at the descending branch after few iterations. With the increase of the maximum interfacial energy, this leaded to delay the debonding. It is of interest to mention that the increase in the predicted debonding load when using high interfacial fracture energy is similar to the assumption of full bond between the concrete and FRPs.

In conclusion, the interfacial fracture energy \( (G_f) \) rather than the interfacial stiffness \( (k_s) \), and interfacial bond strength \( (\tau_{\text{max}}) \) has a significant influence on the global behaviour of FRP shear-strengthened beams.
Figure 7.5: Comparison between the provided and predicted shear stress-slip curves for various values of interfacial fracture energy.
7.2.2 Parameters of Shear-Strengthened Beams

The effect of important parameters such as the FRP elastic modulus, FRP thickness, shear steel stirrups, concrete compressive strength, width ratio between the FRP to the concrete beam is investigated through a parametric study. This study is carried out for side-bonded shear-strengthened beams, and the variation of the axial strain in the FRPs is the focus. The results show the shear force versus axial strain in the FRP composites at mid-depth of the sheet at the centre of the shear span. The governing failure mode is debonding of FRP sheets. The principal objective of this study is to examine and assess the parameters that most influence the behaviour of shear-strengthened beams.

7.2.2.1 Steel Stirrups

The effect of steel stirrups is studied by increasing the amount of steel stirrups along the shear span. The spacing between the shear steel stirrups is assumed to be 300, 200 and 100 mm, while the cross section of steel stirrups was kept identical. The study included an additional beam without shear steel stirrups. The interaction between the shear steel stirrups and externally bonded FRP composite is confirmed. Figure 7.6 shows that the presence of steel stirrups in shear-strengthened beams is found to reduce the contribution of the FRPs to the shear capacity of the section. The figure illustrated that the gain in the axial strain of the FRPs absolutely decreases as the amount of steel stirrups increased. Additionally, the increase of the amount of shear steel stirrups increase the load carrying capacity of the beam, and the tendency for the debonding.

7.2.2.2 Concrete Compressive Strength

The influence of the concrete strength to the axial strain of the FRPs was only studied by [Deniaud and Cheng, 2001a]. It is well established that the bond conditions depends on the tensile strength of concrete, where such value is proportional to the concrete compressive strength. So one may argue that the axial strain in the FRP composites depends on the concrete compressive strength. In this study, five quantities of concrete compressive strength are taken, i.e., 25, 31, 35, 40 and 45 MPa. We can report that from Figure 7.7 the
axial strain in the FRPs is observed to increase proportional to the increase of the concrete compressive strength. Furthermore, the gain the ultimate load carrying capacity tends to increase by delaying the debonding. This is explained by the increase of compressive strength proportionally increases the tensile strength of concrete, which increases the bond between the FRPs and concrete. These observations coincide with those of Deniaud and Cheng [2001a].

7.2.2.3 Effect of FRP Elastic Modulus

Various values of FRP elastic modulus were used to investigate the influence in the FRP axial strain. These values ranged from 233 GPa to 102 GPa. Figure 7.8 shows that the axial strain in the FRP composites is roughly proportional to the FRP elastic modulus, thereby confirming the results reported by other researchers (Triantafillou 2000). The measured shear carrying capacity gain increases as the value of the FRP composites increases. The implication of this argument is that as the FRP composites become stiffer, debonding of FRPs dominates over rupture, and the axial strain in the FRPs is reduced. Therefore, with decreases of the FRP elastic modulus, the axial strain reaches higher values when the failure mode by rupture of FRPs.
Figure 7.7: Effect of concrete compressive strength on the applied load–FRP axial strain relationship

Figure 7.8: Effect of FRP elastic modulus on the applied load–FRP axial strain relationship
7.2. PARAMETRIC STUDIES

7.2.2.4 Effect of FRP Thickness

The considerable effect of the plate thickness on the axial strain in the FRPs is verified in this study. The variation of the axial strain versus the load carrying capacity on a shear-strengthened beam for four different plate thicknesses: 0.165, 0.3, 0.5 and 0.7 mm are shown in Figure 7.9. Similar to the effect of the FRP elastic modulus, the increase of the plate thickness is found to reduce the axial strain in the FRP composites. It is also interesting to note that the shear capacity of the beam increases with the increases of the composite plate thickness. By observing that the increase of the plate thickness accelerates the debonding.

7.2.2.5 Effect of Width Ratio Between the Bonded FRP Plate to the Concrete Member

Although the width ratio between the bonded FRP plate to the concrete member was shown to have a significant effect on the ultimate bond strength, this parameter has not been studied by any of the researchers. This study is perhaps the first to investigate this ratio. Only one sheet was attached at the centre along the shear span with various sheet widths. These are 50, 100, 350 mm, while the shear span length was 750 mm. The effect of the width ratio between the bonded FRP plate to the concrete member on the FRP axial strain and the load carrying capacity is drawn in Figure 7.10. It is observed that the

Figure 7.9: Effect of FRP thickness on the applied load–FRP axial strain relationship

![Figure 7.9: Effect of FRP thickness on the applied load–FRP axial strain relationship](image-url)
CHAPTER 7. PARAMETRIC STUDIES AND DESIGN EQUATIONS

contribution of the FRPs to the ultimate capacity of the beam increased with increasing the ratio, while the axial strain in the FRPs decreased with increasing the ratio. The results for the FRPs axial strain might be misleading, because those results were taken in a point and the sum of strain along the sheet width might increase with increasing sheet width.

7.2.2.6 Effect of Shear Span to Depth Ratio

Figure 7.11 presents the measured applied load carrying capacity versus the axial strain in the FRPs. Four different shear span to depth ratios are considered: 2.6, 3.0, 3.4 and 4.0; this is obtained by changing the shear span length. Two zones can be identified in the figure: (a) the zone corresponding to shear span to depth ratio is between 2.6 to 3.4 (2.6 ≤ a/d ≤ 3.4), where the gain in shear carrying capacity and the axial strain in the FRPs tend to decrease with increasing the ratio; (b) the zone corresponding to shear span to depth ratio greater than 3.4, where the contribution of the composite material (load carrying capacity and FRP axial strain) appears to increase with the increase of the a/d ratio.

The controversial aspect of the results is that the shear span to depth ratio appears to be less effective as far as the contribution of the FRPs axial strain is concerned. It can
be concluded that the shear contribution of the FRP plies has no distinct relation with shear span to depth ratio.

It is deemed useful to examine and analyse the parameters that most influence the behaviour of shear-strengthened beams. The parameters related to the properties of the FRP, and concrete strength are not the only ones having a significant influence on the behaviour of shear-strengthened beams. Shear steel stirrups, width ratio between the bonded FRP plate to the concrete member have influence on that behaviour. No distinct relation to FRP contribution and the shear span to depth ratio is observed.

### 7.3 Design Equations

The complexity of shear-strengthened beams, combined with the limited experimental research available, makes it difficult to develop robust predictive design equation suitable for practical use. Many design equation have been proposed based on extended experimental results. The design equations have been established by different researchers used the same analogy as for steel stirrups with an effective FRP strain that include the effect of the FRP axial rigidity and concrete compressive strength. Other parameters that may influence the behaviour of FRP shear-strengthened beams such as shear steel stirrups are neglected. The shear design equations that will be presented in this section is based on a
regression analysis of a large number of numerical results.

Bond between the FRPs and the concrete is of critical importance to the effectiveness of shear-strengthened beams. If this interfacial bond is compromised before FRP sheet rupture, sheet-debonding failure occurs. Such a failure mode diminishes the strengthening potential of FRP composites. Most of the available design equations give good correlation with experimental results when the failure is due to fracture of the FRPs. Many design standards stated that at that case an effective strain of FRP composites is about 0.7 times the ultimate tensile strain in FRPs. If debonding of the FRPs is the predominant failure, some discrepancies have been observed between the experimental results and design equations. Since the main goal of this research is to provide an appropriate design equation, the present study aims to provide and describe a relatively simple design approach for the contribution of FRP shear reinforcement when failure occurs due to sheet debonding. A modification to the traditional parameters that influence such behaviour will be concerned.

A new methodology is proposed in this study for the effective FRP strain at debonding failure. Such equation uses two separate equations: for sides-bonded beams, and for U-wrap beams. To create a new methodology for the effective FRP strain, the numerical model results are taken into statistical analyses consisting of three steps. Firstly, Response Surface Methodology (RSM) is used to compensate the numerical model. Five parameters are considered; namely, the shear steel stirrups, concrete compressive strength, FRP elastic modulus, FRP thickness and width ratio between the FRP to the concrete member. The influence of the shear span to effective depth ratio is excluded because it has no distinct relation to the shear contribution of FRPs. The RSM is carried out using the Design-Expert [2000] Software. Secondly, Microsoft Office Excel 2003 is used to perform the Monte Carlo simulation. The reason for that is the amount of results produced by RSM is not satisfactory to draw an acceptable design equation. Monte Carlo simulation created thousand random values for the effective FRP strain. In the third step, a nonlinear regression analysis is employed to generate a design equation describing the effective FRP strain. The details of the statistical analyses are given below.
7.3.1 Response Surface Methodology (RSM)

In many experimental situations, it is desirable to group sets of experimental runs together. The number of runs that can be made without concern about variation caused by factors not being studies specifically in the experiment. Response surface methodology (RSM) is a method to explore the relationships between several independent variables and one or more dependent variables. Also, to determine which expletory variables have an impact on the response variables. The method was introduced by Box and Wilson [1951]. The principal idea of RSM is to use a set of designed experiments to obtain an optimal response. This model is only an approximation, but it is easy to estimate and apply, even when little is known about the process. The model has a high accuracy when a first-degree polynomial is obtained.

RSM was applied in optimizing the FRP axial strain response (dependent variable). Five independent variables are taken into account: the shear steel stirrups (F1), concrete compressive strength (F2), FRP elastic modulus (F3), FRP thickness (F4) and width ratio between FRP to the concrete member (F5). These parameters were grouped in three levels: +1 (maximum value), 0 (medium value) and -1 (minimum value). Table 1 details these variables and their ranges of interest. It is intended that the values of these variables to be in practical ranges. As results, a factorial design with \((2^k + 2k + 1)\) number of numerical predictions (where n is the number of dependant variables) is carried out. In this analysis, an appropriate model is selected to fit the relationship between the independent and dependent variables over the entire space. The least squares method is used to shape a model by adjusting the residual errors, which is explained by the sum of the squares of the differences between the actual and the estimated response.

<table>
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<th>Mid</th>
<th>High</th>
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<td>Concrete compressive strength (MPa)</td>
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<td>50</td>
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<td>FRP elastic modulus (GPa/10^5)</td>
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<td>F4</td>
<td>FRP thickness</td>
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<td>0.500</td>
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<tr>
<td>F5</td>
<td>(b/fbc)</td>
<td>0.1</td>
<td>0.5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

* Transverse steel ratio is given as \(100nA_{sv}/bwb_c\), where n, A_{sv}, b_w and d are the number of stirrups along the shear span, cross sectional of stirrups, web width, and effective depth of the beam, correspondingly.
This analysis was aimed to predict the better combination of the independent variables for the axial strain of side-bonded, and U-wrap strengthening schemes. Accordingly, the model for the effective FRP axial strain for the side-bonded beams is obtained by using a third-order polynomial as follows:

\[
\varepsilon_{fe} = 0.0017 + \sum_{i=0}^{5} \sum_{j=0}^{5} [\alpha_i F_i + \beta_i F_i^2 + \xi_{ij} F_{ij}] 
\]  

(7.1)

where \( \alpha = \begin{bmatrix} -0.34 \\ 0.12 \\ -0.077 \\ -0.22 \\ -0.03 \end{bmatrix} \) and \( \beta = \begin{bmatrix} 0.067 \\ -0.023 \\ 0.0018 \\ -0.45 \\ 0.45 \end{bmatrix} \)

and \( \xi = \begin{bmatrix} 0 & -0.46 & 0.12 & 0.16 & 0.089 \\ 0 & 0 & 0.026 & 0.003 & -0.028 \\ 0 & 0 & 0 & -0.029 & -0.011 \\ 0 & 0 & 0 & 0 & -0.02 \end{bmatrix} \)

The RSM model for the effective FRP axial strain for U-wrap strengthening scheme is expressed as:

\[
\varepsilon_{fe} = 0.003 + \sum_{i=0}^{5} \sum_{j=0}^{5} [\alpha_i F_i + \beta_i F_i^2 + \xi_{ij} F_{ij}] 
\]  

(7.2)

where \( \alpha = \begin{bmatrix} -0.167 \\ 0.2 \\ -0.09 \\ -0.46 \\ -0.01 \end{bmatrix} \) and \( \beta = \begin{bmatrix} 0.033 \\ -0.057 \\ 0.0038 \\ 0.11 \\ 0.013 \end{bmatrix} \)
From the regression models shown above the interaction between the associated independent variables in obtaining the best fit. The parameters of RSM models are statistically significant and all the requirements of regression, such as normality of residuals ($R^2$), standard deviation (SD) and coefficient of variance (C.O.V), are fulfilled. Since the regression models predicted by RSM are appropriate to simulate the finite element response, those models can be used to estimate the response instead of the numerical predictions.

### 7.3.2 Monte Carlo Simulation

The number of trials that was used to generate RSM models is not enough to draw a sufficient conclusion for the effective FRP axial strain. The Monte Carlo simulation is employed to generate random combinations for the five independent variables to estimate their corresponding dependant response of the RSM models. A Monte Carlo method is a computational algorithm which relies on repeated random sampling to compute its results. Monte Carlo methods are often used when simulating response model exhibits nonlinear dependance on the unknown variables.

We would like to emphasis that Monte Carlo simulation simplifies the RSM models; however, it generates immense outputs. The simulation approach tends to follow a particular pattern:

- define a domain of possible inputs.
- generate inputs randomly from the domain, and perform a deterministic computation on them.
- aggregate the results of the individual computations into the final result.

In this analysis, Microsoft Office Excel 2003 is carried to perform such simulations.
7.3.3 Nonlinear Regression Analysis

In this analysis, we hope to generate design models that can be used in design specifications. The RSM models are complicated and hard to be used by the designers. Nonlinear regression analysis is performed to simplify the RSM models. Nonlinear regression analysis is beneficial when a model exhibits nonlinear dependence on unknown variables. The DataFit software DataFit [2005] performs statistical regression analysis to estimate the values of independent variables for polynomial RSM models was carried out in this study.

The curve fitting determines the values of the parameters that cause the function to best fit the data. This function defines the differences between the actual and predicted responses. The process of selecting the best function is an iterative process. The iterative process starts with initial estimates and incorporates algorithms to improve the estimates. The new estimates become a starting point for the next iteration. These iterations continue until the values of the text function are converge. Finally, a nonlinear regression analysis is used to predict the values of the parameters defining the proposed numerical model.

7.3.4 Proposed Design Equations

The deficiencies of existing design proposals necessitate the development of new shear strength models. Beams with bonded FRP composites with a large strain capacity may fail in shear with FRP far away from the FRP ultimate strain at beam debonding failure. There is currently insufficient information in the literature to determine a suitable value for the maximum usable FRP axial strain quantitatively at the case of debonding. The fact that the stress in the FRP composites varies along the shear crack, explains why the contribution of FRPs differs from the shear steel stirrups.

Based on the above RSM models and nonlinear regression analysis, the following relatively simple design equations are proposed to estimate the FRP axial strain of shear-strengthened beams. Two different design equations are presented: when beams are shear-strengthened with side-bonded schemes, and when beams are shear-strengthened with U-wrap scheme. This approach is more correct, as it treats the two strengthening schemes separately. These new methodologies focused on FRPs debonding is a dominant
7.3. DESIGN EQUATIONS

The effective FRP axial strain for shear strengthening using side-bonded scheme a design equation is established as a function of the five independent variables, and expressed as:

\[
\varepsilon_{fe} = \exp(-0.0558\rho_{sv} + 0.0231f'_c - 0.0227E_f - 0.019t_f - 0.0085b_f/b_c - 7.29)
\] (7.3)

The proposed design equation has a \( R^2 \) value equal to 0.92, which indicates a good fit with the RSM model. The equation defines the effective FRP axial strain as a function of the shear steel stirrups (\( \% \)), concrete compressive strength (MPa), FRP elastic modulus (MPa/10\(^5\)), and width ratio between the FRPs width to concrete beam width. It is obvious that the FRP axial strain increases with the decrease of the shear steel stirrups ratio (\( \rho_{sv} \)), FRP elastic modulus (\( E_f \)), or width ratio between the FRPs width to concrete beam width (\( b_f/b_c \)). However, it increases with the increase of concrete compressive strength (\( f'_c \)). This is, in fact, the same as what was observed in the parametric studies. The FRP axial strain for the U-wrap strengthening scheme is given by:

\[
\varepsilon_{fe} = \exp(-0.2568\rho_{sv} + 0.00171f'_c - 0.332E_f - 0.207t_f - 0.104b_f/b_c - 5.48)
\] (7.4)

The normality of residual (\( R^2 \)) for this equation has a lower value (0.86) than the previous one. Same as for the other equation, the FRP effective axial strain increases with the decrease of of the shear steel stirrups ratio (\( \rho_{sv} \)), FRP elastic modulus (\( E_f \)), or width ratio between the FRPs width to concrete beam width (\( b_f/b_c \)). However, it increases with the increase of concrete compressive strength (\( f'_c \)).

Finally, we can observe that the FRP axial effective strain in new design equations based on the variation of five independent parameters rather than those predicted by design standards. These equations are only valid for the uncracked FRP shear-strengthened beams.
7.3.5 Comparison with Experimental Results

An extensive literature review is carried out to investigate the accuracy of the proposed equation of the FRP axial strain of shear-strengthened beams with experimental results. Shear-strengthened beams that failed by FRP fracture are not reported here. Table 7.2 and Table 7.3 includes shear-strengthened beams that failed by FRP debonding for both strengthening schemes (side-bonded and U-wrap). This database gathers all relevant data, such as the section type, scheme of strengthening and depth size. The specimens designation correspond to those used in the original references. Table 7.2 and Table 7.3 show the effective FRP axial strain and contribution of the FRP to the shear resistance, respectively. Further details can be found from the original sources. Tests that were not sufficiently well documented are excluded.

For the 61 beams listed in Table 7.2 and Table 7.3, they were tested under symmetric three-point bending or four-point bending. The beams have the following parametric ranges: beam height is between 200 to 600 mm; web thickness is between 100 to 300 mm; concrete compressive strength is between 25 to 52 MPa; shear steel stirrups ratio is between 0 to 1.2(%) ; FRP elastic modulus is between 35 to 210 GPa, FRP thickness is between 0.165 to 2 mm; width ratio between the FRPs width to concrete beam width is between 0.25 to 1.

After the above-presented database of experimental results of shear-strengthened beams, it is of interest to see how the proposed design equation compares with predictions from available design guidelines ACI, ISIS, FIB, BS. The various approaches are briefly described in Chapter 2. In all these design guidelines, their approaches will be adopted individually to obtain the effective FRP axial strain of each of shear-strengthened beams showed in Table 7.2. However, the FRP shear contribution \(V_f\) is calculated by considering the equation of ACI. According to ACI, the shear capacity contributed by FRP can be calculated by the following equation:

\[
V_f = A_f\varepsilon_f E_f (\sin \alpha + \cos \alpha) d_f / s_f
\]

(7.5)

Then, the predicted values from each approach will be compared with experimental results.
Table 7.2: Comparison of FRP axial strain of shear-strengthened beams controlled by debonding

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<th>Specimen</th>
<th>Section</th>
<th>Str</th>
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<th>Exp</th>
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In the calculation of the effective FRP axial strain $\varepsilon_{fe}$ and FRP shear capacity contribution $V_f$, all the necessary safety factors are incorporated. The experimental shear contribution by the FRP is obtained by subtracting the shear capacity of the reference beam, which represents the shear contribution by concrete and steel stirrups. This means that we neglect the superposition effect of the steel stirrups on the FRP shear contribution. Figure 7.12(a)–(e) show the comparison between the experimental results of the effective FRP axial strain and the ACI, ISIS, FIB, BS and proposed design equation, respectively. Also, the test data are compared with design equation in terms of FRP shear contribution is shown graphically in Figure 7.13(a)–(e).

According to Figure 7.12, for the FRP axial strain, the proposed design equation is able to give reasonable predictions for various test data, whereas both the ACI and ISIS equation (Figure 7.12 (a) and (b)) tends to underestimate the effective FRP axial strain. However, the ACI and ISIS equations can be conservative in some cases, but unconservative in others. Based on the results shown in Figure 7.12(c) and (d), it can be stated that the FIB and BS could not estimate the FRP axial effective strain correctly. The difference may be attributed to the various volume of shear steel stirrups in the specimens. The contribution of the FRP axial effective strain is lower in a beam containing shear stirrups when compared to a member without steel stirrups, so the presence of shear stirrups influence the effectiveness of FRP. We should note that FIB design practise does not provide an equation for the side-bonded strengthening scheme as well as the for AFRP and GFRP materials in case of debonding.

For the FRP shear contribution shown in Figure 7.13(a)–(e), either the ACI or ISIS equation can give conservative results, as the predicted results are higher than the test results. The proposed design equation actually gives the closest agreement with test data due to the fact that consideration of parameters most influence the behaviour of shear-strengthened beam. The model predicted the experimental results with relatively good accuracy, which is obvious because the model include the parameters most influence the shear behaviour. It is seen that the model proposed by FIB and ISIS could not assess correctly the FRP shear contribution. The model showed poor correlation with experimental results for both side-bonded and U-wrap strengthening schemes.

Finally, since design guidelines were derived from relatively limited experimental database and did not take into account most of factors affecting the FRP shear contribution, they
Figure 7.12: Comparison of FRP axial strain results between experimental and various design codes: (a) ACI; (b) ISIS; (c) FIB; (d) BS; (e) new design equation
Figure 7.13: Comparison of FRP shear strength between experimental and various design codes: (a) ACI; (b) ISIS; (c) FIB; (d) BS; (e) new design equation
are not able to predict the experimental results with good accuracy. The proposed design equation is believed to be the most appropriate for practical design.

7.4 Prediction of FRP Axial Effective Strain Profile

Little attention is currently available on the actual strain distribution in the FRP. This is partially attributed to the fact that the importance of this behaviour was not realized in previous studies. The strain distribution is difficult to measure in tests because the position of the critical shear crack is unknown a priori and the failure process is brittle and thus rapid. In this section, the developed theory arrived at describing the FRP strain distribution along a shear crack.

It is evident that the axial strain distribution in the FRP along the shear crack is non-uniform. This phenomenon can be attributed to the fact that the shear crack width varies along its length. The axial strain in the FRP intersected by a shear crack is closely related to the width of the crack due to a linear elastic behaviour of FRP composites. The non-uniform strain distribution leads to a fundamental behaviour difference between shear steel stirrups and FRP composites. Shear steel stirrups at yielding can undergo large plastic deformation, so it is reasonable to assume that all shear steel stirrups intersected by a shear crack reaches the yield strength for shear design. For FRP shear reinforcement, the non-uniformity of stress distribution means that the failure process will start once the most highly stressed point in the FRP intersected by the shear crack reaches the ultimate tensile strength of the FRP due to the lack of any post-peak ductility.

Rupture of adjacent parts of the FRP then follows quickly, as the adjacent intact parts of the FRP need to take over the forces released by the ruptured parts. Rupture of FRP, once started, will thus propagate rapidly along the shear crack, leading to catastrophic failure of the beam. In the debonding failure mode, the failure process differs from that of fracture. This because the bond length of the FRP depends on the location of the shear crack relative to the ends of the FRP. The FRP intersected by the shear crack, therefore, are not stressed to the same axial effective strain at instant of failure. As a result, the simple analogous assumption that the FRP intersected by a shear crack will all have the same level of axial effective strain is inappropriate.
Figure 7.14: Predicted FRP axial strain profile

To develop an accurate shear strength model, a rational FRP axial effective strain profile in the FRP needs to be assumed and considered. Let us consider FRP composites attached to beam faces without special anchorages (mechanical fastening) and a shear crack at an angle $\beta$ to the longitudinal axis of the member. A description of the FRP axial effective strain profile along the shear crack is shown in Figure 7.14, which indicates different regions of distribution. The first region is mid-way along the shear crack, the second region is beside the bottom edge of the shear crack, while the third region is near the top end. In the first region, mid-way along the shear crack, the higher strain values are occurred at this area. The experimental and numerical studies conducted in this study predict the same analogy. The FRP axial strain profile decrease towards the bottom edge of the shear crack, second region, to a low values. The same response is observed for the U-wrap strengthening scheme. In the third region, almost zero values are obtained. Notice that fibres are compressed in this region and the reason for that is the negative stresses found at that region.

The assumption of the FRP intersected by a shear crack will all have the same effective FRP axial strain is very conservative. So that imply the maximum effective FRP axial strain measured at the mid-way length of the shear crack is not accurate. The average strain is thus only fracture of that strain. For practical design, the design effective FRP axial strain is assumed uniformly distributed along the shear crack, as shown in Figure 7.15, with a value equal to half of the effective FRP axial strain located in the midway along the shear crack $\varepsilon_{fe}$. The design FRP axial effective strain is defined as:

$$\varepsilon_{fed} = 0.5\varepsilon_{fe}$$  \hspace{1cm} (7.6)
The presence assumption of design FRP axial effective strain is based on the FRP composites are not stressed at the same level along the shear crack. This assumption is unconservative and it leads to close agreement with the experimental data.

7.5 Summary

In this study, a nonlinear finite element model included the FRP/concrete interfacial behaviour was used to clarify the effects of bond properties on the performance of shear-strengthened beams. Through performing such study, the following conclusions can be drawn and be used as a reference for the shear-strengthened beams practical applications. Interfacial stiffness have no effect of the structural stiffness and ultimate load carrying capacity. The interfacial maximum bond strength, to a less extent, influences the ultimate carrying capacity. Furthermore, the interfacial fracture energy is the main parameter influencing the strengthening performance in terms of ultimate load carrying capacity, and ductility. High interfacial fracture energy can be used to better enhance the axial strain in the FRP composites.

In this chapter, the parameters that have the greatest influence on the behaviour of shear-strengthened beams were examined and analysed. Several conclusions were drawn from this study. It was found that the parameters related to the FRP axial stiffness and the concrete compressive strength are not the only ones having an influence on that behaviour. The shear steel stirrups as well as the width ratio between the FRP to the width of the beam also have impact on the response of shear-strengthened beams. The
results of the FRP axial strain are essential to understand the behaviour.

The results of the parametric study indicate that the technique of strengthening, side-bonded or U-wrap, is particularly effective to increase the FRP axial effective strain. Increasing the compressive strength of concrete appreciably increases the effective strain. However, increasing of FRP elastic modulus and thickness decreases the effective strain. This is attributed to the increase of these values accelerates the debonding. Small width of FRP composites compare to that of the beam increases the FRP effective strain. The influence of the shear span to effective depth ratio needs further research.

A simple design approach that used statistical analysis was presented in this study for the calculation of the FRP contribution to the shear capacity. In that analysis, the effect of five parameters (shear steel stirrups ratio, concrete compressive strength, FRP elastic modulus, FRP thickness and width ratio between the FRP and beam) was considered. The statistical analysis includes three steps. In the first step, the RSM models were employed to define the response of the effective FRP axial strain according to the variance of the above parameters. These RSM models are thought beneficial to replace the finite element simulation. Using the RSM model, Monte Carlo simulations were conducted in the second step to generate a large combination of the five variables. Finally, a nonlinear regression analysis was employed to establish relatively simple design equations that best fit the large response data set.

FRP rupture and debonding are the two main shear failure modes identified for shear-strengthened beams. Their different failure mechanisms mean that separate treatment is essential to develop accurate shear strength models. This study is concerned with the development of a new design equation for shear-strengthened beams that fail in debonding. The truss model was used to satisfactorily describe the contribution of FRP composites. The key contribution of the present study was the realization of the fact that the effective FRP axial strain distribution along the shear crack is non-uniform, and explicit account was taken of this behaviour in the proposed design equation. The shear contributions of published experiments was compared with the new design expression, together with other design guidelines of ACI, ISIS, FIB and BS. The new strength model is found to compare very well with a large amount of independent experimental data. The proposed equation is hence the best compromise to be employed for practical design calculations.
This study predicted three regions of axial strain distribution along the shear crack. This is because the maximum axial strain is concentrated along the shear crack. In the first region, mid-way along the shear crack, the higher strain values are occurred at this area. The FRP axial strain profile decrease towards the bottom edge of the shear crack, second region, to a low values. In the third region (near the top end), almost zero values were obtained.
Chapter 8

Conclusions and Recommendations

8.1 Introduction

To conclude this dissertation, some remarks should be made on the modelling of shear-strengthened beams with the consideration of the FRP/concrete interfacial behaviour as a major parameter. This study is an attempt to develop an accurate numerical tool that is able to describe the behaviour of such beams. Numerical analyses were performed with the ADINA finite element commercial package.

A comprehensive theoretical investigation on various shear-strengthening schemes was presented. The research work was divided into two main phases. The first phase is the development of a reliable numerical model to properly simulate the response of shear-strengthened beams. This was achieved through an investigation of various elements for the FRP composites and FRP/concrete interfacial behaviour. The numerical model studied the validity of using shell or truss elements to simulate the behaviour of FRP bonded sheets. On the other hand, through different interface elements, the FRP/concrete interfacial behaviour was verified. These were spring elements, discrete truss elements and continuous truss elements. The objective of this comparison was to identify the appropriate interface elements to accurately predict the FRP/concrete interfacial behaviour. In addition to the vertical interface elements, the current study investigated the influence of including horizontal interface elements. Two arrangements of the interface elements were examined and compared to each other. In the first arrangement, the interface elements
were oriented parallel and perpendicular to the beam axis. In the second arrangement, the interface elements were oriented perpendicular to the beam axis. In order to develop a reliable numerical model, the mesh density was investigated to examine the fluctuation of the slip profiles along the FRP sheet depth. The first phase of this study as well included numerical simulations of the shear-strengthened beams with the DIANA computer software.

In the second phase of this thesis work, the three-dimensional finite element model was applied to various strengthening strategies; namely, beams with vertical and inclined side-bonded FRP sheets, U-wrap FRP strengthening configurations, as well as anchored FRP sheets. It has been shown that the proposed model was capable of accurately predicting the load carrying capacities and load–deflection relationships for a wide variety of FRP shear strengthened applications. However, the numerical model sometimes exhibited a slightly stiffer behaviour than that observed in the experiments. In addition, it can provide valuable insight into quantities that are difficult to measure in the laboratory, such as interfacial slip profiles, shear crack angles and the details related to the initiation and propagation of FRP debonding. The analysis was also capable of predicting the strains along depth of the FRP sheets.

Finally, statistical analysis and parametric studies were carried out to propose a new design expression for the effective FRP axial strain. Different parameters that most influence the behaviour of shear-strengthened beams were considered. The parametric study was followed by a statistical analysis incorporating five variables; namely, shear steel stirrups ratio, concrete compressive strength, FRP elastic modulus, FRP thickness and width ratio between the FRP sheets to the concrete beam. The response surface methodology (RSM) was employed to optimize the accuracy of the statistical model. Based on the results of the statistical models, a design expression was recommended for the effective FRP axial strain in the FRP sheets for shear-strengthened beams at the state of debonding.

Based on the main findings of this thesis, the following sections outline the detailed conclusions resulting from this study and recommendations for future work.
8.2 Conclusion from Development of a Reliable Numerical Model

The effectiveness of two different elements to simulate the behaviour of FRP sheets was studied. By comparing the measured and predicted load-deflection behaviour, it was showed that the numerical model can effectively simulate the global response when shell elements were employed for the FRP composites. In particular, the consideration of the FRP sheet width is a key parameter that influences the behaviour of shear-strengthened beams.

Based on the conclusion that the numerical model with a shell simulation of the FRP composites can successfully predict the response of the experimental beams in terms of ultimate load carrying capacities and load-deflection relationships, the numerical model is expected to give reliable predictions for the slip profiles. The simulation of the FRP/concrete interfacial behaviour was affected by the type of interface element used to represent such behaviour. While the global behaviour was well predicted, meaning accurate load-deflection behaviour was obtained, the nonlinear spring interface elements were unable to accurately capture the debonding failure. Since it is desirable to accurately predict the interfacial slip response and determine the regions of severe cracking, discrete truss elements appear to be more appropriate to achieve this goal. It was also found that the inclusion of horizontal interface elements did not modify the results of the analysis.

Moreover, it can be concluded from the current research that the behaviour of the bond interface must be considered for an accurate prediction of the response of the member. Neglecting the interfacial behaviour would lead to overestimated stiffnesses and ultimate loads. Modelling of the FRP/concrete interface is dependant on the characterization of the bond zone. In the current research, a convergence study was carried out to examine an appropriate mesh density and to ensure that the spatial discretization used did not introduce excessive approximations into the simulations. Although the convergence study showed potential in enhancing the accuracy of the numerical predictions, the increase in mesh density did not yield a significant difference in predicting the response of shear-strengthened beams.
8.3 Conclusion from Size Effects of RC Beams Strengthened with FRP Composites

8.3.1 Experimental Investigations

The experimental program presented in this study pointed the way to provide data on beams on practical sizes as well as to investigate the strengthening effectiveness for small and large specimens. In the present investigation, geometrically similar beams with depths of 200, 400, and 600 mm were retrofitted in shear with carbon FRP strips. Based on our findings, the strengthening effectiveness may significantly decrease with member size. However, it did not meet the aim that the increase in effective depth offers the opportunity to better exploit the FRP composites tensile strength due to the increased of the bonded area. The experimental results also supplied valuable information on the effect of the depth increase. It was found that the increase of depth is consistent with excessive shear cracks prior to failure. It was also observed that the FRP axial strain was not uniformly distributed along the height of the strip, and it increases up to a limit beyond which no further increase was reported.

8.3.2 Numerical Investigations

The proposed numerical model was carried out in parallel to the experimental results to investigate the behaviour of shear-strengthened beams in more detail. The numerical model was able to simulate the characteristics of the shear-strengthened beams, including the interfacial behaviour between the concrete and CFRP strips. The numerical predictions compare very well with the experimental data in terms of ultimate load carrying capacity, load-deflection relationships and failure modes. The maximum CFRP axial strain values were found in the same locations as the ultimate interfacial slip values. This finding herein is different to those of the other researchers. Furthermore, the numerical model was able to capture the shear crack angle along the shear span.
8.4 Conclusion from Various Configurations of FRP Composites

The investigation of the numerical model was extended to another batch of shear-strengthened beams with different strengthening schemes and with different shear span to effective depth ratios. The various shear-strengthened beams considered included strengthening with strips and continuous U-wrap, as well as completely wrapped strengthening schemes. These beams were also used to investigate whether the use of a top horizontal band anchoring for a U-shaped wrapping can prevent debonding. The specimens were strengthened with either GFRP or CFRP sheets to examine the effectiveness of using various strengthening materials.

Finite element analysis was carried out to address the behaviour of the ten shear-strengthened beams. This study clearly showed the importance of appropriately modelling the FRP/concrete interface if accurate predictions of the behaviour are to be obtained. It also demonstrated that reliable numerical models represent very significant tools for gaining insight into phenomena that are difficult to measure experimentally. The validity of the numerical models was investigated against the experimental data. The comparisons between the numerical and experimental results showed very good agreement.

The numerical model was capable of predicting typical failure scenarios as the tested specimens. At the stage of excessive stress transformation from the concrete to the adjacent FRP composites, the delamination was initiated at the intersection between the shear crack and the bonded strips. Debonding of the interface elements at top edge of the strip beside the load was the first indication of the delamination. After removing the debonded interface elements and with the load increase, the interface elements of the adjacent strip were delaminated at the intersection between the shear crack and the bonded strips and continued towards the top edge. The delamination occurred at the corresponding points of maximum slip values. This behaviour progressed until the concrete crushing.
8.5 Conclusion from Design Equations

The calibration study between the experimental and numerical results leads to a successful modelling of different aspects of the finite element analysis so that a good agreement between the numerical models and the available experimental data was achieved. This calibration of the finite element analysis was followed by a parametric study to generate several pre-design cases of FRP shear-strengthened beams.

The data generated of these cases are pre-designed using the Response Surface Methodology (RSM) statistical approach. The impact of five parameters on the FRP shear contribution was investigated. The studied parameters were the shear steel stirrups ratio, concrete compressive strength, FRP elastic modulus, FRP thickness, and width ratio between the FRP to the beam width. The output response was the effective FRP axial strain for side-bonded and U-wrap strengthening schemes. The RSM models were useful in replacing the finite element models. Nonlinear regression analysis was used to develop a relatively simple design equation to predict the effective FRP axial strain based on the five parameters.

The proposed design equation can be very beneficial for design engineers to predict the FRP contribution of FRP shear-strengthened beams. The shear contribution of FRP assessed by the new design equation, together with equations from design guidelines of ACI, ISIS, FIB and BS were compared with the available experimental results. The new design equation appears to be the best compromise to be employed for practical design calculations.

8.6 Recommendations for Future Work

This investigation is a step towards the simulation of FRP shear-strengthened beams and it leads to a better understanding of the behaviour. Based on the findings of the numerical analyses, there is still a need for further studies. The following are some recommendations for future work:

- This study does not consider the behaviour of some parameters that may influence
8.6. RECOMMENDATIONS FOR FUTURE WORK

FRP shear-strengthened beams, such as the spacing between the FRP strips, the longitudinal steel reinforcement, and the shear span to effective depth ratio. These parameters should be examined.

- With the U-wrap strengthening scheme, both failure modes are possible but when each failure mode might occur is a question that has not yet been answered. Special attention should be paid to this question.

- The effective bond length of the FRP plates influences their axial effective strain. The published results regarding this variable are contradictory. In order to determine the suitable sheet height, more investigations are required.

- In this study, the shear-strengthened beams were analysed only under static loading. It appears that no study has been carried out when the loads are applied in a cyclic manner in order to investigate the fatigue in bridge design. To understand such behaviour, further studies are recommended.

- The concrete material modelling available in the current finite element packages are not appropriate for complicated applications. The last proposition on future research concerns the needs of advanced concrete material modelling for use in the finite element analysis.
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Appendix A

ADINA Concrete Constitutive Model

It is well accepted that concrete is a very complex material. The concrete constitutive model provided in ADINA may not be capable to simulate all structural applications. However, it works well for simple applications, such as FRP shear-strengthening of reinforced concrete beams. The concrete model is a hypoelastic concrete model. It is based on three features: concrete in compression; post-cracking model; and failure envelope. The nonlinear stress–strain relation allows for weakening of the material under increasing compressive stresses. Failure envelopes that define failure in tension and crushing of concrete in compression. A fixed smeared crack model is used to describe the post-cracking behaviour of concrete. The objective of this appendix is to provide a brief description of the characteristics of ADINA, version 8.4, concrete material model.

A.1 Concrete in Compression

The general multiaxial stress–strain relations are derived from a uniaxial (incremental) stress–strain relation. When the concrete in tension the stress-strain relation is linear until tensile failure. In this case, an incremental relation between stress ($\dot{\sigma}$) related to the strain ($\dot{\varepsilon}$) through a modulus of elasticity ($[C]$). Such relation can be expressed as:

$$\{\dot{\sigma}\} = [C]\{\dot{\varepsilon}\}$$  \hspace{1cm} (A.1)
When the concrete is under compression and before the cracking initiated, the following relation is used to calculate the matrix of modulus of elasticity:

\[
E_{pi} = \frac{E_c [1 - B(\frac{\varepsilon_{pi}}{\varepsilon_m})^2 - 2C(\frac{\varepsilon_{pi}}{\varepsilon_m})^3]}{[1 + A(\frac{\varepsilon_{pi}}{\varepsilon_m} + B(\frac{\varepsilon_{pi}}{\varepsilon_m})^2 + C(\frac{\varepsilon_{pi}}{\varepsilon_m})^3]^2}
\]  (A.2)

where, \( A = \frac{E_u + (P^3 - 2P^2 E_u E_m - 2P^3 - 3P^2 + 1)}{([P^3 - 2P + 1]P)} \), \( B = [(2E_u - 3) - 2A] \), \( C = [(2E_u - 3) + A] \)

and the values of \( E_s = \frac{\sigma_{nu}}{\varepsilon_m} \), \( P = \frac{\varepsilon_u}{\varepsilon_m} \) and the parameters \( (\varepsilon_u, \varepsilon_m, \sigma_u, \sigma_m) \) are computed in relation to their corresponding uniaxial values \((\varepsilon_u, \varepsilon_m, \sigma_u, \sigma_m)\) as shown in Figure A.1 as:

\[
\varepsilon_u = (C_1 \gamma_1^2 + C_2 \gamma_1) \varepsilon_u
\]  (A.3)

\[
\varepsilon_m = (C_1 \gamma_1^2 + C_2 \gamma_1) \varepsilon_m
\]  (A.4)

\[
\sigma_u = \gamma_1 \sigma_u
\]  (A.5)

\[
\sigma_m = \gamma_1 \sigma_m
\]  (A.6)
where $C_1$ and $C_2$ are taken as 1.4 and -0.4, respectively. The value of $\gamma_1$ is calculated from the biaxial failure envelope of the concrete according to the ratio $(\sigma_{p2}/\sigma_m)$, as depicted in Figure A.2.

In the region of concrete cracking, the stress–strain relations are evaluated differently. The material is considered as orthotropic with the direction of orthotropy being defined by the principal stress directions. The matrix of modulus of elasticity has the following form:

$$
C = \frac{1}{(1 + \gamma)(1 + 2\gamma)} \begin{bmatrix}
(1 - v)E_{p1} & vE_{12} & vE_{13} & 0 & 0 & 0 \\
(1 - v)E_{p2} & vE_{23} & 0 & 0 & 0 & 0 \\
(1 - v)E_{p3} & 0 & 0 & 0 & 0 & 0 \\
0.5(1 - 2v)E_{12} & 0 & 0 & 0 & 0 & 0 \\
0.5(1 - 2v)E_{13} & 0 & 0 & 0 & 0 & 0 \\
0.5(1 - 2v)E_{23} & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

(A.7)

where $E_{p1}$, $E_{p2}$, and $E_{p3}$ are the equivalent multiaxial modulus of elasticity in the principal directions, computed according to the value of principal strain, $\varepsilon_{pi}$. Additionally, the off-diagonal components are computed using:

$$
E_{ij} = \frac{|\sigma_{pi}|E_{pi} + |\sigma_{pj}|E_{pj}}{|\sigma_{pi}| + |\sigma_{pj}|} 
$$

(A.8)

where $\sigma_{pi}$ is the principal stress value, $i \neq j$ and $i,j=1$ to 3.

When the principal stress state lies on the failure envelope, it is assumed that the material strains soften isotropically in all directions. This corresponds to the case of $\varepsilon_{pi} \leq \varepsilon_u$. The stresses, in all principal directions, are assumed to linearly reduce to zero using the following modulus:

$$
E_{pi} = \frac{\sigma_u - \sigma_m}{\varepsilon_u - \varepsilon_m}
$$

(A.9)

### A.2 FE Material Failure Envelopes

After the principal stresses ($\sigma_{pi}$) have been established with ($\sigma_{p1} \geq \sigma_{p2} \geq \sigma_{p3}$), the stresses $\sigma_{p1}$ and $\sigma_{p2}$ are kept constant and minimum stress value that would have to be reached
APPENDIX A. ADINA CONCRETE CONSTITUTIVE MODEL

Figure A.2: Biaxial concrete failure envelope

in the third direction to cause crushing is calculated using the failure envelopes. If we consider that \( \sigma_m \) is the third principal stress cause crushing of concrete. After calculating the value of \( \sigma_{p1}/\sigma_m \), the two-dimensional failure envelope functions for the stresses \( \sigma_{p2} \) and \( \sigma_{p3} \) are evaluated based on the ratio \( \sigma_{p1}/\sigma_m \). If the value of the principal stress components, \( \sigma_{p2} \) and \( \sigma_{p3} \), lies on outside the biaxial failure envelope, then the material failure has occurred. Figure A.2 shows the biaxial failure envelope of the concrete. For the tensile failure envelope, it is assumed that the tensile strength of the concrete in a principal direction does not depend on the tensile stresses in the other principal stress directions.

A.3 Fixed Smeared Crack Model

To identify whether the material has failed, the principal stresses are used to locate the current stress state. Tensile failure occurs if the tensile stress in a particular stress direction exceeds the tensile failure stress. Based on fixed smeared crack model adopted on this package, it is assumed that a plane of failure develops perpendicular to the corresponding principal stress direction. The effect of this material failure is that the normal and shear stiffnesses, \( E_c \) and \( G_c \), respectively, and stresses across the plane of failure are reduced by reduction factors \( \eta_n \) and \( \eta_s \), respectively. The plane stress conditions are assumed to exist.
at the plane of tensile failure. After concrete cracking, the matrix of modulus of elasticity is presented by:

\[
C = \begin{bmatrix}
  E_c & 0 & 0 & 0 & 0 \\
  \frac{1}{(1-v_e^2)} & \frac{E_p^2}{E_p^2} & \frac{vE_{23}}{E_p^2} & 0 & 0 \\
  \frac{vE_{23}}{E_p^2} & \frac{E_p^2}{E_p^2} & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 \\
  \text{symetric} & \frac{\eta_s E_c}{2(1+\nu)} & 0 & 0 & 0 \\
  & & & & \frac{E_c}{2(1+\nu)} \\
\end{bmatrix}
\]  

(A.10)

When the principal tensile strain, \(\varepsilon_t \leq \varepsilon \leq \varepsilon_{mt}\), where \(\varepsilon_{mt}\) and \(\varepsilon_t\) are represented in Figure A.1, the secant elastic modulus \(E_t\) replaces the norm \(E_c\eta_n\) in the material response matrix. Beyond the strain level, \(\varepsilon_m\), the factor \(\eta_s\) is taken 0.0001 (in order avoid the possibility of a singular stiffness matrix). The factor \(\eta_n\), known as the shear reduction factor, is assumed 1.0 in the case of \(\varepsilon \leq \varepsilon_t\). Then \(\eta_s\) is reduced linearly to be 0.5 at a strain level \(\varepsilon_m\) and remains constant to consider several physical factors such as aggregate interlock, reinforcement dowel action and friction between cracks. In Figure A.1, the maximum tensile strain \(\varepsilon_{mt}\) is related to the concrete tensile strain \(\varepsilon_t\) through a factor \(\xi\) by \(\varepsilon_{mt} = \xi \varepsilon_t\). The factor \(\xi\) defines the amount of tension stiffening computed at the each integration point based on the size of the finite element as:

\[
\xi = \frac{2E_cG_f}{f_c^2h}
\]  

(A.11)

where \(h\) is the width of the finite element perpendicular to the plane of tensile cracks, \(G_f\) is the concrete fracture energy released per unit area. If the normal strain across the existing crack becomes greater than that just before crack formation, crack is assumed to open; otherwise, it is closed. For cracked concrete in which all sets of cracks are closed, the concrete is assumed to behave as an uncracked element.