NONLINEAR MICROMECHANICS-BASED FINITE ELEMENT ANALYSIS OF THE INTERFACIAL BEHAVIOUR OF FRP-STRENGTHENED REINFORCED CONCRETE BEAMS

ANALYSE PAR ÉLÉMENTS FINIS BASÉE SUR UNE LOI MICROMÉCANIQUE DE COMPORTEMENT DE L'INTERFACE POUR DES POUTRES EN BÉTON ARMÉ RENFORCÉES DE PRF

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Abstract

In the last two decades, intensive laboratory investigations have been carried out on the feasibility of using FRP laminates to enhance the flexural capacity of reinforced concrete beams. In such investigations, it has been observed that failure is generally governed by debonding of the FRP laminate off the concrete surface either at the end of the laminate or at an intermediate crack. These premature failures caused by debonding often limit the effectiveness of this strengthening scheme and prevent the reinforced concrete beams from attaining their ultimate flexural capacities. To date, our knowledge of the various debonding phenomena and the associated mechanics of the bond between the FRP and concrete is somewhat limited. Numerical studies, particularly those concerned with the interfacial behaviour of the FRP/concrete interface, are necessary for a better understanding of the failure mechanisms. Such studies can give insight into the quantities that are virtually impossible to be captured experimentally, such as the stress/slip distributions and stress concentrations along the interface.

This research work is devoted to theoretical and numerical studies on the flexural behaviour of FRP-strengthened concrete beams. The objectives of this research are to extend and generalize the results of simple experiments, to recommend new design guidelines based on accurate numerical tools, and to enhance our comprehension of the bond performance of such beams. These numerical tools can be exploited to bridge the existing gaps in the development of analysis and modelling approaches that can predict the behaviour of FRP-strengthened concrete beams. The research effort here begins with the formulation of a concrete model and development of FRP/concrete interface constitutive laws, followed by finite element simulations for beams strengthened in flexure. Finally, a statistical analysis is carried out taking the advantage of the aforesaid numerical tools to propose design guidelines.

Although several powerful finite element packages are now available with rich libraries of structural elements, inadequate constitutive laws for concrete and FRP/concrete interfaces are one of the major challenges restricting the detailed analysis of FRP-strengthened beams. Of the various attempts that have been made to date to develop a unified constitutive theory for concrete, the approach taken by Bažant and co-workers in formulating microplane models has undoubtedly led to the greatest success. In this dissertation, an alternative incremental formulation of the M4 microplane model is proposed to overcome the computational complexities associated with the original formulation. Through a number of numerical applications, this incremental formulation is shown to be equivalent to the original M4 model. To assess the computational efficiency of the incremental formulation, the “arc-length” numerical technique is also considered and implemented in the original Bažant et al. [2000] M4 formulation. Finally, the M4 microplane concrete model
is coded in FORTRAN and implemented as a user-defined subroutine into the commercial software package ADINA, Version 8.4. Then this subroutine is used with the finite element package to analyze various applications involving FRP strengthening.

In the first application a nonlinear micromechanics-based finite element analysis is performed to investigate the interfacial behaviour of FRP/concrete joints subjected to direct shear loadings. The intention of this part is to develop a reliable bond–slip model for the FRP/concrete interface. The bond–slip relation is developed considering the interaction between the interfacial normal and shear stress components along the bonded length. A new approach is proposed to describe the entire $\tau-s$ relationship based on three separate models. The first model captures the shear response of an orthotropic FRP laminate. The second model simulates the shear characteristics of an adhesive layer, while the third model represents the shear nonlinearity of a thin layer inside the concrete, referred to as the interfacial layer. The proposed bond–slip model reflects the geometrical and material characteristics of the FRP, concrete, and adhesive layers.

Two-dimensional and three-dimensional nonlinear displacement-controlled finite element (FE) models are then developed to investigate the flexural and FRP/concrete interfacial responses of FRP-strengthened reinforced concrete beams. The three-dimensional finite element model is created to accommodate cases of beams having FRP anchorage systems. Discrete interface elements are proposed and used to simulate the FRP/concrete interfacial behaviour before and after cracking. The FE models are capable of simulating the various failure modes, including debonding of the FRP either at the plate end or at intermediate cracks. Particular attention is focused on the effect of crack initiation and propagation on the interfacial behaviour. This study leads to an accurate and refined interpretation of the plate-end and intermediate crack debonding failure mechanisms for FRP-strengthened beams with and without FRP anchorage systems.

Finally, the FE models are used to conduct a parametric study to generalize the findings of the FE analysis. The variables under investigation include two material characteristics; namely, the concrete compressive strength and axial stiffness of the FRP laminates as well as three geometric properties; namely, the steel reinforcement ratio, the beam span length and the beam depth. The parametric study is followed by a statistical analysis for 43 strengthened beams involving the five aforementioned variables. The response surface methodology (RSM) technique is employed to optimize the accuracy of the statistical models while minimizing the numbers of finite element runs. In particular, a face-centred design (FCD) is applied to evaluate the influence of the critical variables on the debonding load and debonding strain limits in the FRP laminates. Based on these statistical models, a nonlinear statistical regression analysis is used to propose design guidelines for the FRP flexural strengthening of reinforced concrete beams.
Résumé

Au cours des deux dernières décennies, plusieurs travaux expérimentaux ont été effectués sur l'emploi des polymères renforcés de fibres (PRF) pour augmenter la capacité portante en flexion de poutres en béton armé. Dans ces travaux, on a observé que la ruine est généralement régie par le délaminage du stratifié PRF à l'interface avec le béton, soit à l'extrémité du laminé ou à une fissure intermédiaire. Ces ruines prématurées provoquées par délaminage limitent souvent l'efficacité de ce renforcement et empêchent les poutres en béton armé d'atteindre leurs capacités ultimes en flexion. À ce jour, la compréhension des divers phénomènes de délaminage et les mécanismes associés au lien entre le PRF et le béton est quelque peu limitée. Les études numériques, en particulier celles concernant le comportement de l'interface béton-PRF, sont nécessaires pour mieux comprendre les mécanismes de ruine. Les études numériques peuvent fournir de l'information quantitative sur des phénomènes comme les concentrations de contraintes ainsi que le glissement le long de l'interface, et qui sont pratiquement impossibles à mesurer expérimentalement.

Cette recherche est consacrée aux études théoriques et numériques sur le comportement des poutres en béton armé en flexion renforcées de PRF. Les objectifs de cette recherche sont de prolonger et de généraliser les résultats des expériences simples, de recommander de nouvelles directives de conception basées sur des outils numériques précis, et d'augmenter la compréhension du mécanisme de délaminage. Ces outils numériques peuvent être exploités pour combler les vides existants dans le développement de méthodes analytiques et d'approches de modélisation permettant de prédire le comportement des poutres en flexion en béton armé renforcées de PRF. Ce travail de recherche commence par la formulation des lois constitutives pour le béton et la mise au point des modèles d'interface béton-PRF, suivi de simulations par éléments finis de poutres en flexion renforcées du PRF. Finalement, une analyse statistique utilisant les outils numériques mentionnés ci-haut a été réalisée pour proposer des règles de conception.

Bien que plusieurs logiciels d'éléments finis soient maintenant disponibles avec des bibliothèques riches d'éléments structuraux, leurs capacités d'analyse des poutres en béton armé renforcées de PRF sont limitées par des lois constitutives pour le béton et des interfaces béton-PRF insatisfaisantes. Des diverses tentatives qui ont été faites pour développer une théorie constitutive unifiée pour le béton, l'approche adoptée par Bazant et ses collègues en formulant des modèles de microplan a assurément eu le plus grand succès. Dans cette thèse, on propose une formulation alternative incrémentale du modèle microplan M4 pour surmonter les complexités numériques liées à la formulation originale. Pour de nombreuses simulations numériques, cette formulation incrémentale s'est avérée équivalente au modèle M4 original. Pour évaluer l'efficacité numérique de la formulation incrémentale, la technique numérique de la longueur de l'arc ("arc-length") est aussi considérée et mise en œuvre dans le modèle M4 de Bazant et al. [2000]. Finalement, le
modèle du béton microplan M4 est programmé en Fortran est introduit sous forme d'un sous-programme dans le logiciel commercial ADINA, version 8.4.

Premièrement, une analyse non linéaire par éléments finis basée sur les lois micromécaniques est effectuée pour étudier le comportement de l’interface béton-PRF soumis au cisaillement direct. Le but de cette partie est de développer un modèle de glissement fiable pour l’interface béton-PRF. La relation de glissement est développée en considérant l’interaction entre les composantes de la contrainte normale et du cisaillement interfacial sur la longueur collée. Une nouvelle technique mathématique est employée pour décrire le relation totale basée sur trois modèles séparés. Le premier modèle décrit la réponse du cisaillement d’une lamelle de PRF orthotopique. Le deuxième modèle simule les caractéristiques de cisaillement d’une couche adhésive. Le troisième modèle représente la non-linéarité du cisaillement de la couche superficielle du béton, aussi appelée couche interfacciale. Le modèle de glissement proposé reflète les caractéristiques géométriques et matérielles du PRF, du béton, et des couches adhésives.

Des modèles non linéaires d’éléments finis, 2-D et 3-D, basés sur un contrôle des déplacements sont ensuite développés pour étudier le comportement en flexion de poutres en béton armé renforcées de PRF et celui de l’interface béton-PRF. Le modèle d’éléments finis 3-D est employé pour modéliser des poutres renforcées de PRF ayant des systèmes d’ancrage en PRF. Des éléments discrets d’interface sont proposés et employés pour simuler le comportement de l’interface béton-PRF avant et après fissuration. Les modèles éléments finis sont capables de simuler les divers modes de ruine, y compris le délaminage du PRF, à l’extrémité de laminé ou à une fissure intermédiaire. Une attention particulière est portée à l’effet de l’apparition de fissures et de leur propagation sur le comportement de l’interface béton-PRF. Cette étude mène à une interprétation précise et raffinée du mécanisme de délaminage des PRF.

Finalement, les modèles d’éléments finis sont employés pour entreprendre une étude paramétrique qui permet de généraliser les conclusions de l’analyse numérique. Les variables incluses dans l’étude comprennent deux propriétés de matériau; à savoir, la résistance à la compression du béton et la rigidité axiale des lamelles en PRF, et trois caractéristiques géométriques; à savoir, le rapport de renfort, la longueur de la poutre et la profondeur de la poutre. L’étude paramétrique est suivie par une analyse statistique de 43 poutres impliquant les cinq variables mentionnées ci-dessus. La méthode de réponse de surface est utilisée pour optimiser l’exactitude des modèles statistiques avec les plus petits nombres d’exécution des modèles des éléments finis des poutres en béton armé renforcées de PRF. En particulier, une conception à face-centrée est utilisée pour évaluer l’influence des variables critiques sur la charge et la limite de déformation provoquant le décollement des lamelles en PRF. L’analyse de régression non linéaire, basé sur ces modèles, est employée pour proposer des règles de conception pour les poutres en béton armée renforcées de PRF.
To my ADVISORS
To those who still think that research might make a difference...!

"A theory is something nobody believes, except the person who made it. An experiment is something everybody believes, except the person who made it...."

by Albert Einstein,

"If the facts do not fit the theory, change the facts....
God does not play dice...."

by Albert Einstein,

(March 14, 1879 - April 18, 1955)
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List of Symbols

Chapter One

\( B \) strain-displacement matrix
\( C_{ijkl} \) tensor of material moduli
\( f \) external body force
\( i, j, k, \) and \( l \) Greek indices range from 1 to 3
\( K \) stiffness matrix
\( K \) incremental stiffness matrix
\( P \) applied external force vector
\( \dot{P} \) incremental applied external force vector
\( U \) displacement vector
\( \dot{U} \) incremental displacement vector
\( \bar{u}_i \) kinematically admissible virtual displacement field
\( x_i \) cartesian coordinates
\( \varepsilon_{ij} \) strain tensor
\( \sigma_{ij} \) stress tensor
\( \phi_i \) basis function (interpolation function)

Chapter Two

\( A_c \) cross section area of concrete block
\( A_p \) cross section area of bonded FRP plate
\( b \) beam width
\( b_p \) width of the bonded laminate
\( E_a \) Young’s modulus of adhesive layer
\( E_c \) concrete Young’s modulus
\( E_p \) Young’s modulus of FRP laminate
\( f'_c \) concrete compressive strength
\( f_{nmax} \) maximum peeling off stress
\( f_p \) normal stress function of FRP laminate
\( f_t \) concrete tensile strength
\( F \) applied shear force
\( G_f^{\text{b}} \) interfacial fracture energy
LIST OF SYMBOLS

$G_a$  shear modules of adhesive layer
$k_m$  reduction factor for strain in FRP laminates to prevent debonding
$K_n$  interfacial stiffness in peeling off direction
$L_i$  length of bonded plate
$s(x)$  local slip value at location $x$
$S_o$  slip corresponding to maximum shear stress value
$t$  beam thickness
$t_a$  thickness of adhesive layer
$t_p$  thickness of the bonded laminate
$V_c$  shear force in concrete beam
$V_p$  shear force in the FRP laminates
$w$  crack width
$\ell_{frpd}$  development length of FRP laminates
$\varepsilon_{cr}$  cracking strain
$\varepsilon_{fu}$  rupture strain in FRP laminates
$\varepsilon_{ub}$  debonding strain in FRP laminates
$\tau_{max}$  bond strength value
$\tau(x)$  interfacial shear stress value at location $x$

Chapter Three
$\tilde{C}$  modified incremental plane stress moduli
$C_{ijkl}$  incremental material moduli
$c_1 - c_{18}$  microscopic fixed constant
$E_D$  microplane deviatoric elastic moduli
$E_T$  microplane shear elastic moduli
$E_V$  microplane volumetric elastic moduli
$f_D$  deviatoric stress boundary fuction
$f_N$  shear stress boundary fuction
$f_T$  shear stress boundary fuction
$f_V$  volumetric stress boundary fuction
$k_3$  microscopic adjustable parameters
$K_D$  incremental deviatoric modulus
$K_T$  incremental shear modulus
$K_V$  incremental volumetric modulus
$E_D^U$  microplane deviatoric unloading moduli
$E_T^U$  microplane shear unloading moduli
$E_V^U$  microplane volumetric unloading moduli
$m$  total number of microplane
$n_i$  normal unit vector
$W_{macro}$  macroscopic incremental virtual work
$W_{micro}$  microscopic incremental virtual work
$w_N$  integration coefficients over unite hemisphere
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\dot{\epsilon}_{kl})</td>
<td>incremental strain tensor</td>
</tr>
<tr>
<td>(\epsilon_D)</td>
<td>deviatoric strain vector</td>
</tr>
<tr>
<td>(\epsilon_L)</td>
<td>microscopic shear stress component in (l) direction</td>
</tr>
<tr>
<td>(\epsilon_M)</td>
<td>microscopic shear stress component in (m) direction</td>
</tr>
<tr>
<td>(\epsilon_N)</td>
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<tr>
<td>(\epsilon_T)</td>
<td>microscopic shear strain vector</td>
</tr>
<tr>
<td>(\epsilon_V)</td>
<td>volumetric strain vector</td>
</tr>
<tr>
<td>(\delta_{ij})</td>
<td>Kronecker delta</td>
</tr>
<tr>
<td>(\Delta \lambda)</td>
<td>stress multiplier parameter</td>
</tr>
<tr>
<td>(\Delta r)</td>
<td>scalar representing a fixed radius of the constraint surface in arc-length method</td>
</tr>
<tr>
<td>(\mu)</td>
<td>parameter characterizes the effects of damage</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Poisson's ratio</td>
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<tr>
<td>(\sigma_D^b)</td>
<td>deviatoric stress boundary</td>
</tr>
<tr>
<td>(\sigma_N^b)</td>
<td>normal stress boundary</td>
</tr>
<tr>
<td>(\sigma_T^b)</td>
<td>shear stress boundary</td>
</tr>
<tr>
<td>(\sigma_V^b)</td>
<td>volumetric stress boundary</td>
</tr>
<tr>
<td>(\dot{\sigma}_{ij})</td>
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<tr>
<td>(\dot{\sigma}_D)</td>
<td>incremental microscopic deviatoric strain vector</td>
</tr>
<tr>
<td>(\dot{\sigma}_L)</td>
<td>incremental microscopic shear strain vector in (l) direction</td>
</tr>
<tr>
<td>(\dot{\sigma}_M)</td>
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<tr>
<td>(\dot{\sigma}_V)</td>
<td>incremental volumetric strain vector</td>
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<tr>
<td>(\sigma^e)</td>
<td>microscopic elastic stress</td>
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<td>(\sigma_D^e)</td>
<td>microscopic elastic deviatoric stress</td>
</tr>
<tr>
<td>(\sigma_N^e)</td>
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</tr>
<tr>
<td>(\sigma_T^e)</td>
<td>microscopic elastic shear stress</td>
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<tr>
<td>(\sigma_V^e)</td>
<td>microscopic elastic volumetric stress</td>
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### Chapter Four

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(A_p)</td>
<td>cross section area of FRP laminates</td>
</tr>
<tr>
<td>(b_a)</td>
<td>width of adhesive layer</td>
</tr>
<tr>
<td>(b_c)</td>
<td>width of concrete block</td>
</tr>
<tr>
<td>(b_p)</td>
<td>width of FRP laminates</td>
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<td>(c_{14} - c_{18})</td>
<td>microscopic fixed constant</td>
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<td>(d_x)</td>
<td>differential distance in (x)-direction</td>
</tr>
<tr>
<td>(d_y)</td>
<td>differential distance in (y)-direction</td>
</tr>
<tr>
<td>(E_a)</td>
<td>Young's modulus of adhesive layer</td>
</tr>
<tr>
<td>(E_a)</td>
<td>Young's modulus of concrete</td>
</tr>
<tr>
<td>(E_p)</td>
<td>Young's modulus of FRP laminates</td>
</tr>
<tr>
<td>(E_{11})</td>
<td>Young's modulus in 1-1 plane</td>
</tr>
<tr>
<td>(E_{22})</td>
<td>Young's modulus in 2-2 plane</td>
</tr>
<tr>
<td>(E_{33})</td>
<td>Young's modulus in 3-3 plane</td>
</tr>
<tr>
<td>(F)</td>
<td>Applied force on FRP laminates in direct shear test</td>
</tr>
</tbody>
</table>
LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{pu}$</td>
<td>ultimate strength of FRP laminates</td>
</tr>
<tr>
<td>$f_p$</td>
<td>normal stress function in FRP laminates</td>
</tr>
<tr>
<td>$f_t$</td>
<td>uniaxial concrete tensile strength</td>
</tr>
<tr>
<td>$f_{bc}'$</td>
<td>biaxial compressive strength ratio</td>
</tr>
<tr>
<td>$f_c'$</td>
<td>uniaxial concrete compressive strength</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>shear modulus in 1-2 plane</td>
</tr>
<tr>
<td>$G_{13}$</td>
<td>shear modulus in 1-3 plane</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>shear modulus in 2-3 plane</td>
</tr>
<tr>
<td>$G_f'$</td>
<td>interfacial fracture energy of bond-slip model</td>
</tr>
<tr>
<td>$h_{eff}$</td>
<td>thickness interfacial concrete layer</td>
</tr>
<tr>
<td>$k_1 - k_4$</td>
<td>microscopic adjustable parameters</td>
</tr>
<tr>
<td>$L_i$</td>
<td>length of FRP laminates</td>
</tr>
<tr>
<td>$P_u$</td>
<td>ultimate capacity of direct shear test</td>
</tr>
<tr>
<td>$s$</td>
<td>interfacial slip between FRP laminates and concrete</td>
</tr>
<tr>
<td>$S_o$</td>
<td>interfacial slip between FRP laminates and concrete corresponding to interfacial shear strength</td>
</tr>
<tr>
<td>$t_c$</td>
<td>pure shear strength of concrete</td>
</tr>
<tr>
<td>$t_p$</td>
<td>thickness of FRP laminates</td>
</tr>
<tr>
<td>$\varepsilon_a$</td>
<td>normal strain in adhesive layer</td>
</tr>
<tr>
<td>$\varepsilon_c$</td>
<td>normal strain in concrete layer</td>
</tr>
<tr>
<td>$\varepsilon_p$</td>
<td>normal strain in FRP laminates</td>
</tr>
<tr>
<td>$\gamma_a$</td>
<td>shear strain in adhesive layer</td>
</tr>
<tr>
<td>$\nu_{12}$</td>
<td>Poisson's ratio in 1-2 plane</td>
</tr>
<tr>
<td>$\nu_{13}$</td>
<td>Poisson's ratio in 1-3 plane</td>
</tr>
<tr>
<td>$\nu_{23}$</td>
<td>Poisson's ratio in 2-3 plane</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>normal stress in adhesive layer</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>normal stress in concrete layer</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>normal stress in FRP laminates</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>normal stress in x-direction</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>normal stress in y-direction</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>normal stress in z-direction</td>
</tr>
<tr>
<td>$\tau$</td>
<td>interfacial shear stress between FRP laminates and concrete</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>interfacial shear stress inside certain depth in the concrete</td>
</tr>
<tr>
<td>$\tau_{ea}$</td>
<td>interfacial shear stress at concrete/adhesive interface</td>
</tr>
<tr>
<td>$\tau_m$</td>
<td>mean interfacial shear stress between FRP laminates and concrete</td>
</tr>
<tr>
<td>$\tau_{pa}$</td>
<td>interfacial shear stress at FRP/adhesive interface</td>
</tr>
<tr>
<td>$\tau_{max}$</td>
<td>maximum interfacial shear stress (bond strength)</td>
</tr>
<tr>
<td>$\tau_{xy}$</td>
<td>shear stress in x-y plane</td>
</tr>
</tbody>
</table>

Chapter Five

$b_c$ | width of concrete block |

$b_f$ | width of the FRP |
LIST OF SYMBOLS

C material response moduli
$\delta_c$ concrete cover thickness
$E_c$ tangent Young's modules of concrete in uniaxial stress–strain relationship
$E_s$ secant Young's modules of concrete in uniaxial stress–strain relationship
$E_{sl}$ Young's modulus of longitudinal reinforcement steel bars
$E_{pi}$ Young's modules in the principle directions
$f_t$ concrete tensile strength
$G_c$ shear modulus of concrete
$G_f$ concrete fracture energy
$G_f^b$ interfacial fracture energy
$L$ beam span length
$W$ width of the beam cross section
$H$ height of the beam cross section
$f_s$ reinforcement steel stress
$f_y$ yield stress of reinforcement steel bars
$f_u$ ultimate strength of FRP laminates
$s$ interfacial slip between FRP laminates and concrete
$S_o$ interfacial slip between FRP laminates and concrete corresponding to interfacial shear strength
$w$ crack width
$\eta_n$ reduction factor for axial stiffness
$\eta_s$ shear retention factor
$\varepsilon_{fu}$ ultimate strain in FRP laminates
$\varepsilon_{ub}$ debonding strain in FRP laminates
$\dot{\varepsilon}$ incremental strain tensor
$\nu$ Poisson's ratio
$\dot{\sigma}$ incremental stress tensor
$\tau_{max}$ maximum interfacial shear stress (bond strength)
$\xi$ factor defines the amount of tension stiffening
$\rho$ steel reinforcement ratio

Chapter Six

$A_s$ cross sectional area of steel reinforcement bars
$b$ width of the beam cross section
$b_{frp}$ width of FRP laminates
$d$ depth of the beam cross section
$E_{frp}$ Young's modulus of FRP laminates
$F_i$ the input variables
$f_c'$ concrete compressive strength
$l$ beam span
$M_u$ ultimate moment capacity of the beam cross section
$M_u/bd^2$ flexural capacity of the beam cross section
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$P_u$</td>
<td>debonding load</td>
</tr>
<tr>
<td>$t_{frp}$</td>
<td>thickness of FRP laminates</td>
</tr>
<tr>
<td>$X$</td>
<td>test function</td>
</tr>
<tr>
<td>$y$</td>
<td>a response of fitted surface</td>
</tr>
<tr>
<td>$\beta_{ij}$</td>
<td>the parameters of the fitted surface</td>
</tr>
<tr>
<td>$\Delta_u$</td>
<td>deflection at debonding load</td>
</tr>
<tr>
<td>$\varepsilon_{ub}$</td>
<td>debonding strain in FRP laminates</td>
</tr>
<tr>
<td>$\rho$</td>
<td>steel reinforcement ratio</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 FRP strengthening of reinforced concrete beams

In the midst of the previous century, major innovative applications were witnessed in the field of composite materials. However, this development was predominantly in the aerospace/automotive domain with little emphasis on the construction industry. The transfer of the composite material technology from the former to the latter was initiated in the early 1980's owing to the distinct characteristics of fibre reinforced polymer (FRP) materials. FRP sheets were originally used in the construction industry to confine concrete columns to increase their ultimate load capacities and enhance their structural ductility. Subsequently, for reinforced concrete beams, FRP laminates were externally bonded to enhance their performance and increase their flexural as well as shear capacities.

Recently, FRPs have gained a wide acceptance worldwide as a promising alternative material to conventional steel plates for the rehabilitation of concrete beams. The corrosion of steel, a typical adversity, is aggravated when steel is applied externally in the form of plates for concrete element repair. On the contrary, FRP materials have a number of significant advantages over conventional steel plates including their high strength-to-weight ratio, easiness of installation, and their ability to repair and conform to complicated shapes. The lack of knowledge on the long-term response and brittle nature of FRPs are challenges for wider applications of this repair technique in the industry. Over the last two decades, the rehabilitation of concrete structures using FRPs has been studied ex-
tensively [Ritchie et al., 1991; Nanni, 1993; Saadatmanesh and Ehsani, 1996; Bizindavyi, 2000; Kotynia et al., 2008].

Generally, FRP-strengthened beams cannot reach their ultimate load carrying capacities due to the debonding of the FRP laminates off the concrete surface, either at the end of the laminate or at an intermediate crack. The response of flexurally strengthened beams, including the stress transfer from bonded FRP laminates to concrete, and the mechanism of failure of such beams is intrinsically tied to the bond mechanisms between the FRP and concrete. The propagation of cracks in flexural and shear spans debilitates the bond performance and consequently accelerates the debonding mechanism. Further details on the advancements related to the use of FRPs in the rehabilitation of concrete structures can be found in various state-of-the-art reports and review papers [Meier, 1995; Neale, 2000; Bakis et al., 2002; Teng et al., 2001; Karbhari et al., 2007].

In recent years, numerous experimental applications have been carried out on the flexural strengthening of reinforced concrete beams using externally bonded FRPs. Accordingly, the global performance of such applications is now relatively well understood from the experimental point of view. However, our knowledge regarding the local behaviour, such as the mechanism of the debonding mode of failure and the associated bond response between an FRP laminate and concrete is slightly limited. Although significant progress has been achieved through experimental investigations, there is an obvious demand to develop reliable and refined analytical procedures and mathematical simulations for FRP-strengthened reinforced concrete beams. Numerical studies, particularly those that concern the interfacial behaviour of FRP-strengthened concrete beams, are necessary for a more in-depth understanding of the failure mechanisms for such beams.

1.2 Numerical analyses

Two groups of concrete constitutive laws have been developed for numerical simulations of concrete structures: phenomenological (macroscopic) models and micromechanical (microscopic) models. Phenomenological models are based on the fundamentals of elasticity or plasticity. In general, they are inspired from the constitutive relations for metals. These models are developed based on the hypotheses and assumptions observed experimentally
for the macroscopic response of concrete. On the other hand, micromechanics-based constitutive laws of concrete, such as the microplane concrete theories focus primarily on microscopic phenomena, such as microcrack formation at mortar-aggregate interfaces. The macroscopic responses therefore arise from what occurs at the microscopic level. The microplane concrete model has proven to be very effective in representing the behaviour of concrete under a wide range of complex stress and strain histories [Bažant and Oh, 1985; Bažant et al., 2000].

Reliable simulations of FRP-strengthened concrete structures require an in-depth understanding of the interfacial behaviour between the FRP and concrete and the bond mechanisms involved. The mechanical properties of the FRP/concrete interface depend on many attributes of the concrete, such as crack softening and shear compression interaction. Moreover, the concrete response under several states of stress, in compression or in tension, arises from microcracks initiating and propagating on the microstructural level. Considerable efforts have been devoted in this thesis research to formulate an accurate concrete constitutive law based on a microstructural theory, namely the microplane concrete model, and to implement the formulation in a finite element package.

In the finite element simulations of FRP-strengthened concrete structures, the constitutive laws to date have been almost exclusively phenomenological in nature. Although this approach together with a cracking model is accepted and accurate for a wide range of applications, it has been proven to be inadequate in providing unified constitutive relations that accurately reflect the experimental data for arbitrary deformation histories of concrete. The limited number of simulations of FRP-strengthened concrete structures using micromechanics-based models are a result of the computational demands of these analyses. With the advent of supercomputers over the last few years, researchers were able to apply micromechanics-based models and use a huge number of discrete finite elements. The supercomputer employed for this thesis research, known as “Mammoth”, is the most powerful parallel supercomputer in Canada. It is at the Université de Sherbrooke and consists of 576 nodes with a maximum speed of 4147.2 GHz. Each node consists of two processors of 3.6 GHz with a total RAM of 8.0 GB.
CHAPTER 1. INTRODUCTION

1.3 Finite element analysis

Of the various attempts that have been made to date to model FRP-strengthened concrete structures using nonlinear FE analysis, it has been concluded that the proper simulation of crack propagation and the FRP/concrete interface are undoubtedly the main challenges [Neale et al., 2006]. Essentially, the nonlinear response of FRP-strengthened beams under various states of stress and strain is frequently dominated by progressive cracking. These cracks typically result from shear or tensile stresses and they influence the bond mechanism. Two crack models that are commonly used in the FE simulation of FRP-strengthened concrete structures are the smeared crack model [Neale et al., 2006; Lu et al., 2007] and the discrete crack model [Niu and Wu, 2006; Camata et al., 2007]. In general, the discrete crack approach is recommended for cases where one crack or a finite number of cracks propagate within the beam. On the other hand, the smeared crack approach is appropriate when cracks are spread over the entire beam.

As in the case of simulating crack propagation, the proper modelling of the interfacial behaviour between the FRP and concrete, using nonlinear FE analysis, is inherently difficult. Several techniques have been used to simulate the FRP/concrete interface. The full-bond assumption between the FRP sheets and the concrete [Hu et al., 2001], using a very fine mesh to simulate the adhesive layer [Lu et al., 2005b], and employing interface elements that have a predefined bond-slip relationship to link the FRP and concrete nodes [Lu et al., 2007; Niu and Wu, 2006] are among the popular approaches that have been introduced for such simulations. The full-bond assumption overestimates the strains at failure in the FRP sheets. Commonly, finite element models of FRP-strengthened beams with a fine mesh are used to investigate the elastic interfacial shear behaviour at both the FRP/adhesive and adhesive/concrete interfaces. Interface elements between the FRPs and the concrete are commonly used to capture the debonding failure load. However, several researchers concluded that, as a result of the continuity of the interpolation function employed in the smeared crack model, the finite element simulations fail to capture the interfacial stress concentration due to cracks [Lu et al., 2007].

Most of the finite element studies concerning the flexural response of FRP-strengthened reinforced concrete beams attempt to simulate the nonlinearities in the load–deformation relationships. Few studies can be found concerning the local interaction between the FRP
and concrete, such as the effect of FRP characteristics on the tension stiffening model and the concrete shear retention factor.

1.4 Objectives and methodology

The general objective of this thesis research is to develop powerful micromechanics-based nonlinear finite element models to simulate the FRP/concrete interfacial behaviour. This implies other sub-objectives that can be summarized as follows:

- Developing accurate and reliable concrete and FRP/concrete interface constitutive models based on micromechanical simulations;
- Building successful finite element models to investigate the nonlinear behaviour of FRP-strengthened reinforced concrete beams;
- Investigating the relative importance of the nonlinear behaviour of the FRP/concrete interfaces and their interaction through the bond-slip relationship on the response of concrete beams;
- Addressing the effect of the various material and numerical parameters on the ultimate capacities of the strengthened beams and predicting debonding failure modes;
- Assessing the effect of both flexure and shear cracks on the interfacial stress concentrations along the interface;
- Providing practical design guidelines, governing equations and recommendations that can be used in the design of FRP-strengthened concrete beams.

In this study, the microplane concrete model M4 is formulated and written in FORTRAN codes. These are implemented in the finite element package ADINA, Version 8.4 [ADINA, 2004b], as a user-defined subroutine. Two- and three-dimensional nonlinear finite element models are developed. The relevant finite element calculations have been done using the Mammoth supercomputer. A face-centred central composite response surface experimental method and nonlinear regression analysis are used for the statistical analyses.
CHAPTER 1. INTRODUCTION

1.5 Structure of the dissertation

This dissertation is presented in six chapters followed by a chapter summarizing the main findings of the research. It is organized as follows:

- Chapter 2: Presents a literature review of the research that deals with the applicability of using FRP laminates as an external bonding material. A particular attention is paid to research concerning numerical simulations of FRP-strengthened beams;

- Chapter 3: Concerns the details of the microplane concrete model. This involves a brief summary of the basic relations of the M4 version of the microplane model, followed by a proposed incremental formulation of this version;

- Chapter 4: Presents the micromechanical finite element analysis for the case of FRP/concrete joints subjected to direct shear. In this chapter, a local bond shear stress–slip relationship is proposed based on the finite element results;

- Chapter 5: Introduces two and three-dimensional nonlinear finite element analyses for FRP-strengthened beams. The 3-D analysis is presented to accommodate the orthotropic nature of FRP laminates and the cases of anchored FRP systems;

- Chapter 6: Presents a parametric study to generalize the information obtained from the finite element results. The parametric study is followed by a statistical analysis to develop design guidelines and governing equations;

- Chapter 7: Consists of the general conclusions and recommendations for future work.

Finally, at the end of this thesis three appendices are included. The first deals with continuum mechanics models for the shear and normal stress profiles along FRP/concrete joints. In the second appendix, we present a brief recapitulation of the main features of the concrete model that is available in ADINA. The detailed description of the bond–slip models of Lu et al. [2005b] are given in the third appendix.
Chapter 2

Literature Review

2.1 Introduction

Numerous studies have been carried out on using FRP materials for the flexural rehabilitation of concrete beams. In the following sections, a survey of some of these studies is presented. This chapter is organized as follows. First the flexural behaviour of FRP-strengthened reinforced concrete beams is discussed followed by a survey of existing bond-slip models. Then a brief summary is presented for the theoretical studies proposed to develop accurate mathematical closed-form solutions to simulate the interfacial behaviour of FRP-strengthened beams. As a result of the main focus of this dissertation, specific attention is given to finite element simulations for such beams. An emphasis is also given to design guidelines and code specifications.

2.2 Flexural behaviour of reinforced concrete beams retrofitted with FRP laminates

Most studies that have been carried out on FRP-strengthened concrete beams were devoted to evaluate the effectiveness of the strengthening techniques on the flexural and shear responses [Nanni, 1993; Saadatmanesh and Ehsani, 1996; Triantafillou, 1998; Rizkalla and Hassan, 2002; Takahashi et al., 1997; Brena et al., 2003; Adhikary and Mutsuyoshi, 2006].
Some of these studies have investigated the influence of various factors on the overall load carrying capacities and ductilities of the strengthened beams [Ritchie et al., 1991; Missi­houn, 1995; Stallings et al., 2000; Gao et al., 2005; Hosny et al., 2006]. Other studies have addressed the feasibility of using prestressed FRP sheets to enhance the load carrying capacities [Triantafillou and Plevris, 1992; El-Hacha et al., 2003], have investigated the effect of impact and fatigue loads on the flexural responses [Ekenel et al., 2006; Masoud et al., 2006], or have examined the influence of environmental conditions on the overall behaviour [Grace and Singh, 2005; Jia et al., 2005; Maaddawy et al., 2007]. The focus of the present survey is on studies concerning the flexural behaviour of FRP-retrofitted beams under static loads.

### 2.2.1 Effect of retrofitting on flexural performance

From results of several experimental studies, it has been concluded that attaching FRPs to the tension side of a beam increases its flexural capacity [Grace, 2001; Takahashi and Sato, 2003; Kotynia, 2005; Leung, 2006]. However, the reduction in the ductility of the strengthened beams is significant compared to those of unstrengthened beams [Ehsani and Saadatmanesh, 1990; Sharif et al., 1994; Arduini and Nanni, 1997]. Several researchers have strengthened concrete beams in flexure by attaching the FRP sheets or laminates to the beam sides in the tension region [Brena et al., 2003]. This method was attractive due to the improvement in the ductility ratio of the strengthened beams compared to that obtained when attaching the FRPs to the extreme tension side. However, the increase in the overall load carrying capacities in the former case was less. Figure 2.1 shows the two methods of flexural strengthening for concrete beams using FRP sheets.

Because of the fact that the bending moment is not uniformly distributed along the beam length, some experimental studies examined the effect of varying the thickness of the FRP laminates with the bending moment. In the experimental study of M'Bazaa [1995], the reduction of the thickness of the FRP laminate according to the intensity of the bending moment was seen not to have a significant effect on the overall load carrying capacities or stiffnesses, particularly when the FRP laminate lengths were relatively long.

Carbon, aramid, and glass FRPs have been used as strengthening materials for rehabilitation purposes. For the same FRP cross sectional area, using CFRP sheets contributes
to higher ultimate capacities than those obtained when AFRP and GFRP sheets [Ritchie et al., 1991; Joh et al., 2003] are used. It was observed from the numerical results of Thomsen et al. [2001] (Figure 2.2) and other experimental studies [Ritchie et al., 1991; Nanni, 1993; Brena et al., 2003; Takahashi and Sato, 2003] that the ductility ratios of GFRP-strengthened reinforced concrete beams were higher than those of CFRP-strengthened beams. When the length of the strengthening sheets was relatively long, the ductility of the strengthened beams was not affected by a further increase in the sheet length.

With regard to the cracking behaviour of retrofitted beams, the observed crack widths of strengthened beams were narrower than those of unstrengthened beams when both were subjected to the same moment. However, the crack widths for the strengthened beams at the maximum load might be wider than those of the unstrengthened beams [Joh et al., 2003]. It was also observed that, for FRPs having the same axial stiffness, the type of FRP sheet did not affect the crack propagation. The first crack of the strengthened beams occurred at slightly higher loads than those of the corresponding unstrengthened beams. Generally, in the strengthened beams, there were more cracks, more closely spaced, and more uniformly distributed than those for the unstrengthened beams [Ritchie et al., 1991].

A minimum reinforcement ratio was applied in most of the experimental studies as this provided the maximum load capacity enhancement. In the experimental study of Ross et al. [1999], the reinforcement ratio was changed from 0.46 to 3.30%. In this investigation, a significant flexural strength enhancement was obtained in a lightly reinforced beam (300.0% over the unstrengthened beam in the case of a reinforcement ratio of 0.46%) and the beam experienced a debonding mode of failure. Regarding the heavily reinforced beam (3.30%), the flexural capacity was improved by only 10% over that of the unstrengthened
CHAPTER 2. LITERATURE REVIEW

Figure 2.2: Effect of plate length on ductility ratio with two plate types [Thomsen et al., 2001]

beam, and a concrete crushing mode of failure was observed.

Using plate-end FRP anchorage systems can prevent plate debonding and increase the ultimate load carrying capacities by providing a vertical stiffness against the peeling off stresses (Figure 2.3(a)). The ductility ratios for strengthened beams having plate-end FRP anchorage sheets were increased compared to those of the unanchorage-strengthened beams; furthermore, the ultimate load carrying capacities were increased [Ritchie et al., 1991; Brena and Macri, 2004].

FRP anchorage sheets can be used not only at the plate ends but also along the length of the FRP laminate to delay intermediate crack debonding. The FRP-strengthened beams in the work of Chicoine [1997] failed initially by debonding of the FRP laminates at their ends. Two different FRP anchorage configurations were developed to prevent the debonding mode of failure. In the first configuration, depicted in Figure 2.3(b), U-shaped FRP anchorage sheets were used at the two FRP laminate ends. In the second configuration, shown in Figure 2.3(c), unidirectional transverse strips were used along the laminate. The first configuration led to an increase of the flexural capacity of 32.0% over that of the unstrengthened beam, and displaced the debonding zone of FRP sheets towards the mid span. The second configuration enhanced the flexural capacity by 46.0% over that of the unstrengthened beam, and changed the mode of failure to a rupture of the FRP laminate. A similar conclusion was drawn in the work of Spadea et al. [1998] and Brena et al. [2003].

Applying FRP anchorage sheets away from the plate end, only in the flexural regions
2.2. FLEXURAL BEHAVIOUR OF FRP-STRENGTHENED RC BEAMS

![Diagram](image)

(a) FRP sheets at plate cut-off [Ritchie et al., 1991]
(b) Configuration I [Chicoine, 1997]
(c) Configuration II [Chicoine, 1997]
(d) Continuous FRP anchorage sheets [Kotynia et al., 2008]

**Figure 2.3: Different FRP anchorage systems for the FRP-strengthened beams**

as in Figure 2.3(d), was also of interest to several researchers [Leung, 2006; Kotynia et al., 2008]. From the Kotynia et al. [2008] study, not extending the lengths of the FRP U-shaped sheets to cover the ends of the laminates limited the effectiveness of the FRP anchorage system as far as the ultimate load capacities were concerned. This was a result of the fact that the debonding initiated just after the end of the continuous FRP anchorage laminates and propagated towards the plate ends at load levels similar to the failure load of the unanchored beam. Leung [2006] concluded from his experimental program that using FRP anchorage sheets away from the plate end could indeed be more effective in some particular cases, and that using the plate-end anchorage was not necessarily appropriate in all cases.

### 2.2.2 Failure modes of retrofitted beams

Experimental tests have indicated that debonding of the bottom FRP strip from the concrete surface is the most common mode of failure for concrete beams strengthened in flexure. This type of failure generally limits the strength utilization ratio of the strip; i.e., the ratio of the strain in the FRP at failure to its ultimate strain [Kaminska and Kotynia,
2000; Kotynia and Kaminska, 2003]. Depending on the starting point of the debonding process, the debonding modes can be classified into two main categories [Oehlers, 1992; Smith and Teng, 2001b,c]. The first mode occurs in the plate cut-off zone and is known as a plate-end debonding or a concrete cover separation, while the second mode of failure occurs in the vicinity of flexural-shear or flexural cracks and is referred to as intermediate flexural crack-induced debonding (IC debonding).

The bond strength at the FRP/concrete interface plays a main role in the local failure mode mechanism so that the contact area, which depends on the length and the width of the FRP sheet, has a significant effect on the failure mechanism. In addition, the stiffness of the FRP sheet controls the initiation of the local failure modes [Fanning and Kelly, 2001; Teng et al., 2001; Sebastian, 2001; Brena et al., 2003; Brena and Macri, 2004]. Leung [2006] concluded from his experimental results that the common beliefs that high elastic stress concentrations at the plate cut-off points are the reason for the plate-end debonding was not necessarily correct. It was obvious from his experimental investigation that this mode of failure occurred as a result of the shear cracks that initiated near the plate end. The debonding failure modes are depicted in Figure 2.4(b). In Figure 2.4(a), the conventional modes of failure are shown.

**Figure 2.4**: Failure modes of FRP-retrofitted beams
2.2.3 Bond characteristics between the FRP and concrete

The flexural responses of FRP-strengthened concrete beams and the failure mechanisms involved are intrinsically related to the bond characteristics between the FRP and concrete. Debonding modes of failure imply that a full understanding of the bond mechanism between the bonded laminates and concrete is essential. Direct shear tests have been broadly used in laboratory investigations to investigate the bond characteristics of FRP/concrete interfaces. These tests have been performed on both steel-to-concrete joints [Van Gemert, 1980; Theillout, 1983; Swamy et al., 1986; Ranisch and Rostasy, 1986], and FRP-to-concrete joints [Kobataka et al., 1993; Chajes et al., 1995, 1996; Neubauer and Rostasy, 1997; Bizindavyi and Neale, 2001; Nakaba et al., 2001; Dai et al., 2005]. From these experiments, it was observed that the failure modes of direct shear tests could be a rupture of the FRP laminate, shear failure in the adhesive layer, or failure within a thin layer inside the concrete referred to as an “interfacial layer”. Adhesives currently used in FRP strengthening applications are generally stiff enough to ensure that there is no shear failure inside the adhesive layer. Accordingly, with few exceptions, researchers have concluded that with a good concrete surface preparation the main failure mode of an externally bonded FRP laminate in a direct shear test occurs in the concrete. More precisely, failure occurs a few millimeters inside the concrete beneath the adhesive layer due to the high shear stress in the concrete layer [Bizindavyi, 2000; Dai et al., 2005]. Accordingly, a proper simulation of FRP/concrete interfaces requires a reliable concrete model to represent the behaviour within this interfacial concrete layer.

From various flexural and modified beam tests on steel-to-concrete joints, as well as on FRP-to-concrete joints, it has been concluded that when laminate detachment did occur it was due to the high local interfacial bond and peeling-off stresses at the ends of the laminate [Jones et al., 1980; Van Gemert, 1980; Quantrill et al., 1996a]. These stresses were found to depend on the tensile strength of the concrete [Jones et al., 1980; Triantafillou and Plevris, 1992; Quantrill et al., 1996b; Bizindavyi, 2000], the axial stiffness of the laminated section [Triantafillou and Plevris, 1992], the concrete surface preparation [Jones et al., 1980; Van Gemert, 1980; Ziraba et al., 1995], the strength and thickness of the adhesive layer [Jones et al., 1980; Quantrill et al., 1996b], and to a lesser extent on the laminate aspect ratio \( b_p/t_p \) where \( b_p \) and \( t_p \) are the width and thickness of the bonded laminate, respectively [Jones et al., 1980; Quantrill et al., 1996b].
CHAPTER 2. LITERATURE REVIEW

Bond–slip relationships have generally been obtained from direct shear tests by using the measured strain readings, $\varepsilon_p$, along the bonded FRP laminates. The shear stress, $\tau(x)$, and the local slips, $s(x)$, can be calculated using the first derivative of the FRP laminate axial strain function, $\varepsilon_b$ as follows:

$$\tau(x) = E_p t_p \frac{d\varepsilon_p}{dx}$$

$$s(x) = \int \varepsilon_p dx$$

In the aforesaid two equations, $E_p$ and $t_p$ are the Young's modulus and thickness of the FRP plate, respectively. Figure 2.5 shows typical local bond–slip relationships determined experimentally for different FRP strengthening laminates. The local bond strength (i.e., the maximum shear stress on these curves) increases with an increase in the concrete compressive strength and the FRP sheet stiffness [Nakaba et al., 2001; Dai and Ueda, 2003]. In Figure 2.5(b), the local $\tau$–s profiles at three different locations are depicted along the bonded length [Dai et al., 2005]. Here it is observed that the local bond–slip relationships vary along the bonded length depending on the location from the loaded end. The same conclusion was reported by other researchers [Sato et al., 2000; Nakaba et al., 2001]. To date, no persuading explanation in the literature has been given for such differences in the shape of these $\tau$–s relationships or in the associated values of the maximum shear stresses (2.6 to 17.3 MPa) along the laminate length. Several researchers ascribed this to either the microstructure composition of the concrete including the distribution of the fine and coarse aggregates along the concrete surface, or to the local bending contribution of the bonded FRP laminate [Dai et al., 2005; Lu et al., 2005b].

2.3 Survey of existing bond–slip relations

Various mathematical models have been proposed to represent the local bond–slip profiles, as well as to predict the associated values of the bond strength. Figure 2.6 shows typical bilinear and nonlinear local bond–slip relations, where $\tau_{\text{max}}$ denotes the bond strength; i.e., the maximum shear stress, and $S_o$ is the corresponding slip. The area under the bond–slip curve represents the interfacial fracture energy $G_f$; that is, the energy required for complete debonding. The bilinear bond–slip models are characterized by $S_f$, the slip
at complete debonding. Some of the most recent research on bond–slip models has been critically reviewed by Lu et al. [2005b] and Karbhari et al. [2007]. Six relations have been investigated by Lu et al. [2005b] in terms of characteristics of the bond–slip model. Karbhari et al. [2007] compared between six relations in terms of the bond strength \( \tau_{\text{max}} \) and interfacial fracture energy \( G_f \). In this section, nine bond–slip relations from the literature are selected for the sake of comparison. Most of these models were collected from Lu et al. [2005b] and Karbhari et al. [2007]. An overview of these nine relations is summarized in Table 2.1. In this table, the relations for the ascending and descending branches of the bond–slip model are given. The expressions of \( \tau_{\text{max}}, S_0 \) and \( S_f \) are shown in the last three columns of the table. In these bond–slip relations, \( G_a \) and \( t_a \) are the shear modulus and thickness of the adhesive layer, respectively. For the FRP laminates, \( E_p \) and \( t_p \) denote the Young’s modulus and the thickness, respectively. The interfacial fracture energy is indicated by \( G_b \), which is defined as a function of the concrete tensile \( (f_t) \) and compressive \( (f_c) \) strengths. \( \beta \) is a width factor that correlates the effect of the FRP laminate width \( b_b \) and concrete block width \( b_c \).

Several shapes for the bond–slip curves are associated with the mathematical models in Table 2.1; they are presented schematically in Figure 2.7. Neubauer and Rostasy [1999], Monti et al. [2003] and Lu et al. [2005b] (the bilinear model) suggested a linear function to describe the ascending branch of the \( \tau–s \) relationship. In this linear function, the initial slope of the \( \tau–s \) profile is \( \tau_{\text{max}}/S_0 \). In the Dai and Ueda [2003] model, and the simplified or precise models of Lu et al. [2005b], nonlinear functions were employed to
describe the ascending branch of the \( \tau - s \) curve with an initial slope being infinite. The only two relations among the nine models in Table 2.1 that used a nonlinear function for the ascending branch with a reasonable initial slope were the models presented by Nakaba et al. [2001], and Dai et al. [2005]. The former uses a third order function with an initial slope of \( \frac{3\tau_{\text{max}}}{2S_0} \), while the latter model assumes an exponential function with an initial slope of \( 2U^2G_f \). (The function of \( U \) is presented in Table 2.1).

In several models, the bond strength \( \tau_{\text{max}} \) is linearly proportional to the concrete tensile strength \( f_t \) [Neubauer and Rostasy, 1999; Brosens and Van Gemert, 1999; Monti et al., 2003; Lu et al., 2005b], to \( f_c^{0.19} \) [Nakaba et al., 2001; Savioa et al., 2003] or to \( f_c^{0.67} \) [Ulage et al., 2003] where \( f_c \) represents the concrete compressive strength. It is now well accepted that the bond strength depends on the FRP [Bizindavyi and Neale, 2001; Dai et al., 2005] and the adhesive characteristics [Dai et al., 2005], and not only on the concrete parameters as assumed in the above bond strength models. The bond strength model presented by Dai and Ueda [2003] and that introduced by Dai et al. [2005] were the only two models that have included the influence of the the FRP, concrete and adhesive layer characteristics on the value of \( \tau_{\text{max}} \).

In Figure 2.8(a), a comparison is given among the nine bond-slip models in terms of the associated value of \( \tau_{\text{max}} \). In this comparison, the values of the concrete tensile strength, \( f_t \), and the Young's modulus, \( E_c \), are calculated based on the CSA-A23.3 [2004] as follows:

\[
f_t = 0.62 \sqrt{f_c'} \quad (MPa)
\]
Table 2.1: Summary of bond–slip models

<table>
<thead>
<tr>
<th>No</th>
<th>Bond–slip model</th>
<th>Ascending branch</th>
<th>Descending branch</th>
<th>$\tau_{\text{max}}$</th>
<th>$S_o$</th>
<th>$S_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Neubauer and Rostasy [1999]</td>
<td>$\tau_{\text{max}} \left( \frac{S}{S_o} \right)$</td>
<td>0</td>
<td>$1.8\beta f_c$</td>
<td>0.202$\beta$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\beta = \sqrt{1.125 \frac{2-b_p/b_c}{1+b_p/400}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Nekaba et al. [2001]</td>
<td>$\tau_{\text{max}} \left( \frac{S}{S_o} \right) \left( 2 + \left( \frac{S}{S_o} \right)^3 \right)$</td>
<td>$3.5 f_c^{0.19}$</td>
<td>0.065</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Monti et al. [2003]</td>
<td>$\tau_{\text{max}} \left( \frac{S}{S_o} \right)$</td>
<td>$\tau_{\text{max}} \left( \frac{S_f-S}{S_f-S_o} \right)$</td>
<td>$1.8\beta f_c$</td>
<td>$2.5\tau_{\text{max}} \left( \frac{f_c + 50}{E_y} \right)$</td>
<td>$0.33\beta$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\beta = \sqrt{1.5 \frac{2-b_p/b_c}{1+b_p/100}}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Savioa et al. [2003]</td>
<td>$\tau_{\text{max}} \left( \frac{S}{S_o} \right) \left( 1.86 + \left( \frac{S}{S_o} \right)^{2.86} \right)$</td>
<td>$3.5 f_c^{0.19}$</td>
<td>0.051</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Dai and Ueda [2003]</td>
<td>$\tau_{\text{max}} \left( \frac{S}{S_o} \right)^{0.575}$</td>
<td>$\tau_{\text{max}} e^{-\beta(S-S_o)}$</td>
<td>$-1.575\alpha K_o + \sqrt{2.481\alpha^2 K_o^2 + 6.3\alpha^2} / K_o G_j$</td>
<td>$2\beta$</td>
<td>$\tau_{\text{max}} / (\alpha K_o)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\alpha = 0.028(E_p t_p / 1000)^{0.234}$</td>
<td>$K_o = G_o / t_u$</td>
<td>$G_j = 7.554 K_o^{-0.419} f_c^{0.243}$</td>
</tr>
<tr>
<td>6</td>
<td>Dai et al. [2005]</td>
<td>$2UG_j \left( e^{-0.8} - e^{-20.8} \right)$</td>
<td>$UG_j / 2$</td>
<td>$0.693 / U$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No</td>
<td>Bond-slip model</td>
<td>Ascending branch</td>
<td>Descending branch</td>
<td>$S_0$</td>
<td>$S_f$</td>
<td></td>
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<td>----</td>
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<td></td>
</tr>
<tr>
<td>7</td>
<td>Lu et al. [2005] (Simplified model)</td>
<td>$\tau_{\text{max}} = 1.5\beta f_s$</td>
<td>$\alpha = 0.30G_f \sqrt{f_s}$</td>
<td>$S_0 = 0.0195\beta f_s$</td>
<td>$S_f = 2G_f f_s$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Lu et al. [2005] (Bilinear model)</td>
<td>$\tau_{\text{max}} = 1.5\beta f_s$</td>
<td>$\alpha = 0.30G_f \sqrt{f_s}$</td>
<td>$S_0 = 0.0195\beta f_s$</td>
<td>$S_f = 2G_f f_s$</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Lu et al. [2005] (Precise model)</td>
<td>$\tau_{\text{max}} = 1.5\beta f_s$</td>
<td>$\alpha = 0.30G_f \sqrt{f_s}$</td>
<td>$S_0 = 0.0195\beta f_s$</td>
<td>$S_f = 2G_f f_s$</td>
<td></td>
</tr>
</tbody>
</table>

$G_a$ and $f_s$ are the shear modulus and the thickness of the adhesive and concrete, respectively, $G_f$ is the fracture energy, $\tau_{\text{max}}$ is the maximum interfacial shear stress, $S_0$ is the slip corresponding to maximum shear stress value, $\beta$, $f_c$, and $f_t$ are the concrete compressive strength, $E$ is a width factor that correlates effect of FRP laminate and concrete block widths, $b_c$ and $b_p$ are the widths of FRP and concrete block, respectively.
2.3. SURVEY OF EXISTING BOND-SLIP RELATIONS

![Figure 2.7: Several \( \tau-s \) relationships available in the literature \( f'_c=35 \, \text{MPa}, \, E_a=2000 \, \text{MPa}, \, G_a=717.28 \, \text{MPa}, \, t_a=1.00 \, \text{mm}, \, E_p=100,000 \, \text{MPa} \) and \( t_p=0.33 \, \text{mm}, \, b_c=100 \, \text{mm}, \, b_p=25 \, \text{mm} \)](image)

\( E_c = 4500 \sqrt{f'_c} \quad (\text{MPa}) \) \hspace{1cm} (2.4)

In Figure 2.8(a) the bond strength varies from 6.1 MPa in the Brosens [2001] model to 17.6 MPa in the Monti et al. [2003] relation (at \( f'_c = 60.0 \, \text{MPa} \)). Furthermore, the rate of increase of the bond strength with the concrete compressive strength is different in all the models. The predictions from the Monti et al. [2003] model have a maximum rate, while the lowest rate of increase is associated with the Nakaba et al. [2001] and Dai et al. [2005] models.

As far as the slip corresponding to the bond strength, \( S_0 \), is concerned, a comparison between the predicted values of \( S_0 \) is shown in Figure 2.8(b). The model presented by Neubauer and Rostasy [1999] gives the largest prediction of \( S_0 \) (a constant value of 0.374 mm) while the smallest value is obtained from the Brosens [2001] model (a value of 0.018 mm at \( f'_c=60 \, \text{MPa} \)). In Figure 2.8(b), Neubauer and Rostasy [1999] and Savioa et al. [2003] assumed constant values for \( S_0 \), while Lu et al. [2005b] assumed a linear function with the concrete tensile strength. In the Monti et al. [2003], Dai and Ueda [2003], Dai et al. [2005], Brosens and Van Gemert [1999], and Brosens [2001] models, \( S_0 \) is a function of the FRP, concrete and adhesive layer mechanical characteristics. Brosens and Van Gemert [1999] assume \( S_0 \) as \( \tau_{\text{max}} \left( 2.6 \frac{E_a}{E_c} + 2.32 \frac{t_{\text{ref}}}{E_c} \right) \), where \( t_{\text{ref}} \) is the thickness on the interfacial layer, and assumed as 2.5-3 maximum aggregate size. A recent modification
Figure 2.8: Effect of the concrete compressive strength on the bond characteristic of several bond–slip models \( [E_a = 2000 \text{ MPa}, G_a = 717.28 \text{ MPa}, t_a = 1.00 \text{ mm}, E_p = 100,000 \text{ MPa}, \text{ and } t_p = 0.33 \text{ mm}, b_c = 100 \text{ mm}, b_p = 25 \text{ mm}] \)

has been given by Brosens [2001] to include the geometric and material properties of FRPs as

\[
\tau_{\text{max}} = \left( 2.5 \frac{t_a}{E_a} + 2.4 \frac{t_{\text{ref}}}{E_c} + \frac{t_f}{G_f} \right).
\]

The discrepancies in the shape of the bond–slip curves (Figure 2.7), or in the associated bond strength values (Figure 2.8(b)) could be a result of the fact that the profiles observed experimentally are not constant along the bonded length (Figure 2.5(b)). Although significant discrepancies have been observed among the above models, their proponents claim that they all predict with a reasonable accuracy the ultimate loads of the direct shear tests. This is probably because of the relatively small amount of experimental data considered in the calibration of the bond–slip models. Ebead and Neale [2007] examined the Lu et al. [2005b] model by considering a large amount of experimental data. They found that, although the model predicted the experimental ultimate load with an average prediction ratio of the order of 101.1%, a significant scatter was observed in the model predictions (a standard deviation of 14.63% and coefficient of variation of 14.4%). This scatter in the predictions was mainly due to the fact that the bond–slip model did not account for the characteristics of the adhesive layer. Moreover, the adhesive layer thicknesses and the mechanical characteristics were not always reported in the original experimental work.
Although significant progress has been achieved with regard to the development of bond-slip models, there is an obvious demand for more reliable and refined procedures for defining the $\tau-s$ relationships in order to enhance our understanding of the interfacial bond behaviour. An accurate bond-slip model has to reflect the characteristics of the concrete, adhesive and FRP, and to reflect the differences observed in Figure 2.5(b). A new bond-slip model that accounts for the various parameters of the interface and reflects the experimentally observed trends will be presented in Chapter 4 of this thesis.

### 2.4 Closed-form solutions for FRP-strengthened beams

Over the last ten years, various mathematical and numerical models have been developed to predict the debonding failure modes and analyze the cracking behaviour of FRP-strengthened reinforced concrete beams [Malek et al., 1998; Shen et al., 2001; Rabinovitch and Frostig, 2001a,b]. In the literature, two different approaches have been proposed.

**The first approach** used a mathematical model that was able to calculate the interfacial stresses at the FRP plate end or at the flexural cracks, and compared these values with the ultimate bond strengths. Several models were derived based on a simplified technique [Malek et al., 1998; Arduini and Nanni, 1997; Taljsten, 1997; Bizindavyi and Neale, 2001], or a closed-form high order (CFHO) model [Rabinovitch and Frostig, 2000; Shen et al., 2001].

A simple closed-form solution for the interfacial stresses at an FRP/concrete interface was derived in the work of Malek et al. [1998] as follows:

\[
\tau(x) = \tau_p [b_3 \sqrt{A} \cosh(\sqrt{Ax}) - b_3 \sqrt{A} \sinh(\sqrt{Ax}) + 2b_1 x + b_2]
\]

\[
\tau_{\text{max}} = \tau_p (b_3 \sqrt{A} + b_2)
\]

where, \(A = \frac{G_a}{t_a t_p E_p} \), \(b_1 = \frac{Y a_1 E_p}{I_{tr} E_c} \), \(b_2 = \frac{y E_p}{I_{tr} E_c} (2a_1 L_0 + a_2) \), and

\(b_3 = E_p \left[ \frac{Y}{I_{tr} E_c} (a_1 L_0^2 + a_2 L_0 + a_3) + 2b_1 \frac{t_a t_h}{G_a} \right] \). In these equations, \(t_p \) and \(E_p \) are the FRP sheet thickness and Young’s modulus. The shear modulus and the adhesive layer thickness are denoted by, respectively, \(G_a \) and \(t_a \). The parameters \(a_1 \), \(a_2 \) and \(a_3 \) are determined...
by applying the boundary conditions of the beam. The moment of inertia and centre of gravity of the strengthened beam are indicated by $I_{tr}$ and $Y$, respectively. For the peeling off stress, we have:

$$f_{n_{\text{max}}} = K_n \frac{V_p}{E_p I_p} - \frac{V_c + \beta M_0}{E_c I_c}$$

(2.7)

where $K_n = \frac{E_a}{t_a}$. $V_p$ and $V_c$ in Equation 2.7 are shear forces in the FRP sheet and concrete beams, respectively. Although this model was developed based on the uncracked beam section analysis, the results were in a good agreement even for the case of cracked sections. However, according to this model, the peak interfacial stress occurs only at the plate cut-off point. Therefore the model fails to predict the stress concentrations at the flexural cracks.

Another simplified model for the stresses at the interface based on numerical techniques was developed by Arduini and Nanni [1997] using a discrete model. The maximum shear and normal stress values can be calculated using the following equations.

$$(\tau_{a_f,j})_{\text{max}} = (N_{j+1} + N_{a,j+1} - N_j - N_{aj}) \frac{2}{b.d_x}$$

(2.8)

$$(\sigma_{a_f,j})_{\text{max}} = (N_{j+1} - N_j) \left( dv_j + t_a + t_f/2 \right) + \left( N_{a,j+1} - (N_{aj}) \left( dv_j + t_a/2 \right) \frac{6}{b.d_x^2} \right)$$

(2.9)

where $N$ is the normal horizontal force, $b$ and $t$ are the beam width and thickness, respectively, $a,c,f$ are the indices for the adhesive layer, concrete and FRP sheet, respectively, $a_f$ is the index for the adhesive layer/FRP sheet interface, and $dv_j$ indicates the increment of vertical displacement of the $j + 1$ segment with respect to $j$ segment. In this discrete model, the concrete beam was divided into several segments and for each segment the shear and normal stresses at the adhesive/concrete interface and adhesive/FRP interface were assumed to be triangular. The Arduini and Nanni [1997] model is simple and can be used to calculate the stresses at the interface. However it is still unable to predict the values at the vicinity of the flexural cracks or produce a realistic stress distribution along the interface.

A more precise simplified model for the interfacial shear stress along the bonded FRP/concrete joint is the following presented in the work of Bizindavyi and Neale [2001]:

$$\tau(x) = B \cosh (\lambda x) + C \sinh (\lambda x)$$

(2.10)
2.4. CLOSED-FORM SOLUTIONS FOR FRP-STRENGTHENED BEAMS

The parameters $B$ and $C$ are related to the average shear stress $\tau_m$ and mechanical properties of the concrete, adhesive and FRP laminates according to the following equation:

$$B = \tau_m \frac{\lambda L_i}{\tanh(\lambda x)} , \quad C = -\lambda L_i \tau_m$$

(2.11)

Here $\lambda = \sqrt{\frac{G_a}{E_p t_p t_a} (1 + \eta \rho)}$, $\eta = \frac{E_p}{E_c}$ and $\rho = \frac{A_p}{A_c} = \frac{df_c(x)}{df_p(x)}$ where $L_i$, $b_o$ and $t_p$ are the length, width, and thickness of the bonded plate. Although this model is more accurate than that presented by Malek et al. [1998] and that proposed by Arduini and Nanni [1997], it is based on the direct shear test, so it must be calibrated before being applied to concrete beams. Moreover, the model was based on a linear elastic analysis so it cannot be applied for cracked concrete. A more advanced model for the interfacial shear stresses at the FRP/concrete interface based on the analysis of FRP-strengthened concrete beams was developed by Taljsten [1997]. However, this model assumes as well a linear elastic behaviour for the concrete and adhesive layer.

Other accurate mathematical relations are the closed-form high order (CFHO) models based on sandwich panel theories, which simulate the adhesive layer as an elastic core with linear properties. These have been derived by many researchers [Rabinovitch and Frostig, 2000, 2001b; Shen et al., 2001]. In one of the CFHO models, the adhesive layer is treated as a linear orthotropic elastic medium with negligible stiffness in the longitudinal direction, and it is assumed that the interfacial stresses are constant over the adhesive layer thickness [Rabinovitch and Frostig, 2000]. Another CFHO model was derived by taking into account the longitudinal stiffness for the adhesive layer and assuming a linear shear stress distribution along the adhesive layer-thickness to avoid the divergence of the stresses between the adhesive/concrete interface and the adhesive/FRP interface for thin plate [Shen et al., 2001].

All the aforementioned models can be used successfully to calculate the maximum stresses at the interface and compare their values with the ultimate bond strength. However, these relations are valid only for the elastic materials, so they diverge in the case of cracking, crack propagation or in the case of failure when brittle materials are used.

The second approach to predict debonding failure modes is based on linear or non-linear fracture mechanics theories. In this approach, the stress intensity factor or fracture energy criteria are used to predict the interfacial stress concentrations at the vicinity of the
cracks [Rabinovitch and Frostig, 2000]. Figure 2.9 shows the fracture mechanics model of Rabinovitch and Frostig [2000]. This model is a six order differential model that is rather complicated to generalize and solve for the case of multi-crack distributions along the interface. In Figures 2.10(a) and 2.10(b), the normal and shear stress distributions for the case of a cracked section resulting for the Rabinovitch and Frostig [2000] model are depicted, respectively.

Although the approach based on fracture mechanics theory is able to accurately predict the interfacial stresses at the cracked and uncracked sections, it simulates the beams using a plane stress analysis. Generally, this approach is accepted and has a good accuracy. However, when the fibre orientation or the beam and the FRP sheet widths are considered, the accuracy of this model is diminished. Despite the fact that all the above models showed a reasonable accuracy to represent the problem of the FRP/concrete interface, they fail to predict the effect of crack propagations on the interfacial behaviour.
2.5 Finite element simulation of FRP-strengthened concrete beams

Various finite element studies have been presented to simulate the flexural behaviour of FRP-strengthened concrete beams. These studies have endeavoured to enhance the modelling technique [Nitereka and Neale, 1999; Kishi et al., 2005; Lu et al., 2007], or to generalize the experience of experimental investigations through parametric studies [Hu et al., 2001; Wu and Hemdan, 2005; Pham and Al-Mahaidi, 2005]. The parameters under investigation in the literature were the geometrical characteristics of the concrete beams [Hu et al., 2001; Wu and Hemdan, 2005], and geometrical or material parameters of the bonded FRP laminates [Hu et al., 2001; Thomsen et al., 2001; Wu and Hemdan, 2005; Camata et al., 2007].

Commercial finite element packages have been broadly employed to carry out finite element calculations including ABAQUS [Pesic and Pilakoutas, 2003; Hu et al., 2001], ANSYS [Gao et al., 2006], ADINA [Ross et al., 1999], DIANA [Kishi et al., 2005; Wu and Hemdan, 2005; Niu and Wu, 2006], MSC.MARC [Lu et al., 2007], FEAP [Thomsen et al., 2001], and LUSAS [Rahimi and Hutchinson, 2001; Smith and Teng, 2001a]. Other studies consisted of finite element codes based on a layer-by-layer model [Nitereka and Neale, 1999], or two-dimensional elements [Yang et al., 2003; Camata et al., 2007].

Figure 2.10: Stresses in adhesive layer at flexural crack formation [Rabinovitch and Frostig, 2000]
CHAPTER 2. LITERATURE REVIEW

One of the first analytical works on the behaviour of FRP-strengthened beams was that presented by Ehsani and Saadatmanesh [1990]; this was based on linear elastic analyses, and thus limited to the interfacial behaviour before cracking. A more advanced approach, the layer-by-layer numerical technique, takes into consideration the material nonlinearities of the concrete before and after cracking [Takahashi et al., 1997; Nitereka and Neale, 1999; Shin and Lee, 2003]. Despite the fact that layer-by-layer approaches have meant to account for the mechanical changes of the various materials throughout the beam depth, the out-of-plane stress components and geometrical characteristics of the beams were ignored.

Various constitutive laws have been adopted in the simulations of FRP-strengthened beams. These models included nonlinear elastic models [Arduini and Nanni, 1997; Pesic and Pilakoutas, 2003; Pham and Al-Mahaidi, 2005; Niu and Wu, 2006; Camata et al., 2007; Lu et al., 2007], and plasticity-based models whether perfect plasticity models [Hu et al., 2001; Wu and Hemdan, 2005], or elastic-plastic models [Hu et al., 2001; Kishi et al., 2005]. The failure of concrete under a general state of stress has been represented using a two-parameter criterion, such as the Drucker-Prager [Wu and Hemdan, 2005], or Mohr-Coulomb criteria [Hu et al., 2001]. In addition, a five-parameter model was also employed to represent the failure surface of concrete [Li et al., 2006]. To date, damage models have been rarely used to simulate the concrete nonlinearities; only the Rahimi and Hutchinson [2001] FE model applied an isotropic damage model for the concrete. Micromechanics-based constitutive laws have not yet been employed to simulate the mechanical characteristics of FRP-strengthened beams. All the previous elasticity or plasticity concrete relations successfully represented the nonlinearities in the load–deflection behaviour of flexurally strengthened beams. This is due to the fact that the flexural responses of the strengthened beams depend mainly on the tensile and cracking behaviour of the concrete rather than the compressive behaviour.

The nonlinear response of FRP-strengthened beams under various states of stress and strain is frequently dominated by progressive cracking, resulting from shear or tensile stresses and the influence of these cracks on the bond behaviour. In the literature, two approaches have been proposed to simulate crack initiation and propagation in the FRP-strengthened beams; namely, the discrete crack [Rahimi and Hutchinson, 2001; Kishi et al., 2005; Wu and Hemdan, 2005; Niu and Wu, 2006; Camata et al., 2007], and smeared crack approaches [Arduini and Nanni, 1997; Ross et al., 1999; Pesic and Pilakoutas, 2003; Hu
et al., 2001; Pham and Al-Mahaidi, 2005; Li et al., 2006; Lu et al., 2007]. The discrete and smeared crack models for cracked concrete are shown in Figure 2.11(a).

In a discrete crack model, the crack discontinuity is simulated using separated points of a finite element mesh, as shown in Figure 2.11(a) [Ngo and Scordelis, 1967]. This model was originally used to represent cracks at any point when the principal stress reaches the limit tensile strength. This approach requires continuous re-meshing of the elements at the vicinity of the cracks. Accordingly, the finite element analysis using this crack model demands a high computational capacity and frequently causes numerical errors. Researchers who have encountered these inherent difficulties and inconsistencies in FRP-strengthened beams proposed predefining the positions and directions of the crack growth within the model using interface elements at prescribed locations. Some of these studies considered only flexural cracks [Wu and Hemdan, 2005; Niu and Wu, 2006], while others addressed critical shear cracks [Kishi et al., 2005]. The use of predefined discrete crack models was generally combined with a smeared crack approach to describe the tensile behaviour between cracks. Although this technique gives a more realistic representation at the crack discontinuity, predefining the locations of the crack initiations and forcing these cracks to propagate along specific paths might not necessarily lead to accurate results. An application using the predefined discrete crack model is shown in Figure 2.11(b).

![Discrete crack model](image1) ![Smeared crack model](image2)

Figure 2.11: Smeared and discrete crack approaches

In the smeared crack approach, the discontinuity associated with cracks was represented using a cracking strain and smearing out this cracking strain over a volume element, \( \varepsilon_{cr} = \frac{w}{l} \) (\( w \) is the crack width and \( l \) is an equivalent element length). Consequently, a mesh-bias dependency of the flexural response was observed [Shayanfar et al., 1997].
In general, for a proper simulation of the crack width in concrete beams employing a smeared crack approach, the element size must be about one to three times the aggregate size. Then the crack strain can be considered as uniform over the element [Kwak and Filippou, 1990]. Figure 2.12 shows the variation of the predicted ultimate load with the number of elements in a nonlinear finite element analysis of a concrete beam. Here it was observed that although the results converge to a specific value, this value may not necessarily be the accurate value. Generally, reducing the element size underestimates the results. In the smeared crack approach, two different representations have been presented in the literature: the fixed crack model [Ross et al., 1999; Lu et al., 2007] and the rotating crack model [Pham and Al-Mahaidi, 2005; Camata et al., 2007].

![Figure 2.12: Effect of number of elements on the predicted ultimate load [Shayanfar et al., 1997]](image)

Finite element analyses of FRP-strengthened beams with a refined mesh in the plate end regions are in general used to investigate the elastic interfacial shear behaviour of the FRP/adhesive and adhesive/concrete interfaces. Such a mesh is shown in Figure 2.13.

![Figure 2.13: A fine finite element mesh [Smith and Teng, 2001a]](image)
2.5. FINITE ELEMENT SIMULATION OF FRP-STRENGTHENED CONCRETE BEAMS

[Smith and Teng, 2001a; Gao et al., 2006; Sarazin and Newhook, 2007]. To simulate debonding of the FRP laminates off the concrete surface, interface elements having a predefined bond–slip relation are used to link the FRP and concrete nodes [Wong and Vecchio, 2003; Niu and Wu, 2006; Lu et al., 2007]. These interface elements have no physical dimensions (Figure 2.14(a)). Although the debonding load can in several cases be predicted using such interface elements or using a predefined discrete crack model, stress concentrations along the FRP/concrete interface at the vicinity of a crack cannot be predicted. Lu et al. [2007] concluded that, when using continuous interface elements, the continuous interpolation function used for the interface forces the stress fields along the interface to a linear function, which contradicts the fact that there are stress concentrations at the cracks tip [Lu et al., 2007]. To remedy this deficiency, Lu et al. [2007] used two different bond–slip models at the crack and away from the crack as depicted in Figure 2.14(d). At the crack, the bond stress suddenly descends to zero at a slip value, $S_0$, corresponding to the bond strength. Lu et al. [2007] assumed that local debonding occurred when the crack width, $w$, in the concrete element exceeded $2S_0$. Thus, their FE model represented the stress/slip concentrations only for crack widths greater than $2S_0$. In Chapter 4 of this dissertation, a new technique to model the interface is presented to remedy all the aforementioned complexities in simulating the interfacial stress concentrations and fluctuations along the interface.

Several studies have been presented using a nonlinear fracture mechanics-based discrete crack analysis to investigate the interaction of bond properties on the performance of strengthened beams [Yang et al., 2003; Niu and Wu, 2006]. By contrast, little literature can be found concerning the interaction between concrete cracking and the behaviour at FRP/concrete interfaces. Niu and Wu [2006] concluded that the characteristics of the bond–slip model in terms of the bond strength $\tau_{\text{max}}$ and the initial slope of the curve have a negligible influence on the load–deflection behaviour of FRP-strengthened beams. However, the fracture energy of the bond–slip relationship does have a significant effect on the debonding load. In general, a low interfacial fracture energy will result in early debonding. In the Niu and Wu [2006] study it was concluded that the interfacial stiffness had a very slight effect on the yield load, but had no effect on the ultimate load carrying capacity.

To date, no refined finite element studies have been reported in the literature con-


**Figure 2.14:** Numerical model using interface elements
2.5. FINITE ELEMENT SIMULATION OF FRP-STRENGTHENED CONCRETE BEAMS

cerning the effect of crack initiation and propagation on the interfacial stresses in FRP-strengthened beams. One attempt to capture this behaviour was that presented by Rabinovitch and Frostig [2000] to analyze the interfacial stress/slip at the vicinity of the cracks using a nonlinear fracture mechanics analysis, as shown in Figures 2.14(b) and 2.14(c). They predicted that, before cracking, the interfacial shear-slip profile was continuously distributed over the element, as shown in Figure 2.14(b). In the cracked region (Figure 2.14(c)) an abrupt change in the slip occurred through the element. This was because of the relative movement of the two crack sides, thus leading to fluctuations of the interfacial slip. (Their fracture mechanics-based model was shown in Figure 2.9.).

Although interface elements have been intensively employed for FRP/concrete interfaces, most studies have ignored the bond-slip behaviour between the steel reinforcement and concrete. This has been justified by the fact that the reduction of the structure stiffness arising from the relative slip between the steel reinforcement and concrete can be generally included in the tension stiffening model of the concrete. This model attempts to model the stiffening effect by increasing or decreasing the stiffness of the cracked concrete. On the other hand, introducing relative slip between reinforcement bars and surrounding concrete elements reduces the overall stiffness of the structure. The finite element models accounting for the steel/concrete interfacial behaviour have used either a bilinear bond-slip model [Pham and Al-Mahaidi, 2005], or an elastic-plastic model [Kishi et al., 2005; Wu and Hemdan, 2005; Niu and Wu, 2006] to represent the steel/concrete interface.

Several discrete finite elements have been used to model various components of FRP-strengthened beams. In two-dimensional simulations, 4- or 8-node plane stress elements have been applied to represent the concrete, and 2-D plane stress or truss elements have been employed for the FRP laminates and steel reinforcement [Camata et al., 2007; Lu et al., 2007]. In three-dimensional analyses, 8- or 20-node brick elements have been used for the concrete, and 2- or 3-node truss elements for the steel reinforcement and FRP laminates [Kishi et al., 2005; Li et al., 2006]. One 3-D model, in the literature, smeared the steel reinforcement throughout the element section assuming a full bond between the concrete and steel reinforcement [Hu et al., 2001].
In order to prevent debonding of the FRP laminate off the concrete surface, most code specifications reduce the permissible strain values in the FRP reinforcement. Equation 2.12 below gives an expression for the reduced ultimate strain in the FRP, \( \varepsilon_{ub} \), as a function of the ultimate FRP strain value, \( \varepsilon_{fu} \) [ACI-440.2R-02, 2002]:

\[
\varepsilon_{ub} = k_m \varepsilon_{fu}
\]

(2.12)

The reduction factor \( k_m \) is not greater than 0.90. In the document ACI-440.2R-02 [2002], it is given as (SI units):

\[
k_m = \begin{cases} 
\frac{1}{60\varepsilon_{fu}} \left( 1 - \frac{nE_p t_p}{360,000} \right) & \leq 0.90 \text{ for } nE_p t_p \leq 180,000 \\
\frac{1}{60\varepsilon_{fu}} \left( \frac{90,000}{nE_p t_p} \right) & \leq 0.90 \text{ for } nE_p t_p > 180,000
\end{cases}
\]

(2.13)

where \( n \) is the number of plies, \( E_p \) is the modulus of elasticity of the FRP and \( t_p \) is the thickness of an FRP ply. A recent modification has been proposed in ACI-440.2R-07 [2007] as (SI units) (not published yet):

\[
\varepsilon_{ub} = 0.41 \sqrt{\frac{f_c'}{nE_p t_p}} \leq 0.9 \varepsilon_{fu}
\]

(2.14)

The Fédération internationale du béton (fib) [fib, 2001] has a similar formula to that in the ACI specifications; considers only the concrete tensile strength and the axial stiffness of the FRP laminates as parameters affecting the debonding strain. This design equation is given by:

\[
\varepsilon_{ub} = \alpha c_1 k_c k_b \sqrt{\frac{f_t}{nE_p t_p}}
\]

(2.15)

where \( \alpha \), \( k_c \), and \( k_b \) are factors accounting for the influence of cracks on the bond strength, the state of compacting of the concrete, and the width factor, respectively. \( c_1 \) is a factor determined experimentally. The factor \( k_b \) is expressed as:

\[
k_b = 1.06 \sqrt{\left( 2 - \frac{b_f}{b_c} \right) / \left( 1 + \frac{b_f}{400} \right)} \geq 1
\]

(2.16)

In most normal cases \( \alpha = 1.0 \), \( k_c = 1.0 \) and \( c_1 = 0.64 \).
2.7. CONCLUSION

The Chinese FRP code gives another design equation for the debonding strain considering the shear span of the beam as a factor controlling the debonding strain:

\[ \varepsilon_{ub} = \lambda k_b \frac{f_t}{2} \left( \frac{1}{\sqrt{nE_p t_p}} - \frac{0.2}{L_d} \right) \]  \hspace{1cm} (2.17)

Here \( \lambda \) is a factor that accounts for the anchorage FRP system (equal to 1.0 in cases without anchorage sheets) and \( k_b \) is a width factor expressed as:

\[ k_b = \sqrt{\left( 2.25 - \frac{b_f}{b_c} \right) / \left( 1.25 + \frac{b_f}{b_c} \right)} \]  \hspace{1cm} (2.18)

\( L_d \) in Equation 2.17 is the distance from the plate end to the section where the FRP laminate is fully utilized.

The Australian standard controls the debonding mechanism using the shear stress along the interface as:

\[ \varepsilon_{ub} = \sqrt{\tau_p \delta_p \sqrt{L_{per}}} \]  \hspace{1cm} (2.19)

The term \( \tau_p \delta_p \) is the maximum interfacial shear stress and it is computed as:

\[ \tau_p \delta_p = 0.73 \left( \frac{1}{b_f + 2} \right)^{0.5} (f_c)^{0.67} \]  \hspace{1cm} (2.20)

where \( b_f \) is the width of the FRP laminates.

(Note that, the design guidelines according to the Chinese and Australian codes are taken from Ye et al. [2005] and Oehlers et al. [2006], respectively.) Although the debonding of the FRP off the concrete surface is mainly a function of the cracking behaviour of the concrete and the amount of steel reinforcement and FRP, all the code specifications ascribe the debonding strain only to the FRP characteristics. There is an apparent need for more accurate code specifications that consider the interaction of the steel reinforcement ratio and cracking behaviour on the debonding phenomena.

2.7 Conclusion

The flexural behaviour of FRP-strengthened concrete beams has been reviewed in this literature survey. A particular attention has been paid to numerical studies carried out
CHAPTER 2. LITERATURE REVIEW

on these beams, due to the relative lack of numerical modelling work in comparison to the innumerable experimental investigations, and due to the potential for numerical modelling to explain the interfacial behaviour of FRP strengthened concrete beams.

Although the debonding phenomena have been investigated both experimentally and analytically by a number of authors, there is an obvious gap in the development of analysis and modelling approaches that can accurately simulate all of the characteristics of debonding. It was found that many parameters affect the response of the FRP-strengthened concrete beams. These parameters included the FRP sheet length, width and stiffness, beam geometry, concrete properties, loading type, FRP anchorage system type, and interfacial fracture energy.

Attention has been paid by many researchers to experimentally study the effect of these parameters on the ultimate load carrying capacities of FRP-strengthened reinforced concrete beams. In comparison, much fewer research works have been found concerning numerical modelling. In addition, most of the numerical investigations considered a limited number of parameters, or restricted to simulate only the nonlinearities in the load-deflection relationships. It is evident that the main complexities in numerical studies are simulating the FRP/concrete interface and the concrete characteristics. Some numerical works have attempted to find an accurate mathematical closed-form solution to simulate the nonlinear flexural response of FRP-strengthened beams. Most of these fail to simulate the influence of crack initiation and propagation on the interfacial stress distribution.

Code specifications investigating the debonding phenomena of FRP-strengthened beams have generally been based on conclusions drawn from the direct shear test observations or on elastic analyses neglecting the effect of crack initiation. Accordingly, there is a need for design guidelines that consider the influence of the reinforcement ratio and cracking behaviour on debonding phenomena.

As a result of the above observations, the focus of this thesis work will be on the numerical modelling of FRP-strengthened concrete beams. Several material and numerical parameters will be considered. In addition, this research will include a contribution to formulate a constitutive law for concrete and FRP/concrete interfaces using a micromechanics-based approach; namely, the microplane model for concrete. The ultimate objective is to propose practical design guidelines for strengthened beams that adequately
2.7. CONCLUSION

reflect the effect of crack initiation and propagation on the debonding phenomena, and properly account for the various parameters involved.
Chapter 3

Microplane Concrete Model

Of the various attempts that have been made to date to develop a unified constitutive theory for concrete, the approach taken by Bažant and co-workers in formulating microplane models has undoubtedly led to the greatest success. In particular, the so-called "M4" version of the microplane model [Bažant et al., 2000] has proven to be very effective for representing the behaviour of concrete under a wide range of complex stress and strain histories. In this chapter, an alternative incremental formulation of the M4 microplane model is proposed. Through a number of numerical applications, this incremental formulation is shown to be equivalent to the original M4 model. To assess the computational efficiency of the incremental formulation, the "arc-length" numerical technique is also considered and implemented in the original Bažant et al. [2000] M4 formulation. Comparisons of the incremental and arc-length results show that, from the computational viewpoint, the new incremental approach is significantly more efficient. The M4 model is coded in FORTRAN, and implemented as a user-defined subroutine into the commercial software package ADINA, Version 8.4 [ADINA, 2004a].

3.1 Introduction

Over the years there has been considerable interest in the constitutive modelling of brittle-plastic materials such as concrete. Essentially two approaches have been employed; namely, the phenomenological and the micromechanics-based models. The former have
been inspired by the classical macroscopic theories of plasticity and damage, and attempt to account for general triaxial states of stress and strain. However, they have generally proven to be inadequate in providing unified constitutive relations that accurately reflect experimental data for arbitrary deformation histories. To overcome these deficiencies, Bažant and Oh [1985] introduced an alternative microstructural approach, referred to as the “microplane model”, based on concepts in the slip theory of plasticity previously proposed by Batdorf and Budiansky [1949]. This model was meant, basically, to describe the tensile behaviour of concrete. Over the past twenty years, Bažant and co-workers have presented a series of progressively improved versions of the microplane model. Arguably, this approach has proved to be the most successful to date for modelling the behaviour of concrete under general conditions of stress and strain. In particular, the M4 formulation of the microplane model [Bažant et al., 2000] has been shown to yield predictions that are in excellent agreement with experimental data for a wide range of complex stress and strain histories [Caner and Bažant, 2000].

The complexities of the microplane model arise from the large number of parameters involved in the model (18 fixed constants, and 6 adjustable parameters) without a necessarily sound physical meaning for most of those parameters. Although the values of these parameters were determined by Caner and Bažant [2000] to fit well with most of the available experimental data, some debates regarding their values still exist. Another key issue of this model occurs when dealing with the fracture and damage behaviour of concrete since the M4 model is a local model in nature; i.e., the stress at a particular point depends only on the strain at this particular point regardless the strain fields in the neighbouring points. To overcome this problem, Bažant and Luzio [2004] proposed a new nonlocal microplane model based on nonlocality of the inelastic part of the strain. They showed that their nonlocal model worked well for the early part of the post-peak response, but did not fit well at very large strains across the cracking band.

The basic approach in the microplane model is to first project the macroscopic strain tensor into components on a certain number of microplanes that are used to characterize a material element. The material element here represents the microstructure in a smeared manner where each microplane represents the damage or weak plane at the microstructural level, such as contact layers between the aggregate pieces in the concrete. Stress–strain relations on each microplane are then employed to determine the respective micro-stress
components. Finally, using the principle of virtual work, a numerical integration over the total number of microplanes representing the material point is performed to determine the macroscopic stress components. The M4 microplane model as originally proposed by Bažant et al. [2000] involves highly nonlinear stress–strain relations. Caner and Bažant [2000] have given the outline of an explicit algorithm for implementing the model in numerical applications.

The M1 and M2 versions of the microplane model were originally presented in incremental forms in the pioneering work of Bažant and his co-workers [Bažant and Oh, 1985; Bažant and Prat, 1988]. Kuhl and Ramm [1998] exploited these incremental forms and presented their numerical examples based on the tangent stiffness matrix for the M2 version of the microplane model. That was relatively simple to do since the M1 and M2 versions have continuous smooth curves for defining the microscopic stress–strain relations. In the M3 and M4 versions of the microplane model, as a result of introducing a new concept for the microscopic stress boundaries, the microscopic stress–strain relationships became non-continuous functions. In addition, in the M4 version, a new concept of the microscopic normal stress boundary was introduced in order to redistribute the microscopic volumetric and deviatoric stresses after the initiation of microcracks to prevent their separate localization. This means that the relatively simple derivations of the incremental formulas of the M1 and M2 versions of the microplane model were no longer applicable.

This chapter is organized as follows. First, a brief summary of the basic relations of the M4 version of the microplane model is given. We then present our proposed incremental formulation of the M4 microplane model. This involves the development of explicit expressions for the incremental microscopic stress–strain moduli. Subsequently, we derive a general expression for the three-dimensional incremental moduli $C_{ijkl}$, as well as the relations for plane stress applications. As examples of application we consider the cases treated in Caner and Bažant [2000], and show that the proposed incremental formulation leads to results that are essentially identical to those presented in that reference. Finally, to demonstrate the computational performance of the incremental formulation, we consider the implementation of the “arc-length” numerical technique [Riks, 1972, 1979] in conjunction with the original Bažant et al. [2000] formulation. This exercise serves first as a verification of the incremental formulation and its implementation, and also demon-
strates its computational efficiency. Finally, the microplane model is coded in FORTRAN and linked to the ADINA finite element package as a user-defined subroutine.

3.2 Objectives

In this chapter, we present an alternative incremental formulation of the M4 microplane model, which is entirely equivalent to that given by Bažant et al. [2000]. Accordingly, we consider a relation of the form \( \dot{\sigma}_{ij} = C_{ijkl} \dot{\varepsilon}_{kl} \) in terms of the macroscopic stress-rate and strain-rate tensors \( \dot{\sigma}_{ij} \) and \( \dot{\varepsilon}_{ij} \), and obtain explicit expressions for the tensor of incremental moduli \( C_{ijkl} \). One advantage of this formulation is its ease of implementation for cases involving “mixed” boundary conditions (i.e., when the prescribed quantities combine both incremental stress and strain components). Another advantage arises for plane stress applications since, for this case, the plane stress incremental moduli are readily obtainable from the three-dimensional \( C_{ijkl} \). A user-defined subroutine has been developed to incorporate the M4 version of the microplane into the commercial finite element program ADINA, Version 8.4, to take advantage of the rich libraries of structural elements involved. The description of this FORTRAN subroutine is presented.

3.3 Review of the microplane concrete model M4

Full details concerning the underlying hypotheses, basic relations, and inherent advantages and difficulties of the M4 version of the microplane model can be found in Bažant et al. [2000] and Caner and Bažant [2000]. A brief recapitulation of its main features is presented below.

With this constitutive theory, a representative volume of material is viewed at the microstructural level, and is considered as a three-dimensional element defined by a set of microplanes of different orientations arranged in a regular pattern. A typical representation is shown in Figure 3.1(a), which depicts a material element characterized by 28 equally-distributed planes per hemisphere. These planes represent the damage or weak planes at the microstructural level (aggregate-mortar interface) or planes of microcrack
CHAPTER 3. MICROPLANE CONCRETE MODEL

formation as depicted in Figure 3.1(b). The orientation of each plane is specified by a unit normal vector, having components \( n_i \). (Throughout this chapter the usual notations and conventions for Cartesian vectors and tensors are employed.) A key assumption in the theory is that the macroscopic strain tensor, \( \varepsilon_{ij} \), can be projected into a microscopic normal strain vector \( \varepsilon_N \) and shear strain vector \( \varepsilon_T \) on each microplane as illustrated schematically in Figure 3.1(c). The normal vector is given by:

\[
\varepsilon_N = N_{ij} \varepsilon_{ij},
\]

where \( N_{ij} = n_i n_j \). To better control the triaxial behaviour, Bązant and Prat (1988) have proposed splitting this vector into volumetric \( \varepsilon_V \), and deviatoric \( \varepsilon_D \) strain vectors; i.e.,

\[
\varepsilon_N = \varepsilon_V + \varepsilon_D
\]

The volumetric component characterizes the hydrostatic behaviour of the concrete; it is considered to be an invariant, such that [Bązant et al., 1996]:

\[
\varepsilon_V = \varepsilon_{ij} \delta_{ij} \frac{3}{3}
\]

where \( \delta_{ij} \) is the Kronecker delta. With regard to the microscopic shear strain component \( \varepsilon_T \), it is further decomposed into two components with respect to perpendicular directions \( l \) and \( m \) in the plane as follows:

\[
\varepsilon_M = M_{ij} \varepsilon_{ij}, \quad \varepsilon_L = L_{ij} \varepsilon_{ij}
\]

Figure 3.1: Representative volume element in microplane model
3.3. REVIEW OF THE MICROPLANE CONCRETE MODEL M4

where \( M_{ij} = (m_i n_j + m_j n_i)/2 \) and \( L_{ij} = (l_i n_j + l_j n_i)/2 \). The magnitude of \( \varepsilon_T \) is given by:

\[
\varepsilon_T = \sqrt{\varepsilon_T^2 + \varepsilon_M^2}
\]  

(3.5)

The inelastic macroscopic response of concrete is considered to arise from microcracks initiated at the microstructural level. The concept of microscopic stress boundaries is introduced to capture the microscopic behaviour after cracking, as well as to simulate the softening behaviour of concrete [Bažant et al., 1996, 2000]. Figures 3.2(a) to 3.2(d) depict the microscopic stress boundary curves for the microscopic volumetric, deviatoric, normal, and shear stresses, respectively [Bažant et al., 2000]. Inside the boundaries, the response is assumed linear elastic; it is governed by incremental or rate equations of the form:

\[
\dot{\sigma}_V = E_V \dot{\varepsilon}_V, \quad \dot{\sigma}_D = E_D \dot{\varepsilon}_D
\]  

(3.6)

\[
\dot{\sigma}_M = E_T \dot{\varepsilon}_M, \quad \dot{\sigma}_L = E_T \dot{\varepsilon}_L
\]  

(3.7)

\[\text{Figure 3.2: Microscopic stress boundaries}\]
Here, $E_V$, $E_D$ and $E_T$ represent the microplane elastic moduli. They are related to the macroscopic Young's modulus $E$ and Poisson's ratio $\nu$ as follows [Bažant et al., 2000]:

$$E_V = \frac{E}{1-2\nu}, \quad E_D = \frac{5E}{(2+3\mu)(1+\nu)}, \quad E_T = \mu E_D$$  \hspace{1cm} (3.8)

where $\mu$ is a parameter that characterizes the effects of damage.

Along the boundaries, the various microstress–microstrain relations are [Bažant et al., 2000]:

$$\sigma_V^+ (+\varepsilon_V) = \frac{E_V k_1 c_{13}}{[1 + (c_{14}/k_1) < \varepsilon_V - k_1 c_{13}>^2]} \quad \text{if } \sigma_V \geq 0$$  \hspace{1cm} (3.9)

$$\sigma_V^- (-\varepsilon_V) = -E k_1 k_3 \exp \left( \frac{-\varepsilon_V}{k_1 k_4} \right) \quad \text{if } \sigma_V < 0$$

$$\sigma_D^+ (+\varepsilon_D) = \frac{E k_1 c_5}{1 + (<\varepsilon_D - c_9 c_1 k_1>/k_1 c_{18} c_7)^2} \quad \text{if } \sigma_D \geq 0$$  \hspace{1cm} (3.10)

$$\sigma_D^- (-\varepsilon_D) = -\frac{E k_1 c_8}{1 + (<\varepsilon_D - c_9 c_1 k_1>/k_1 c_{18} c_7)^2} \quad \text{if } \sigma_D < 0$$

$$\sigma_N^+ (\varepsilon_N) = E k_1 c_1 \exp \left( -\frac{<\varepsilon_N - c_1 c_2 k_1>/k_1 c_3 + <-c_4 (\sigma_V/E_V)>}{k_1 c_3} \right)$$  \hspace{1cm} (3.11)

$$\sigma_T^+ (-\sigma_N) = \frac{E T k_1 k_2 c_{10}}{E T k_1 k_2 + c_{10}} \quad \text{if } \sigma_T < -\sigma_N + \sigma_N'$$  \hspace{1cm} (3.12)

where $\sigma_V^+$, $\sigma_D^+$, $\sigma_N^+$, and $\sigma_T^+$ are the volumetric, deviatoric, normal and shear stress boundaries respectively; $k_1$ to $k_4$ are the microplane adjustable parameters, $c_1$ to $c_{18}$ are the microplane fixed constants, and the term $\sigma_N'$ is given by:

$$\sigma_N' = \frac{E T k_1 c_{11}}{1 + c_2 < \varepsilon_V>}$$  \hspace{1cm} (3.13)

The volumetric stress is computed as the minimum of the previous computed value and the average of the microscopic normal stress over the unit hemisphere expressed as:

$$\sigma_V = \int_{\Omega} \frac{\sigma_N d\Omega}{\pi}$$  \hspace{1cm} (3.14)

In general, the microscopic stress components are computed separately on the various planes, which are treated independently of one another. As a result, the equilibrium condition between the microscopic stress components and the macroscopic stress tensor...
is not generally satisfied using the projection method. To satisfy overall equilibrium, the principle of virtual work is invoked, which leads to the following expression for the macroscopic stress tensor [Bažant and Prat, 1988; Bažant et al., 1996, 2000]:

$$\sigma_{ij} = \sigma_V \delta_{ij} + \frac{3}{2\pi} \int_{\Omega} \left[ \sigma_D \left( N_{ij} - \frac{\delta_{ij}}{3} \right) + \sigma_L L_{ij} + \sigma_M M_{ij} \right] d\Omega \quad (3.15)$$

Numerically, this integration is carried out over a unit hemisphere $\Omega$ using an optimal Gaussian integration formula [Bažant and Oh, 1985; Bažant and Prat, 1988]. This leads to an expression of the form:

$$\sigma_{ij} \approx \sigma_V \delta_{ij} + 6 \sum_{N=1}^{N=m} w_N \left[ \sigma_D \left( N_{ij} - \frac{\delta_{ij}}{3} \right) + \sigma_L L_{ij} + \sigma_M M_{ij} \right]_N \quad (3.16)$$

where $m$ is the number of microplanes and the $w_N$ represent the integration coefficients. The values of these coefficients for the 28-plane representative volume element of Figure 3.1(a) can be found in Bažant and Oh [1985].

### 3.4 Numerical schemes using the microplane model

Three different numerical techniques will now be presented in the following sections. The first technique is our proposed incremental formulation of the M4 version of the microplane model, followed by examples of numerical applications. In the second numerical approach, we consider the implementation of the “arc-length” numerical technique [Riks, 1972, 1979] in conjunction with the original Bažant et al. [2000] formulation. Finally, the numerical implementations using a user-defined subroutine incorporating the M4 version of the microplane model into the commercial finite element package ADINA, Version 8.4, are introduced. We show that the three aforementioned numerical techniques lead to identical numerical results, which are essentially the same as those presented in Caner and Bažant [2000].
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3.4.1 Proposed incremental formulation of the microplane model

In this section an incremental formulation of the microplane model M4 is presented, leading to a relation of the form \( \dot{\sigma}_{ij} = C_{ijkl} \dot{\epsilon}_{kl} \) in terms of the macroscopic stress-rate and strain-rate tensors, \( \dot{\sigma}_{ij} \) and \( \dot{\epsilon}_{ij} \). (Note that, as the basic constitutive law is inherently rate-independent, we interchangeably use the terms “rate” and “incremental” throughout this thesis.) Explicit expressions are derived for the tensor of incremental moduli, \( C_{ijkl} \). This formulation is attractive in that it is easily implemented in a straightforward numerical integration scheme to solve for arbitrary applied stress and strain histories, as will be illustrated for various applications in the next section. Another advantage of this approach is that the reduced moduli for plane stress applications are readily obtainable from the three-dimensional moduli \( C_{ijkl} \).

In the proposed incremental formulation, the microscopic material response is assumed linear elastic until the boundary limits are reached, after which incremental tangential relations are used along each boundary such that the microscopic stress components remain on the boundary surface. For conditions of unloading, incremental unloading moduli on the affected microplanes are used in the incremental microscopic stress-strain relationships. The criteria for defining the microscopic elastic and inelastic regimes are as follows:

\[
\| f \| - \| \sigma^e \| > 0 \quad \text{for the elastic regime (inside the boundary)}
\]

\[
\| f \| - \| \sigma^e \| \leq 0 \quad \text{for the inelastic regime (along the boundary)}
\]

where \( f \) represents the boundary value and \( \sigma^e \) is the relevant elastic microscopic stress (e.g., volumetric, \( \sigma^e_v \), deviatoric, \( \sigma^e_D \), or shear, \( \sigma^e_T \)). Similar expressions for the inelastic regime are developed in the following sections.

3.4.1.1 Incremental microscopic volumetric and deviatoric stress-strain relationships

In the inelastic regime, the incremental microscopic volumetric and deviatoric stress-strain relationships are of the form:

\[
\dot{\sigma}_V = K_V^T \dot{\epsilon}_V, \quad \dot{\sigma}_D = K_D^T \dot{\epsilon}_D
\]

where \( K_V^T \) and \( K_D^T \) represent the incremental volumetric and deviatoric microscopic moduli, respectively. These correspond to the first derivatives of the microscopic boundary
functions given by Equation 3.9 and Equation 3.10, respectively; i.e.,

\[ K_V^T = \frac{d\sigma_v^b}{d\varepsilon_v}, \quad K_D^T = \frac{d\sigma_D^b}{d\varepsilon_D} \] (3.19)

Along the compressive volumetric boundary we have:

\[ K_V^T(-\varepsilon_V) = \frac{d\varepsilon_V}{d\varepsilon_V} = \frac{E k_3}{k_4} \exp \left( -\frac{\varepsilon_V}{k_1 k_4} \right) \] (3.20)

whereas the expression associated with the tensile volumetric boundary is as follows:

\[ K_V^T(+\varepsilon_V) = \frac{d\varepsilon_V}{d\varepsilon_V} = \begin{cases} 0 & \text{if } \varepsilon_V < k_1 c_{13} \\ \frac{2 E v c_{13} c_{14}}{(1 + c_{14} - \varepsilon_V - k_1 c_{13})^3} & \text{if } \varepsilon_V > k_1 c_{13} \end{cases} \] (3.21)

The physical significance of the non-zero term in the above relation is volumetric softening behaviour due to the presence of microscopic cracks.

The microscopic inelastic deviatoric moduli characterize the behaviour of lateral cracks normal to the microplane. As the microscopic inelastic deviatoric stress decreases with an increase in the microscopic deviatoric strain, this leads to a negative microscopic deviatoric modulus. From Equation 3.10, the incremental deviatoric moduli for the compressive deviatoric strains become:

\[ K_D^T(-\varepsilon_D) = \frac{d\sigma_D^c}{d\varepsilon_D} = \begin{cases} 0 & \text{if } \varepsilon_D < k_1 c_9 c_{12} \\ \frac{2 E c_8 (\varepsilon_D - c_8 c_9 k_1)}{k_1 c_7 \left[ 1 + \left( \frac{\varepsilon_D - c_8 c_9 k_1}{k_1 c_7} \right)^2 \right]^{\frac{3}{2}}} & \text{if } \varepsilon_D > k_1 C_3 C_9 \end{cases} \] (3.22)

while for the tensile deviatoric strains we have:

\[ K_D^T(+\varepsilon_D) = \frac{d\sigma_D^c}{d\varepsilon_D} = \begin{cases} 0 & \text{if } \varepsilon_D < k_1 c_9 c_6 \\ \frac{2 E c_5 (\varepsilon_D - c_5 c_6 k_1)}{k_1 (c_7 c_{18})^2 \left[ 1 + \left( \frac{\varepsilon_D - c_5 c_6 k_1}{k_1 c_7 c_{18}} \right)^2 \right]^{\frac{3}{2}}} & \text{if } \varepsilon_D > k_1 c_9 c_6 \end{cases} \] (3.23)

### 3.4.1.2 Incremental microscopic normal stress–strain relationship

The microscopic normal boundary plays an important role in the redistribution of the microscopic volumetric and deviatoric stresses after the initiation of microcracks in order
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to prevent the separate localization of the volumetric and deviatoric strains. When the microscopic normal stress reaches the boundary, the deviatoric stress is concentrated on particular planes while other planes unload. Thus, the microscopic normal modulus is used to update the volumetric and deviatoric moduli. The inelastic incremental microscopic volumetric stress rate can be updated using the microscopic normal stress as follows:

$$\Delta \sigma_V = \frac{1}{2\pi} \int_{\Omega} \sigma_N d\Omega - \sigma_V$$  \hspace{1cm} (3.24)

and the normal stress rate $\dot{\sigma}_N$ is given by:

$$\dot{\sigma}_N = \dot{\sigma}_V + \dot{\sigma}_D \quad \text{for elastic regime} \quad (||f_N|| - ||\sigma_N|| > 0)$$  \hspace{1cm} (3.25)

$$\dot{\sigma}_N = \dot{f}_N \quad \text{for inelastic regime} \quad (||f_N|| - ||\sigma_N|| \leq 0)$$  \hspace{1cm} (3.26)

The incremental microscopic volumetric stress is taken as the minimum value of those computed from Equation 3.18 and Equation 3.24. Accordingly, the incremental microscopic volumetric modulus is updated using $K_V^T = \dot{\sigma}_V/\dot{\varepsilon}_V$. Furthermore, using the derivative of Equation 3.11, we get:

$$f_N = \frac{\partial f_N}{\partial \varepsilon_N} \varepsilon_N + \frac{\partial f_N}{\partial \sigma_V} \sigma_V$$  \hspace{1cm} (3.27)

where

$$\frac{\partial f_N}{\partial \varepsilon_N} = \begin{cases} 0 & \text{if } \varepsilon_N < c_1 c_2 k_1 \\ E k_1 c_1 \exp \left( \frac{\langle \varepsilon_N - c_1 c_2 k_1 \rangle}{k_1 c_3 + \langle -\frac{c_4 \sigma_V}{E_V} \rangle} \right) & \text{if } \varepsilon_N > c_1 c_2 k_1 \end{cases}$$  \hspace{1cm} (3.28)

and

$$\frac{\partial f_N}{\partial \sigma_V} = \begin{cases} 0 & \text{if } \sigma_V > 0 \\ E k_1 c_1 \exp \left( \frac{\langle \varepsilon_N - c_1 c_2 k_1 \rangle}{k_1 c_3 + \langle -\frac{c_4 \sigma_V}{E_V} \rangle} \right) \frac{\langle \varepsilon_N - c_1 c_2 k_1 \rangle}{\langle -\frac{c_4 \sigma_V}{E_V} \rangle} \left( \frac{c_4}{E_V} \right) & \text{if } \sigma_V < 0 \end{cases}$$  \hspace{1cm} (3.29)
Writing $\sigma_N = K_N^T \dot{\varepsilon}_N$ gives $K_N^T = d\sigma_N / d\varepsilon_N$.

Subsequent to computing the above incremental moduli, the microscopic deviatoric modulus is updated by substituting the following relations into Equation 3.25:

\[ \dot{\sigma}_N = K_N^T \dot{\varepsilon}_N, \quad \dot{\sigma}_V = K_V^T \dot{\varepsilon}_V \quad \text{and} \quad \dot{\sigma}_D = K_D^T \dot{\varepsilon}_D \quad (3.30) \]

This gives:

\[ K_D^T = K_N^T + (K_N^T - K_V^T) \frac{\dot{\varepsilon}_V}{\dot{\varepsilon}_D} \quad (3.31) \]

Since

\[ \dot{\varepsilon}_V = \frac{1}{3} \delta_{ij} \varepsilon_{ij}, \quad \dot{\varepsilon}_N = N_{ij} \dot{\varepsilon}_{ij}, \quad \dot{\varepsilon}_D = \left( N_{ij} \frac{1}{3} \delta_{ij} \right) \dot{\varepsilon}_{ij} \quad (3.32) \]

then

\[ K_D^T = K_N^T + (K_N^T - K_V^T) \frac{\delta_{ij}}{3 \left( N_{ij} - \frac{1}{3} \delta_{ij} \right)} \quad (3.33) \]

### 3.4.1.3 Incremental microscopic shear stress–strain relationships

When the microscopic shear stress reaches the boundary, $\| f_T \| - \| \sigma_T^b \| = 0$, the incremental microscopic shear stress is computed as the minimum value of the incremental elastic value, $E_T \dot{\varepsilon}_T$, and the incremental shear boundary value, $\dot{f}_T$. The latter is expressed as the time rate of Equation 3.12 as follows:

\[ \dot{f}_T = \frac{\partial f_T}{\partial \sigma_N} \dot{\sigma}_N + \frac{\partial f_T}{\partial \varepsilon_V} \dot{\varepsilon}_V \quad (3.34) \]

where \( \frac{\partial f_T}{\partial \varepsilon_V} = \frac{\partial f_T}{\partial \sigma_N} \frac{\partial \sigma_N}{\partial \varepsilon_V} \).

\[ \frac{\partial f_T}{\partial \sigma_N} = \begin{cases} - \left[ \frac{E_T k_1 k_2 c_{10}}{E_T k_1 k_2 + c_{10} < -\sigma_N + \sigma_N^b >} \right] & \text{if } \sigma_N < \sigma_N^b \\ \left[ \frac{E_T k_1 k_2 c_{10}}{E_T k_1 k_2 + c_{10} < -\sigma_N + \sigma_N^b >} \right]^2 & \text{if } \sigma_N > \sigma_N^b \end{cases} \quad (3.35) \]
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\[
\frac{\partial f_T}{\partial \sigma_N} = \begin{cases} 
\frac{E_T k_1 k_2 c_{10}}{E_T k_1 k_2 + c_{10} < -\sigma_N + \frac{\sigma}{N}} - \frac{E_T k_1 k_2 c_{10}^2 < -\sigma_N + \frac{\sigma}{N}}{(E_T k_1 k_2 + c_{10} < -\sigma_N + \frac{\sigma}{N})^2} & \text{if } \sigma_N < \frac{\sigma}{N} \\
0 & \text{if } \sigma_N > \frac{\sigma}{N}
\end{cases}
\]

and

\[
\frac{\partial \dot{\sigma}_N}{\partial \varepsilon_V} = \begin{cases} 
0 & \text{if } \varepsilon_V < 0 \\
- \frac{E_T k_1 c_{11} c_{12}}{(1 + c_{12} < \varepsilon_V)^2} & \text{if } \varepsilon_V > 0
\end{cases}
\]

The microscopic incremental inelastic shear modulus is calculated as follows:

\[
K_T^N = \min \left( E_T, \frac{\dot{f}_T}{\dot{\varepsilon}_T} \right)
\]

Once \( K_T^N \) is determined, it is then used to compute the microscopic shear stress rates in the \( l \) and \( m \) directions using the expressions:

\[
\dot{\sigma}_L = K_T^N \dot{\varepsilon}_L, \quad \dot{\sigma}_M = K_T^N \dot{\varepsilon}_M
\]

Finally, the microscopic stress–strain relationships are expressed in an incremental form using the microscopic elastic, inelastic or unloading moduli as follows:

\[
\dot{\sigma}_V = K_V^T \dot{\varepsilon}_V, \quad \dot{\sigma}_D = K_D^T \dot{\varepsilon}_D, \quad \dot{\sigma}_T = K_T^T \dot{\varepsilon}_T
\]

where the incremental microscopic volumetric modulus, for example, is expressed as:

\[
K_V^T = \begin{cases} 
E_V & \text{for elastic stage } (\|f_V\| - \|\sigma_V\| > 0) \\
K_V^T & \text{for inelastic stage } (\|f_V\| - \|\sigma_V\| < 0) \\
E_V^U & \text{for unloading state of stress } (\sigma_V \varepsilon_V < 0)
\end{cases}
\]

Similarly, the expression for the incremental microscopic deviatoric moduli, \( K_D^T \) and \( K_T^T \) are:

\[
K_D^T = \begin{cases} 
E_D & \text{for elastic stage } (\|f_D\| - \|\sigma_D\| > 0) \\
K_D^T & \text{for inelastic stage } (\|f_D\| - \|\sigma_D\| < 0) \\
E_D^U & \text{for unloading state of stress } (\sigma_D \varepsilon_D < 0)
\end{cases}
\]
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and

\[ K_T = \begin{cases} 
E_T & \text{for elastic stage } (\|f_T\| - \|\sigma_T\| > 0) \\
K_T^T & \text{for inelastic stage } (\|f_T\| - \|\sigma_T\| < 0) \\
E_T^U & \text{for unloading state of stress } (\sigma_T \dot{\varepsilon}_T < 0) 
\end{cases} \tag{3.43} \]

The various microplane unloading moduli \( E_T^U \), \( E_D^U \) in Equation 3.41 to Equation 3.43 and \( E_T^U \) are given in the original work of Bažant et al. [2000] as:

\[ E_T^V = \begin{cases} 
E_V \left( \frac{c_{15}}{c_{15} - \varepsilon_V} + \frac{\sigma_V}{c_{15}c_{16}E_V} \varepsilon_V \right) & \text{if } \varepsilon_V \leq 0 \text{ and } \sigma_V \leq 0 \\
\min \left[ (\sigma_V / \varepsilon_V), E_V \right] & \text{if } \varepsilon_V > 0 \text{ and } \sigma_V > 0 
\end{cases} \tag{3.44} \]

\[ E_D^U = (1 - c_{17})E_D + c_{17}E_D^a \tag{3.45} \]

\[ E_T^U = (1 - c_{17})E_T + c_{17}E_T^a \tag{3.46} \]

3.4.1.4 General tensor of incremental moduli, \( C_{ijkl} \)

We now consider the incremental macroscopic stress-rate–strain-rate constitutive relation of the form:

\[ \dot{\sigma}_{ij} = C_{ijkl}\dot{\varepsilon}_{kl} \tag{3.47} \]

resulting from the M4 version of the microplane model, and derive explicit expressions for the tensor of incremental moduli, \( C_{ijkl} \). Relations of this form were first used in the work of Bažant and Prat [1988] for the M2 version of the microplane model, based on the principle of virtual work. Here, this idea is generalized to account for the added intricacies of the M4 microplane model associated with the introduction of the various stress boundaries. This is accomplished by employing a rate formulation of the virtual work principle, where the total stress quantities in Bažant et al. [2000] are replaced by rate quantities. The expressions developed earlier for the incremental microscopic moduli \( K_T^V \), \( K_D^T \) and \( K_T^T \) enter into the formulation.

As in Bažant et al. [2000], a surface \( \Omega \) of a unit hemisphere is considered, for which the macroscopic incremental virtual work is given by:

\[ W_{\text{macro}} = \frac{2\pi}{3} \dot{\sigma}_{ij}\delta \varepsilon_{ij} \tag{3.48} \]
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with \( \delta \varepsilon_{ij} = \dot{\varepsilon}_{ij} \delta t \), where \( \delta t \) is an arbitrary time increment. The microscopic incremental virtual work becomes:

\[
W_{\text{micro}} = \int_{\Omega} (\dot{\sigma}_N \delta \varepsilon_N + \dot{\sigma}_L \delta \varepsilon_L + \dot{\sigma}_M \delta \varepsilon_M) d\Omega
\]  

(3.49)

On the microplane level, the material response is incrementally linear, so that

\[
\dot{\sigma}_V = K_T^V \dot{\varepsilon}_V, \quad \dot{\sigma}_D = K_T^D \dot{\varepsilon}_D \\
\dot{\sigma}_L = K_T^L \dot{\varepsilon}_L, \quad \dot{\sigma}_M = K_T^M \dot{\varepsilon}_M
\]

Substituting the above relations in Equation 3.49, and then setting Equation 3.48 equal to Equation 3.49, gives:

\[
\dot{\varepsilon}_{ij} = \frac{3}{2\pi} \int_{\Omega} \left\{ \left[ K_T^V \frac{\delta_{kl}}{3} + K_T^D \left( N_{kl} - \frac{\delta_{kl}}{3} \right) \right] N_{ij} + K_T^L L_{ij} L_{kl} + K_T^M M_{ij} M_{kl} \right\} \dot{\varepsilon}_{kl} d\Omega
\]  

(3.50)

Thus, from Equation 3.50 we have:

\[
C_{ijkl} = \frac{3}{2\pi} \int_{\Omega} \left[ (K_T^V - K_T^D) \frac{\delta_{kl}}{3} N_{ij} + K_T^D N_{ij} N_{kl} + K_T^L L_{ij} L_{kl} + K_T^M M_{ij} M_{kl} \right] d\Omega
\]  

(3.51)

From the above expression, it is clear that the tensor of incremental moduli is symmetric with respect to the indices \( i \leftrightarrow j \) and \( k \leftrightarrow l \); however, in general, it lacks the \( ij \leftrightarrow kl \) symmetry properties due to the term \( \delta_{kl} N_{ij} \). In special cases, such as for uniform hydrostatic compression where \( \dot{\varepsilon}_{11} = \dot{\varepsilon}_{22} = \dot{\varepsilon}_{33} \), the deviatoric strain is equal to zero. In this case and other similar situations, the macroscopic incremental modulus tensor does possess the \( ij \leftrightarrow kl \) symmetry characteristics. Also, within the elastic regime, where all the microscopic stresses are inside the boundary curves, the incremental deviatoric modulus \( K_T^D = E_I \) is given by Equation 3.8. As a result,

\[
\int_{\Omega} K_T^D \delta_{kl} N_{ij} d\Omega = \frac{2\pi}{3} K_T^D \delta_{kl} \dot{\varepsilon}_{ij} = \frac{10\pi E}{3 \left( \frac{2 + 3\mu}{(2 + 3\mu)(1 + \nu)} \right)} \delta_{ij} \delta_{kl}
\]  

(3.52)

which leads to \( ij \leftrightarrow kl \) symmetry.
3.4.1.5 3.5. Plane stress applications

For plane stress applications, it is obviously advantageous to have explicit expressions for the incremental moduli associated with this simplification. In this case, the three-dimensional relation, Equation 3.47, is reduced to the form:

\[ \sigma_{\alpha\beta} = \tilde{C}_{\alpha\beta\gamma\tau} \dot{\varepsilon}_{\gamma\tau} \]

where the Greek indices range from 1 to 2. From the assumption that \( \sigma_{13} = \sigma_{23} = \sigma_{33} = 0 \), the modified incremental plane stress moduli can be written as follows [Hutchinson, 1974]:

\[ \tilde{C}_{\alpha\beta\gamma\tau} = C_{\alpha\beta\gamma\tau} - \frac{C_{\alpha\beta\gamma\lambda} C_{3\gamma\lambda}}{C_{333}} \]  

(3.54)

Furthermore, the normal strain increment in the \( x_3 \)-direction is given by:

\[ \dot{\varepsilon}_{33} = -\frac{C_{33\gamma\lambda}}{C_{333}} \dot{\varepsilon}_{\gamma\tau} \]

(3.55)

3.4.2 Applications of the incremental formulation

We now examine a number of numerical applications of the proposed incremental formulation of the M4 version of the microplane model. Various macroscopic stress histories are imposed, and the corresponding strain responses are determined. A straightforward numerical integration scheme is employed for the computations. As a basis of comparison for computational efficiency, the iterative arc-length technique [Riks, 1972, 1979] (described in Subsection 3.4.3) has also been implemented in the original Bazant et al. [2000] formulation of the M4 model. (Details of the arc-length scheme are presented in Section 3.4.3.) In all the simulations, the differences between the plotted results obtained with the incremental and the arc-length numerical analyses are indistinguishable. The cases considered correspond to those treated by Caner and Bazant [2000], and they include triaxial, biaxial and uniaxial macroscopic stress states.

3.4.2.1 Triaxial states of stress

For fully triaxial states of stress and strain, the implementation of the microplane model is rather straightforward as all of the strain components are known. That is, for each
prescribed macroscopic strain history, the microplane strains are readily obtained. From these, the microplane stresses and resulting overall macroscopic stresses can be directly computed. Typical results are presented in Figures 3.3(a) and 3.3(b), which correspond to uniform hydrostatic compression and confined compression. In these figures, the curves labelled “Incremental” refer to our numerical results (obtained from both the arc-length algorithm and incremental formulation). Those denoted by “Caner and Bažant” refer to the results presented in their 2000 paper, while the dots represent the experimental data. These figures show that our numerical predictions are in excellent agreement with those of Caner and Bažant [2000], which in turn quite accurately simulate the experimental trends.

\[
\begin{align*}
\sigma_1/ f'_c &= 48.4 \text{ MPa} \\
E &= 35163 \text{ MPa} \\
\sigma_1 &= 4.93 \text{ MPa} \\
E &= 41369 \text{ MPa}
\end{align*}
\]

(a) Hydrostatic compression  
(b) Confined compression

Figure 3.3: Triaxial states of stress

In the following sections, we present results of the implementation of our incremental technique for other special cases; namely, biaxial and uniaxial states of stress.

3.4.2.2 Biaxial states of stress

We first consider the case of a biaxial test where \( \sigma_{11} \) and \( \varepsilon_{11} \) are applied along the axis of a specimen, \( \sigma_{22} \) is equal the lateral pressure, and \( \sigma_{33} = 0 \). In the incremental solution,
the value of $\dot{\varepsilon}_{22}$ depends on the value of $\dot{\sigma}_{22}$ according to the following relation:

$$\dot{\varepsilon}_{22} = \frac{C_{2211}}{C_{2222}} \dot{\varepsilon}_{11} + \frac{\dot{\sigma}_{22}}{C_{2222}}$$  (3.56)

To numerically implement the biaxial state of stress, the incremental values of $\dot{\varepsilon}_{22}$ and $\dot{\varepsilon}_{33}$ for a prescribed value of $\dot{\varepsilon}_{11}$ are first computed according to Equation 3.56 and Equation 3.55, respectively, to form the incremental strain vector. Afterwards, this strain vector is used as an input for the microplane model to obtain the associated stress increments. Curves are plotted in Figure 3.4(a), where a comparison is made between the results from the incremental formulation and those of Caner and Bažant [2000]. As with the previous cases, we again observe that both approaches lead to essentially the same results.

Figure 3.4(b) shows the results for the case of shear-compression states of stress. Here we observe that there is a region with slight differences in the two predictions, but that in this area the incremental scheme leads to predictions that appear to be in better agreement with the shear-compression test data. More details concerning this discrepancy are discussed in Section 3.5.
3.4.2.3 Uniaxial states of stress

In the case of a uniaxial test, all stress components except for \( \sigma_{11} \) are equal to zero. This case can be analyzed by setting \( \sigma_{22} = 0 \) in Equation 3.56. Thus,

\[
\varepsilon_{22} = -\frac{C_{2211}}{C_{2222}} \varepsilon_{11}
\]  \( 3.57 \)

Substituting for \( \varepsilon_{22} \) in Equation 3.54 leads to the following relation for the incremental modulus corresponding to a uniaxial state of stress:

\[
\ddot{C}_{1111} = \ddot{C}_{1111} - \frac{C_{2211}}{C_{2222}} \ddot{C}_{1122} \]

In Figures 3.5(a) to 3.5(d) comparisons are made between our numerical results and those presented by Caner and Bazant [2000] for the cases of uniaxial tension and compression. In Figures 3.5(a), 3.5(c), and 3.5(d), both formulations are seen to produce essentially the same predictions. However, some discrepancies occur in the predictions of the tensile volumetric strains (Figure 3.5(b)) in the regions when these exceed 0.001. Nevertheless, there is a reasonable fit with the experimental data. A plausible reason for these differences is that the value of the parameter \( c_{18} \) in our simulation and that of Caner and Bazant [2000] may not have been the same. These discrepancies may result from numerical aspects, as will be discussed in Section 3.5.

3.4.2.4 Comparisons with the arc-length method

As mentioned above, for all the cases considered the results obtained using the iterative arc-length method were identical to those obtained with the incremental formulation. However, the incremental approach has proven to be much more efficient from the computational viewpoint. Table 3.1 summarizes the required total number of iterations, at the same state of stress and strain, and for the equivalent degree of accuracy, with the two schemes for four selected cases: uniaxial compression, uniaxial tension, biaxial compression, and the shear-compression test. It can be seen that, in all cases, the number of required iterations to reach the same stress and strain state is significantly reduced when
3.4. NUMERICAL SCHEMES USING THE MICROPLANE MODEL

Figure 3.5: Uniaxial tensile and compressive states of stress
Table 3.1: Comparison of arc-length and incremental methods

<table>
<thead>
<tr>
<th>No.</th>
<th>State of stress</th>
<th>Total steps</th>
<th>Arc-length method</th>
<th>Incremental method</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Uniaxial compression</td>
<td>100</td>
<td>4,524</td>
<td>801</td>
</tr>
<tr>
<td>2</td>
<td>Uniaxial tension</td>
<td>50</td>
<td>1,923</td>
<td>452</td>
</tr>
<tr>
<td>3</td>
<td>Biaxial compression</td>
<td>500</td>
<td>16,325</td>
<td>3,245</td>
</tr>
<tr>
<td>4</td>
<td>Shear-compression</td>
<td>500</td>
<td>13,589</td>
<td>2,108</td>
</tr>
</tbody>
</table>

the incremental formulation is employed due to the use of tangent moduli rather than initial moduli as in the case for the arc-length technique. This suggests that our proposed incremental technique is computationally much more efficient than the iterative arc-length technique.

3.4.3 Iterative arc-length numerical scheme

The arc-length numerical scheme was proposed by Riks [1972, 1979] in order to trace nonlinear equilibrium paths, particularly for those cases having descending parts in their load-deflection relationships. This technique has been adapted here for the analysis of stress–strain histories based on the M4 version of the microplane model of Bažant et al. [2000]. The governing equation of the arc-length numerical scheme can be written as:

\[ g(\Delta \lambda, \Delta \varepsilon) = \sigma^*(\varepsilon) - \Delta \lambda \sigma \]  

(3.59)

where \( \sigma^* \) is the incremental stress vector calculated using the microplane model and \( \sigma \) is the reference incremental stress vector assumed at the beginning of each time step. In Equation 3.59, \( \Delta \lambda \) is a scalar representing a stress multiplier parameter, \( g \) is the out-of-balance stress vector, and \( \Delta \varepsilon \) is the incremental strain vector. The value of the stress multiplier parameter \( \Delta \lambda \) is unknown and is used to increase or decrease the value of the incremental stress vector \( \sigma \) at each iteration. In the current implementations, the unknowns in Equation 3.59 are the six components of the incremental stress vector resulting from the microplane model, \( \sigma^* \), as well as the stress multiplier parameter \( \Delta \lambda \).

A constraint equation governing the relation between \( \Delta \lambda \) and \( \Delta \varepsilon \) is used to solve for
3.4. NUMERICAL SCHEMES USING THE MICROPLANE MODEL

the seven unknowns; it takes the form [Crisfield, 2000]:

\[ a = \Delta S^2 - \Delta r^2 \] (3.60)

where

\[ \Delta S^2 = \Delta \varepsilon^T \Delta \varepsilon + \Delta \lambda^2 \sigma^T \sigma \] (3.61)

and \( \Delta r \) is a scalar representing a fixed radius of the constraint surface. The value of "a" in Equation 3.60 is within a prescribed tolerance approximating zero at the converged points. After some simplifications, Equation 3.59 and Equation 3.60 can be written as follows:

\[
\begin{bmatrix}
\delta \varepsilon \\
\delta \lambda
\end{bmatrix} = - \begin{bmatrix}
K & -\sigma \\
2\Delta \varepsilon^T & 2\Delta \lambda \sigma^T \sigma
\end{bmatrix}^{-1} \begin{bmatrix}
g_{old} \\
a_{old}
\end{bmatrix} \] (3.62)

where \( \delta \varepsilon \) is a vector representing the iterative change in the incremental strain vector \( \Delta \varepsilon \) and \( \delta \lambda \) is a scalar that is the iterative change in the stress multiplier parameter \( \Delta \lambda \). In Equation 3.62, \( K \) represents the initial material stiffness matrix; \( g_{old} \) and \( a_{old} \) denote the out-of-balance stress vector and arc-length, respectively; their values are computed using Equation 3.59 and Equation 3.60.

**Sequence of analysis**

The arc-length scheme has two aspects: the first being the predictor phase, while the second is the corrector phase. In the predictor phase, suitable starting values of \( \Delta \lambda \) and \( \Delta \varepsilon \) are calculated and used afterwards in computing the initial guess for the corrector phase. In the corrector phase, an iterative technique is employed to determine the incremental strain vector \( \Delta \varepsilon \), which is input into the microplane model to compute the incremental stress vector \( \sigma^* \) in order to match the prescribed stress vector.

The sequence of analysis, depicted in Figure 3.6, can be summarized as follows:

1. Assume initial values for the components of the prescribed incremental stress vector \( \sigma \). This vector remains constant for all iterations.

2. Calculate the initial value for the stress multiplier parameter \( \Delta \lambda \) (\( \Delta \lambda_1 \) in Figure 3.6) using Equation 3.60. In this equation \( \Delta r \) is assumed constant for all time steps and the components of the incremental \( \sigma \) and incremental \( \varepsilon \) vectors are used from the
last converged point (Point “o” in Figure 3.6). In the first time step, the value of $\Delta \lambda$ is taken equal to that of $\Delta r$.

3. Calculate the incremental strain vector $\Delta \varepsilon$ ($\Delta \varepsilon_1$ in Figure 3.6) using Hooke's law:

$$\Delta \varepsilon = K^{-1} \Delta \lambda \sigma$$  \hfill (3.63)

4. The incremental strain vector $\Delta \varepsilon$ is used afterwards as an input vector for the microplane model to calculate the corresponding incremental stress vector $\sigma^*$ (Point “$b_1$” in Figure 3.6). The total microscopic stress and strain components are stored from the last converged point and considered as the input for the microplane model.

5. Compute the out-of-balance stress vector $g$ (distance “$b_1c_1$” in Figure 3.6) using Equation 3.59.

6. Apply the convergence criterion to check the accuracy as follows:

$$\| g \| = \| \sigma^* - \lambda \sigma \| < \text{prescribed tolerance}$$  \hfill (3.64)
If the convergence criterion is not satisfied, the corrector phase is initiated by computing the iterative vector $\delta \varepsilon$ and the iterative scalar $\delta \lambda$ using Equation 3.62. Subsequently, $\delta \varepsilon$ and $\delta \lambda$ are used to update the incremental strain $\Delta \varepsilon$ and the stress multiplier parameter $\Delta \lambda$ for the new iteration as follows:

$$\Delta \varepsilon_{i+1} = \Delta \varepsilon_i + \delta \varepsilon$$  \hspace{1cm} (3.65)

$$\Delta \lambda_{i+1} = \Delta \lambda_i + \delta \lambda$$  \hspace{1cm} (3.66)

where the subscript "i" denotes the iteration number. The iterative technique (corrector phase, steps 4–6) continues until a new converged point that satisfies the required accuracy according to Equation 3.64 is reached.

The arc-length scheme is graphically represented in Figure 3.6. The thick solid curve represents the stress-strain relationship resulting from the microplane model that is being sought, while the dashed thick curve represents the constraint surface expressed by Equation 3.60. The iterative technique starts from the last converged point $[\varepsilon_0, \sigma_0]$ until we reach a new converged point $[(\varepsilon_0 + \Delta \varepsilon), (\sigma_0 + \Delta \lambda \sigma)]$. The values of $\Delta \lambda_1$ to $\Delta \lambda_3$ and $\Delta \varepsilon_1$ to $\Delta \varepsilon_3$ are computed using Equation 3.65 and Equation 3.66, where the subscript (1, 2, 3) represents the iteration number.

### 3.4.4 User-defined material subroutine for ADINA, Version 8.4

The microplane concrete model has been implemented as a user-supplied subroutine in ADINA to take advantage of the rich library of elements and the powerful numerical tools available in this commercial finite element package.

For each increment size (time $\Delta t$), the total strain increment $(\Delta \varepsilon)$ is divided into sub-increments and the microplane model is called, for each sub-increment, to update the sub-increment microscopic and macroscopic stress components. This procedure for computing the incremental stress $(\Delta \sigma)$ using the sub-increments stains, as can be seen in Figure 3.7, is recommended due to the high nonlinearities associated with the microplane model. The stress integration is performed by forward integration, where the stress increment value is updated at the end of each sub-increment.

At each integration point, one array is constructed to store 112 independent variables...
CHAPTER 3. MICROPLANE CONCRETE MODEL

Figure 3.7: Sub-increment technique utilized to compute the incremental stress

Figure 3.8: Required input data for the user-defined subroutine
including 28 microscopic values defining $\varepsilon_M$, $\varepsilon_L$, $\varepsilon_N$, and $\varepsilon_D$ to characterize the history of the microscopic concrete behaviour. These values are updated at the end of each sub-increment. The material properties consisting of six adjustable parameters and eighteen fixed constants are specified in an array set for each integration point. The six macroscopic stress and strain components are stored in two separate arrays. The necessary input variables to define the microstructure response of concrete are listed in Figure 3.8.

The microplane user-defined subroutine is called for all integration points to perform four types of operations during the various phases. At the beginning of the analysis the user-defined subroutine is used to initialize the variables stored at each integration point (macroscopic and microscopic stress and strain components), and is performed only once during the input phase. Usually the variables are initially set to zero. During the solution phase, the subroutine is called at each sub-increment to update the microscopic and macroscopic stress and strain components. After updating the various variables associated with each integration point, the subroutine is called to compute the tangent stiffness matrix. We have provided a special code inside the user-defined subroutine to print out the microscopic stress components (volumetric, deviatoric, normal, and shear microscopic stress components) in the output file.

In all the simulations presented in Figure 3.2, Figure 3.3 and Figure 3.4, the differences between the predicted results obtained with the incremental, arc-length numerical analyses, and user-defined subroutine in ADINA are indistinguishable. Other implementation examples using the user-defined subroutine in ADINA are now considered. The two applications in Figures 3.9(a) and 3.9(b) are the cases of triaxial compression under low and high confining pressures treated by Caner and Bažant [2000]. The case in Figure 3.9(c) is a uniaxial cyclic compression test previously simulated by Caner and Bažant [2000]. Our analyses are carried out using 8-node brick elements. An excellent match is achieved for the first three cases; the little discrepancy observed in Figure 3.9(c) might be ascribed to differences between the input loading paths in our simulation and that of Caner and Bažant [2000].

Figure 3.9(d) presents the predicted results for the lateral load–displacement relationship of a shear wall tested by Miao et al. [2006] having dimensions of 1000.0 mm in width, 2150.0 mm in height, and 100.0 mm in depth. Eight-node brick elements with a total of 288 elements were used to represent this shear wall, while 2-node truss elements were em-
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Figure 3.9: Numerical implementations using ADINA
3.5 Numerical aspects of the microplane model

The emphasis of this section is on numerical aspects of the microplane model in terms of increment size (increment strain value) and iteration tolerance (convergence factor) in order to address discrepancies such as those observed in Figures 3.4(b) and 3.5(b). Figures 3.10(a) to 3.10(d) depict the influence of the number of increments on the uniaxial compressive and tensile responses. The increment size is quite important for the compressive behaviour, in particular for the softening branch of the stress–strain curve. The effect of increment size on the compressive strength of the specimen of Figure 3.5(d) is shown in Figure 3.10(a). In general, the larger the increment size the higher the predicted strength. Thus the increment size affects the peak compressive strength value as well as the post peak softening part of uniaxial stress–strain relationship. This can perhaps be ascribed to the effect of increment size on the propagation of microcracks (simulated numerically by the redistribution of microscopic deviatoric stress components after cracking). The physical causes of increment size sensitivity are not completely understood; however, they could be related to microcrack initiation and propagation. The numerical simulations of the softening behaviour and failure processes using the microplane model indirectly described the initiation and propagation of microcracks, which leads to the localization of microscopic stress components on particular microplanes. A small increment size allows these microcracks to localize on more planes, after which fracture rapidly occurs. On the other hand, a large increment size prevents crack localization or reduces the number of localized cracks. This results in an increased strength at a higher increment size. For this example, the difference between the predicted compressive strength using a larger increment size (25 steps) and that using a small increment size (1000 steps) is around 7.5%.

In an attempt to understand the effect of increment size on the micromechanical characteristics, we have examined the effect of each equation in the microplane formulation on the sensitivity of the results to the increment size. We have found that removing the
CHAPTER 3. MICROPLANE CONCRETE MODEL

Figure 3.10: Effect of number of increments

condition corresponding to Equation 3.14 or Equation 3.24 eliminates the sensitivity of the results to the increment size value. Note that this condition was originally proposed in the work of Bazant et al. [2000] to redistribute volumetric and deviatoric stresses after microcrack initiation (i.e., to localize the deviatoric stress components on particular microplanes that contain microcracks). However, removing this condition over-predicts the results in terms of compressive strength. Generally, the increment size sensitivity arises from the dependency of the microcrack propagations and their localized microscopic stress values on the strain increment value. This phenomenon does not exist in phenomenological concrete models as these laws are not meant to consider microcrack propagation. It should be recognized that since this phenomenon is well supported experimentally, it is an advantage of the microplane model that it is able to capture this behaviour.
Figure 3.10(b) represents the stress–volumetric strain relationships for several increment sizes for the case considered in Figure 3.4(b). The solid thick line in Figure 3.10(b) (strain increment of 0.00005) exactly matches the simulation of Caner and Bažant [2000] in Figure 3.4(b), and the dashed thick line (strain increment of 0.0002) is our simulation presented in Figure 3.4(b). This explains the discrepancies observed in Figure 3.4(b) between our prediction and that of Caner and Bažant [2000].

In the case of the uniaxial tensile test, the concrete exhibits a critical increment size or strain increment threshold above which significant increases in strength are exhibited, as depicted in Figures 3.10(c) and 3.10(d). The same conclusion regarding the sensitivity of predicted uniaxial tensile strengths on the strain increment value has been reported in the work of Nemecek et al. [2002]. In general, the effect of increment size on the predicted uniaxial tensile stress–strain relationship is not significant (around 5% over the experimental value).

With regard to the effect of the iteration tolerance employed in the arc-length numerical technique to converge to an equilibrium path, Table 3.2 shows the effect of iteration tolerance on the accuracy of the predicted compressive and tensile strengths. The iteration tolerance has an almost negligible influence on the compressive and tensile stress–strain relationships. The cumulative errors (summation of the square roots of the difference between the results of using two iteration tolerance values) between predictions using an iteration tolerance value of \(1.0^{-10}\) and that of a value of \(1.0^{-3}\) are 0.21\(^{-6}\) and 0.37\(^{-7}\) for the compressive and tensile tests, respectively. It should be mentioned that the comparisons in Table 3.1 are carried out for an iteration tolerance value of \(1.0^{-5}\). Despite the fact that using a low iteration tolerance of the order of \(1.0^{-3}\) reduces the computational demand of the arc-length technique without affecting the accuracy, the incremental technique is still more efficient.

3.6 Conclusion

An incremental formulation of the M4 version of the microplane model of Bažant et al. [2000] has been developed. With this formulation, a rate form of the constitutive law relating the macroscopic stress increments and strain increments has been proposed in
which explicit expressions are obtained for the tensor of incremental moduli. The proposed incremental approach has been applied to the various cases previously analyzed by Caner and Bažant [2000]. In general, the incremental scheme was seen to lead to essentially the same predictions as those reported by Caner and Bažant [2000]. The well-known iterative arc-length method was also implemented for all the stress and strain histories considered. This numerical technique led to predictions identical to those obtained with the incremental formulation. However, the incremental approach proved to be significantly more efficient in terms of computational performance as a result of using incremental tangent moduli instead of the initial moduli of the arc-length technique.

The M4 version of the microplane model was implemented in a commercial finite element package as a user-defined subroutine. In all the simulations presented in this chapter the differences between the predicted results obtained with the incremental formulations, arc-length numerical technique, and user-defined subroutine are indistinguishable.

The softening branch of the uniaxial compressive stress-strain curve of concrete and the corresponding uniaxial compressive strength value predicted using the microplane model were found to be dependent on the increment size. However, the change in the predicted uniaxial compressive strength value with increment size was less than 10%. The increment size sensitivity might be ascribed to the localization of microscopic stress components on particular microplanes; a smaller increment size allows microcracks to localize on more planes.

The finite element package with the microplane concrete model will be employed in the next chapter to carry out a nonlinear micromechanics-based finite element analysis for the case of FRP/concrete joints subject to direct shear loadings.
Chapter 4

Nonlinear Micromechanics-Based Finite Element Analysis of FRP/Concrete Joints

In the literature, various bond-slip ($\tau-s$) models have been proposed to simulate the experimentally observed shear-slip relationships of FRP/concrete joints, as those summarized in Table 2.1. However, the analysis of these models assumes a unified bond-slip relation along the bonded length, which contradicts the experimental observations (Figure 2.5(b)). The micromechanics-based finite element results, as we shall see in this chapter, show that the bond-slip relationships and the associated bond strength values vary along the bonded length. Furthermore, the slope of the descending part of these relationships as well the residual stresses; i.e., the stresses at high slip values, change according to the location from the loaded end. This peculiarity of the $\tau-s$ profiles has been reported in several experimental investigations in the case of direct shear tests [Sato et al., 2000; Nakaba et al., 2001] and has not been considered yet in any of the available mathematical models. Note that, throughout this chapter we interchangeably use the terms “bond strength”, “maximum shear stress” and “$r_{\text{max}}$” to refer to the maximum local interfacial shear stress of the bond-slip relation, as depicted in Figure 2.6.

In this chapter, 3-D nonlinear micromechanics-based finite element analyses are carried out to investigate the interfacial behaviour of the FRP/concrete joints subject to direct shear loadings. The current study is intended to give a persuading explanation for differ-
ences in the shape of the $\tau$–$s$ relationships. Subsequently, the finite element results are used to develop a nonlinear bond–slip model for the FRP/concrete joints. This model is developed considering the interaction between the interfacial normal stress components $\sigma_{ij}$ along the bonded length in the value of $\tau_{\text{max}}$; i.e., $\tau_{\text{max}} = g(\sigma_{ij}, f_t)$, where $f_t$ is the concrete tensile strength. Accordingly, the bond strength varies along the bonded length according to the state of stress. Then a new mathematical approach is proposed to describe the entire $\tau$–$s$ relationship based on three separate models. The first model captures the shear response of the orthotropic FRP laminate. The second model simulates the shear characteristics of the adhesive layer. The third model represents the shear nonlinearity of a thin layer inside the concrete, referred to as an “interfacial layer”. The proposed bond–slip model reflects the geometrical and material characteristics of the FRP, concrete, and adhesive layers.

In the FE simulations, the element sizes of the FRP, adhesive, and concrete beneath the FRP laminates are chosen very small; 0.5 mm cube for elements in the top 30 mm and 2.0–5.0 mm cube for the other elements, so that the debonding behaviour can be properly captured. The FE simulations are carried out using 4-node 3-D tetrahedral elements with the average number of elements being around 250,000. The FE calculations with these large numbers of discrete elements and associated microstructure parameters have been carried out on the most powerful supercomputer in Canada, the “Mammoth” parallel supercomputer at the Université de Sherbrooke.

The finite element results are compared to those of the experiments of 40 direct shear tests categorized in three series, and they show very good agreement. To assess the efficiency of the proposed bond–slip model, comparisons are carried out with a large experimental database (results of 118 specimens) in terms of the ultimate load carrying capacities and show a satisfactory agreement. Furthermore, comparisons are made between the characteristics and predictions of the proposed model and those of the Lu et al. [2005b] and Dai et al. [2005] bond–slip models.
4.1 Introduction

To produce accurate finite element and analytical models for FRP-strengthened concrete structures, appropriate FRP/concrete interface bond–slip relationships are required. Such relations are indispensable for a better understanding of the debonding failure mechanisms of FRP-strengthened concrete structures. In the literature, the local bond–slip relationships are usually determined experimentally using two different approaches. The first involves computing the \( \tau-s \) relationship using the measured strain values along the bonded FRP laminates [Nakaba et al., 2001]. As a result, many strain readings along the bonded length are necessary to achieve a satisfactory accuracy using this approach (e.g., Equation 2.1 and Equation 2.2 in Chapter 2). In the second approach, the \( \tau-s \) relationship is described using the first derivative of the FRP laminate axial strain function, \( f_p(s) \), with respect to the interfacial slip, \( s \), as follows [Dai et al., 2005]:

\[
\tau(x) = E_p f_p \frac{df_p(s)}{ds} f_p(s) \quad (4.1)
\]

In this second approach, the accuracy of the extracted \( \tau-s \) relationships relies on the accuracy of the derived FRP-axial strain function.

Of particular interest is the fact that the experimentally determined local bond–slip relationships are not identical along the bonded length, as reported in the literature and depicted in Figure 2.5(b). The bond strength decreases with an increase of the distance from the loaded end. This peculiarity of the \( \tau-s \) relationships has been reported in several experimental investigations in the case of direct shear tests [Sato et al., 2000; Nakaba et al., 2001]. To date, no persuading explanation has been given for such differences in the shape of these relationships or in the value of the bond strength (2.6 to 17.3 MPa in Figure 2.5(b)). Several researchers ascribed this either to the microstructural composition of the concrete, including the distribution of the fine and coarse aggregates along the concrete surface, or to the local bending contribution of the bonded FRP laminate [Dai et al., 2005; Lu et al., 2005b]. Since the \( \tau-s \) relationships are not the same along the interface, it is not appropriate to simply pick one of these relationships to represent the entire bond–slip behaviour for the bonded FRP laminate. We shall see from the results of this chapter that the FRP/concrete joint subjected to direct shear is not the case of a pure shear test and that a failure criterion for the interface defining the relation between \( \tau_{\text{max}} \) and the state of stress along the bonded length has to be first defined before deriving...
the bond-slip model.

In the literature, various mathematical models have been developed to simulate the experimentally observed bond-slip relationships. However these models assume a unified bond-slip relation along the bonded length, which contradicts the experimental observations. These mathematical models have been developed based on regression analyses of the experimental data [Nakaba et al., 2001; Dai et al., 2005], fracture mechanics theories [Sato et al., 2000; Yin and Wu, 1999; Chen and Teng, 2001; Niu and Wu, 2003], and finite element simulations [Lu et al., 2005b]. Using the fracture mechanics theories usually restricts the derived $r-s$ relations to a limited number of parameters of the interface and to a linear or bi-linear function. Defining the bond-slip relation based on finite element simulations requires using very small elements to trace the interfacial crack propagation; therefore comprehensive 3-D analyses are scarce. In the present study, the proposed bond-slip model will be developed based on the 3-D finite element results with a refined mesh.

As far as FE analysis is concerned most studies have simulated the FRP/concrete joints using plane stress elements disregarding the influence of the out-of-plane stress components on the behaviour of the bond-slip relationship. These FE simulations have been restricted to the plane stress assumption to avoid the high computational demands of the 3-D analyses. The so called “meso-scale finite element model” was introduced to describe the interfacial behaviour of only the top 45 mm of the FRP/concrete joints using plane stress analysis [Lu et al., 2005a]. In addition, all finite element simulations have disregarded the influence of the interfacial normal stress components on the shear stress behaviour as a result of employing a smeared crack model for the cracked concrete. In the smeared crack model, the shear response of the cracked concrete is simulated using an empirical shear retention factor based on the tensile principal strain value regardless of the interaction of the other two principal strain values.

### 4.2 Objectives

This study aims to derive an accurate and realistic bond-slip model for FRP/concrete joints subjected to direct shear loadings by introducing a failure criterion for the interface.
This failure criterion defines the interaction between $\tau_{\text{max}}$ and the interfacial normal stress components along the bonded length in the form $\tau_{\text{max}} = g(\sigma_{ij}, f_i)$. Accordingly, the value of $\tau_{\text{max}}$ is not a constant along the bonded length and varies according to state of stress. Then in order to introduce a more convenient expression for the bond strength, an equivalent form of $\tau_{\text{max}}$ will be developed as a function of the geometrical and material parameters of the interface. A unified function that best fits with the finite element results will be proposed to define the bond–slip curve in terms of the geometrical and material characteristics of the interface. The value of $\tau_{\text{max}}$ can be computed at any location along the bonded length, and then it will be used with the proposed function to build up the entire bond–slip model at that location. We assume that the bond–slip curves have the same overall shape along the bonded length, however they have different values of $\tau_{\text{max}}$.

In addition, our numerical study is intended to overcome the deficiencies of the previous finite element simulations of FRP/concrete joints and to incorporate a refined model employing a micromechanics-based concrete theory. Furthermore, this is the first attempt, to date, to address the ability of the microplane concrete model to represent the interfacial shear behaviour of FRP/concrete joints. In this chapter, we present 3-D finite element analyses for FRP/concrete joints for the complete concrete block instead of only the top interfacial layer.

This chapter is organized as follows. First, the micromechanics-based finite element models are introduced. This involves a detailed description of the specimens that are analyzed, the structural and material modelling, computational aspects, and the finite element results. We then introduce our proposed expression for the bond strength $\tau_{\text{max}}$. Finally the bond–slip model is introduced. This involves a comparison between results of the proposed model with the available experimental data and with the Lu et al. [2005b] and Dai et al. [2005] models.
4.3 Finite element analysis

4.3.1 Description of the specimens analyzed

Experimental results of 40 specimens, carried out by different researchers, are used to validate the finite element analyses. The database is versatile in terms of geometrical characteristics and material properties of the concrete, adhesive layer, and FRP composites. Table 4.1 lists the dimensions and different parameters of the specimens, which correspond to those tested by Chajes et al. [1996], Bizindavyi and Neale [2001], and Dai et al. [2005]. The identifications of the specimens (IDs) listed in Column 2 of Table 4.1 correspond to the notations employed in the original references, with the exception of the specimens tested by Chajes et al. [1996]. In our analysis, we refer to the Chajes et al. [1996] specimens as S1 through S4, as they do not have specific notations in the original reference.

The 40 specimens have been selected from the large database based on the fact that they have clear and accurate reported data for the materials used. These studies have followed strict procedures to verify the accuracy of the reported material properties. In the Dai et al. [2005] study, the thicknesses of the adhesive layers were measured using a microscope with six-order digit accuracy. In the Chajes et al. [1996] work, biaxial tests were carried out to measure the five FRP orthotropic parameters.

Four specimens constitute the specimen set of Chajes et al. [1996], which have different bonded lengths of FRP laminates ranging from 50.8 mm to 203.2 mm. The specimen set tested by Bizindavyi and Neale [2001] involved 10 specimens investigating the influence of thicknesses and bonded lengths of the FRPs in conjunction with two types of FRP, GFRP and CFRP. In this set of specimens, the thicknesses of the FRP laminates vary from 0.33 mm to 2.0 mm with the bonded lengths varying from 50.0 mm to 180.0 mm. The last specimens are 26 FRP/concrete joints with a relatively long bonded length (330.0 mm) [Dai et al., 2005]. Three types of adhesive and FRP laminate (AFRP, GFRP and CFRP) are considered in the Dai et al. [2005] study. The Young's moduli of the FRPs in these specimens (Column 6 in Table 4.1) have been calculated based on the reported values (in the original reference) for the axial stiffness, $E_p t_p$, and thickness, $t_p$, of the FRP laminates. The geometrical parameters of the three sets of specimens analyzed are
### 4.3. Finite Element Analysis

#### Table 4.1: Characteristics of the specimens analyzed

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### Table 4.1: Specimen Set

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<td>3.5</td>
<td>400</td>
</tr>
<tr>
<td>CR3L2</td>
<td>0.67</td>
<td>0.5</td>
<td>25.0</td>
<td>3550</td>
<td>0.67</td>
<td>0.45</td>
<td>11.8</td>
<td>35</td>
<td>3.5</td>
<td>400</td>
</tr>
<tr>
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<td>0.67</td>
<td>0.5</td>
<td>25.0</td>
<td>3550</td>
<td>0.67</td>
<td>0.45</td>
<td>11.8</td>
<td>35</td>
<td>3.5</td>
<td>400</td>
</tr>
<tr>
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<td>0.5</td>
<td>25.0</td>
<td>3550</td>
<td>0.67</td>
<td>0.45</td>
<td>11.8</td>
<td>35</td>
<td>3.5</td>
<td>400</td>
</tr>
</tbody>
</table>

The values of Poisson's ratio are assumed similar to that reported in the literature.

1. $t_p$, $L$, $E_p$, $f_{pm}$, and $f_c$ are the thickness, width, Young's modulus, and ultimate strength of FRP laminates.

2. $f_c$ is the ultimate compressive strength of concrete block.

3. $h_c$ is the ultimate strength of concrete block.
4.3. FINITE ELEMENT ANALYSIS

FRP laminates \( (t_x b_x L) \)

(a) Bizindavyi and Neale [2001]

(b) Chajes et al. [1996]

(c) Dai et al. [2005]

Figure 4.1: Geometrical characteristics of specimens analyzed

depicted in Figure 4.1.

4.3.2 Structural modelling

Although plane stress analyses have generally been employed in finite element simulations of FRP/concrete joints subjected to direct shear, 3-D analyses are more accurate, especially for considering the influence of the width ratio of the FRP laminates \( (b_p/b_c) \). In fact, the experimental tests on FRP/concrete joints have shown 3-D modes of failure in the concrete block. In addition, several studies have modelled only the top 45.0 mm of the concrete block based on the assumption that only a zone of a few millimeters inside the concrete adjacent to the interface contributes to the overall interfacial behaviour. Although this assumption is acceptable for analyzing the global behaviour, it is not suitable when the local behaviour, such as the bond–slip response, is being investigated. Accordingly, in the present study the complete concrete block, adhesive layer, and FRP laminates are modelled.

Four-node tetrahedral 3-D elements with three translational degrees of freedom per
node are used to represent the concrete, FRP laminate and adhesive layer (Figure 4.2(c)). The element sizes of the concrete prism in the top 30.0 mm beneath the FRP laminates (Zone “I”) are taken as 0.5 mm cube. For the concrete elements adjacent to the FRP in the top 30.0 mm (Zone “II”), the element sizes are increased to 2.0 mm at a distance equal to half of the FRP laminate width. For the remainder of the concrete elements, the element sizes are taken as 5 mm cube. Figure 4.2(a) shows the finite element model for the specimens BN5 tested by Bizindavyi and Neale [2001] as an example. Figure 4.2(b) details the finite element mesh for the top 30 mm.

Two elements are taken through the depth of the adhesive layer. For the FRP laminates, the element size is taken as 0.5 mm cube. For the cases when the thickness of the FRP is less than 0.5 mm, the element size is taken as \( t_p \times 0.5 \times 0.5 \) mm. The average numbers of elements used in the present analysis are about 34,000, 280,000, and 136,000 elements for the specimens tested by Chajes et al. [1996], Bizindavyi and Neale [2001], and Dai et al. [2005], respectively. By using these small element sizes, we expect to accurately trace the propagation of the debonding cracks.

In order to capture the softening branch of the load–displacement profile and the post-debonding behaviour, a displacement-controlled technique is used in the finite element analysis. The applied displacement increment is taken as 0.02 mm with a total of 100 steps. In the simulations, only one-half of each direct shear specimen is modelled due to
4.3. FINITE ELEMENT ANALYSIS

the geometrical and loading symmetries. The nodes on the bottom surface of the concrete prism are prevented from vertical movement (z-direction in Figure 4.2). Across the plane of symmetry, the finite element nodes are prevented from movement in the perpendicular direction (y-direction in Figure 4.2). The translational degrees of freedom of nodes on the surface “a” are restrained in the direction of the applied load (x-direction in Figure 4.2).

4.3.3 Material modelling

4.3.3.1 Constitutive models for the concrete

As discussed in Chapter 3, the microplane concrete model has been implemented as a user-defined subroutine in the finite element package ADINA, Version, 8.4. The values of the non-dimensional material constants (their values are constant for all concrete batches) defining the microplane model (c1 to c18) are taken from the original work of Caner and Băzant [2000]; their values are as follows:

\[
c_1 = 0.62; c_2 = 2.76; c_3 = 4.00; c_4 = 70.00; c_5 = 2.50; c_6 = 1.30; \\
c_7 = 50.00; c_8 = 8.00; c_9 = 1.30; c_{10} = 0.73; c_{11} = 0.2; c_{12} = 7,000.00; \\
c_{13} = 0.20; c_{13} = 0.50; c_{15} = 0.02; c_{16} = 0.01; c_{17} = 0.40; c_{18} = 0.12
\] (4.2)

The non-dimensional material adjustable parameters (k1 to k4) as well as the macroscopic Young’s modulus, \(E_c\), and Poisson’s ratio, \(\nu\), are specified individually for each concrete batch. Since the confining pressure is not high enough in our simulations to cause pore collapse in the concrete, the values of the parameters \(k_2\) (defining the triaxial compressive behaviour under high confining state of stress), \(k_3\) and \(k_4\) (describing the behaviour of the concrete under hydrostatic compression state of stress) are taken constant for all specimens as:

\[
k_2 = 110.0, \quad k_3 = 12.0, \quad k_4 = 38.0
\] (4.3)

Poisson’s ratio \(\nu\) is taken as 0.18 in all the simulations. The value of the parameter \(k_1\) is computed for each individual concrete batch as:

\[
k_1 = 2.45 \times 10^{-4} f'_c \frac{E_c}{4625}
\] (4.4)

The values of \(E_c\) and \(f'_c\) in Equation 4.4 are in GPa and MPa, respectively. In cases where the concrete tensile strength \(f_t\) and the initial modulus of elasticity \(E_c\) are not
experimentally reported, they are evaluated according to the CSA-A23.3 [2004] equations (Equation 2.3 and Equation 2.4 in Chapter 2).

The fixed constants, $c_1$ to $c_{18}$ in Equation 4.2, lead to the following ratios controlling the macroscopic mechanical characteristics of the concrete:

$$
\frac{f_t}{f_c} = 0.068; \quad \frac{f_{bc}}{f_c} = 1.135; \quad \frac{\tau_c}{f_c} = 0.068
$$

where $f_c$, $f_t$, $f_{bc}$ and $\tau_c$ are the uniaxial compressive strength, uniaxial tensile strength, biaxial compressive strength, and pure shear strength of the concrete. Although Equation 4.5 leads to a value of $f_t$ within the ranges of the experimental results, in all the simulations, the concrete tensile strength is taken from the original published experimental studies or calculated from Equation 2.3 and Equation 2.4. Accordingly, to match the experimental value of $f_t$, the value of the constant $c_1$ that controls the uniaxial tensile strength is altered. We find that the value of $c_1$ is linearly proportional to the value of $f_t$, however, it does not affect the value of the uniaxial compressive strength $f_c$.

### 4.3.3.2 Constitutive models for the adhesive

Two approaches have been employed to simulate the mechanical characteristics of the adhesive layer. The first technique assumes uses an isotropic linear elastic material. The second approach considers the ultimate tensile strength of the adhesive layer as a failure criterion; in other words, the adhesive layer is no longer able to transfer stresses when the tensile stresses in the adhesive layer exceed the admissible ultimate tensile strength. No discrepancies have been observed between the two approaches in terms of the predicted ultimate load capacities. Generally, the debonding or rupture of the FRP laminate takes place at low stress levels in the adhesive layer, and this requires no further consideration of the failure criterion of the adhesive layer in our simulations. One exception is that of the eight specimens tested by Dai et al. [2005]. Here the adhesive layer is modelled as an elastic-perfectly plastic material since the yield strength of the adhesive layer in these particular specimens is relatively low (11.8 and 15.9 MPa). These specimens are CR2L1 to CR2L3, CR3L1 to CR3L3, AR2L3, AR3L3, GR2L3, and GR3L3. The mechanical characteristics of the adhesive layer are summarized in Table 4.1.
4.3.3 Constitutive models for the FRPs

FRP materials used in FRP-strengthened concrete structures, in general, are orthotropic linear elastic materials. The stress-strain relation for orthotropic materials is given by:

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{31} \\
\sigma_{12}
\end{bmatrix} = \frac{1}{\Delta} \times \begin{bmatrix}
\frac{1 - \nu_{23}\nu_{32}}{E_{22}E_{33}} & \nu_{21} + \nu_{31}\nu_{23} & \nu_{31} + \nu_{21}\nu_{32} & 0 & 0 & 0 \\
\frac{E_{22}E_{33}}{1 - \nu_{21}\nu_{32}} & \frac{E_{22}E_{33}}{1 - \nu_{21}\nu_{32}} & \frac{E_{22}E_{33}}{1 - \nu_{21}\nu_{32}} & 0 & 0 & 0 \\
\frac{E_{33}E_{11}}{1 - \nu_{21}\nu_{32}} & \frac{E_{33}E_{11}}{1 - \nu_{21}\nu_{32}} & \frac{E_{33}E_{11}}{1 - \nu_{21}\nu_{32}} & 0 & 0 & 0 \\
1 - \nu_{21}\nu_{32} & 1 - \nu_{21}\nu_{32} & 1 - \nu_{21}\nu_{32} & 2G_{23} & 0 & 0 \\
\nu_{31} & \nu_{31} & \nu_{31} & 2G_{31} & 0 & 0 \\
\nu_{23} & \nu_{23} & \nu_{23} & 2G_{12} & 0 & 0
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{23} \\
\varepsilon_{31} \\
\varepsilon_{12}
\end{bmatrix}
\]

(4.6)

Here \(\Delta = \frac{1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{12}\nu_{23}\nu_{31}}{E_{11}E_{22}E_{33}}\), and \(E_{11}, E_{22}, E_{33}, G_{12}, G_{13}, G_{23}, \nu_{12}, \nu_{13}\) and \(\nu_{23}\) are the nine independent material constants, where \(E, G,\) and \(\nu\) denote Young’s modulus, the shear modulus and Poisson’s ratio, respectively. The subscripts 1, 2, and 3 refer to the three orthogonal directions (local directions of the lamina). In the case of a unidirectional oriented FRP composite (the case of our simulations), the material becomes transversely isotropic, where five independent material constants are sufficient to construct the elastic stress–strain relationship. If the fibres are in the 1-2 plane, the material in the 2-3 directions is treated as an isotropic material; i.e., \(E_{22} = E_{33}, \nu_{12} = \nu_{13}\) and \(G_{12} = G_{13}\), and \(G_{23}\) is computed as a function of \(E_{22}\) and \(\nu_{12}\) as:

\[
G_{23} = \frac{E_{22}}{2(1 + \nu_{12})}
\]

(4.7)

Due to the orthotropic nature of composite plates, shear strains can arise from tensile stresses and vice versa. This interaction between normal and shear stress components known as tensile-shear interactions [Hahn and Tsai, 1972]. As far as the failure criterion is concerned, the maximum stress criterion is employed where failure occurs if any stress component reaches a value equal to its ultimate strength. More details regarding the constitutive relation for orthotropic FRPs can be found in Labossiere and Neale [1989] and Jones [1975].

For the specimens tested by Bizindavyi and Neale [2001] and those tested by Dai et al. [2005], \(E_{22}\) is assumed to be one-tenth of that in the direction of the fibres, \(E_{11}\).
Throughout this chapter, we will refer to "$E_{11}$" as "$E_f$". This assumption has been adopted since the data from the literature do not clearly address the exact values of the transverse elastic moduli. For the Chajes et al. [1996] specimens, the value of $E_{22}$ is taken as 11.58 GPa. Poisson's ratio in the fibre direction, $v_{11}$, is assumed as 0.268 and in the other two perpendicular directions, $v_{12}$ and $v_{13}$, as 0.025. These values are reported in the experimental work of Chajes et al. [1996] and have been assumed for the other specimens.

4.3.4 Computational aspects

The finite element calculations lead to significant demands in terms of both computational power and storage, due to the necessity of tracking an enormous number of microscopic variables at each integration point. The limited computational capacities of personal computers are generally unable to supply the computer power required for such applications. However, with supercomputers and parallelization techniques, such demands are easily met. In our simulations, the "Mammoth" parallel supercomputer is used to run the finite element calculations. It is a Dell Power Edge SC1425 consisting of 576 nodes with a maximum speed of 4147.2 GHz. Each node consists of two processors of 3.6 GHz with a total RAM of 8.0 GB. This supercomputer is designed to solve 650,000 equations per second when running at full capacity. The Mammoth supercomputer is shown in Figure 4.3.

Figure 4.3: Mammoth supercomputer, 576-node Dell 1425SC (1152 CPU Intel Xeon, 3.6 GHz) 8 GB RAM/node, network Infiniband 4X

In this study, the parallel computations are performed by concurrently executing differ-
4.3. Finite Element Analysis

Current problems on a different number of processors, while the overall finite element problem is submitted to one processor. This type of parallelization is referred to as a "multi-input, multi-output technique". The total calculation time for the 40 specimens was about 5804 hrs (around 242 days) with an average solution time for each problem ranging from 32 to 48 hrs. Because of the synchronous execution, the entire computer solution time (5804 hrs) was done in a period of about two weeks.

4.3.5 Finite element results

The following sections include a validation of the numerical model through comparisons between predicted and experimental results in terms of the ultimate load carrying capacities and strain profiles in the FRP laminates. Subsequently, the local bond-slip relationships are presented based on the results of the finite element analyses.

4.3.5.1 Validation of the finite element results

The ultimate load carrying capacities and normal strain distributions along the bonded FRP laminates are used here to validate the finite element model and, accordingly, to give credibility to the other finite element results.

1- Ultimate loads:

The comparisons between the finite element predictions and experimental results for all 40 specimens, in terms of the ultimate load carrying capacities, are summarized in Table 4.2 and depicted in Figure 4.4. In this table, the ratio of the numerical-to-experimental load capacity is given for each specimen.

As seen from Table 4.2 and Figure 4.4, there is a very good agreement between the numerically predicted load capacities and experimental results for all the test specimens. The average numerical-to-experimental load ratio is 0.98 with a corresponding standard deviation of 0.095, which indicates an excellent predictive capability of the finite element model. The scatter in the predicted results (9.5%) is within the admissible range for the direct shear test. For example, the specimen CR1L1 has two experimental values
Table 4.2: Comparison between the numerical and experimental results

<table>
<thead>
<tr>
<th>Specimen set</th>
<th>Specimen ID</th>
<th>Load (kN)</th>
<th>Ratio Num./Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Exp.</td>
<td>Num.</td>
</tr>
<tr>
<td>Bizindavyi and Nesle [1999]</td>
<td>BN2</td>
<td>9.67</td>
<td>8.84</td>
</tr>
<tr>
<td></td>
<td>BN5</td>
<td>10.42</td>
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</tr>
<tr>
<td></td>
<td>BN6</td>
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</tr>
<tr>
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<td></td>
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<td>BN27</td>
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</tr>
<tr>
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<td>BN28</td>
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</tr>
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<td>Chajes et al. [1996]</td>
<td>S1</td>
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<tr>
<td></td>
<td>S2</td>
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<tr>
<td></td>
<td>S3</td>
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<tr>
<td></td>
<td>S4</td>
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<tr>
<td></td>
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</tr>
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<td>GR3L3</td>
<td>33.4</td>
<td>37.43</td>
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</tbody>
</table>
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Figure 4.4: Comparison between the numerical and experimental load capacities of the ultimate carrying capacities (23.1 kN, and 24.9 kN) with a scatter of 8%. For the specimens BN22, BN24, and BN25 tested by Bizindavyi and Neale [2001], where the ultimate capacity of the FRP sheet is 8.4 kN ($f_{pu} \times b_y \times t_y = 1014 \times 25.4 \times 0.33 = 8.4kN$), the experimental load values of these three specimens are 8.73 kN, 8.90 kN, and 10.42 kN, respectively. This implies a significant scatter in the material characteristics of the FRP laminates (at least 25%), which could possibly explain why the finite element predictions are greater than the experimental results (i.e., values above the 45° line in Figure 4.4). These predictions cannot necessarily be considered as unsafe in that the values do not exceed the experimental values by more than 10% (i.e., less than the normal average scatter for results of the FRP/concrete joint tests).

2- Strain distributions along the FRP laminates:

The experimental results of the Chajes et al. [1996] specimens are chosen to validate the results of the finite element analysis in terms of the strain distributions along the bonded FRP laminates. The comparisons show a satisfactory agreement between the numerical predictions and experimental results in terms of the FRP strain distributions, as shown in Figures 4.5(a) to 4.5(d). In Figure 4.5, the strain in the FRP plates is measured from the loaded end, where the zero point indicates a starting point of the FRP bonded to the concrete. The load levels in this figure vary up to the capacities of the joints. As can be observed in Figure 4.5, for a given load level the strains in the FRP decrease progressively along the length of the bonded area. This indicates that the load is being transferred from the FRP laminate to the concrete.
Figure 4.5: FRP strain distributions: Numerical vs. experimental [Chajes et al., 1996]
There is a good match between the numerical predictions and experimental results in terms of the strain distributions at load levels less than 80% of the ultimate capacity. Since the finite element analysis is based on a displacement-controlled technique rather than a load-controlled procedure, an interpolation is required to find the strain profiles at load levels that match the exact experimental load level. This could explain the slight discrepancy between the numerical predictions and experimental results for load levels around the debonding failure load, in particular for Specimen S1 (Figure 4.5(a) at the load level of 8.1 kN). Moreover, it is difficult experimentally, around the debonding loads, to obtain accurate measurements of the strains in the CFRP sheets at specific load levels because of debonding. At the debonding load, a small variation in load causes a significant change in the corresponding slip value. As a result, it is not possible to find exactly the same load level from the finite element analysis that matches the experimental one. The fluctuation in the predicted strain profiles (in particular, in Figure 4.5(c) at the load level of 11.9 kN) arises from the interfacial cracks along the bonded FRP length.

4.3.5.2 Local bond-slip relationships

In this section, the $\tau-s$ curves based on the previous finite element simulations are presented. These curves for the specimen CR1L1 are depicted in Figure 4.6. In Figure 4.6(a), the $\tau-s$ profiles are given at five different locations along the adhesive/concrete interface. The bond strength in this figure has a value of 22.3 MPa, 7.4 MPa, 8.9 MPa, 6.3 MPa and 4.4 MPa at the loaded end and, respectively, at 2.0 mm, 4.0 mm, 6.0 mm and 165.0 mm (the middle point) from the loaded end. The bond-slip relationships in Figure 4.6(a) are characterized by a residual stress with an average value around 10–15% of the bond strength. We observe that the slope of these relationships is almost independent of the distance from the loaded end. The maximum shear stress $\tau_{\text{max}}$ occurs at high values of slip (Figure 4.6(a)). In Figure 4.6(b), the $\tau-s$ relationships are presented for various points through the depth of the concrete at the loaded end. The maximum value of the bond strength occurs at the adhesive/concrete interface, with a value of 22.3 MPa. From Figure 4.6, one can observe that the bond-slip curves have the same overall shapes; however they have different values of the bond strength $\tau_{\text{max}}$.

Although the input value of the pure shear strength of the concrete in the finite element
calculations has been set at 2.4 MPa, the output values of the maximum shear stress \( \tau_{\text{max}} \) varies from a value of 22.3 MPa to a value of 4.4 MPa. Thus, the discrepancy between the output bond strength values and the input pure shear strength indicates that the FRP/concrete joints subject to direct shear loading do not represent the case of a pure shear test. This can be explained by considering the equilibrium equations of the discrete element, \( dxdy \), shown in Figure 4.7 as follows:

\[
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \tag{4.8}
\]

\[
\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} = 0 \tag{4.9}
\]

As a result of the shear stress variation (\( \tau_{xy} \) in the above two equations) through the depth of the concrete or along the bonded FRP laminate, normal stress components (\( \sigma_x \) and \( \sigma_y \)) necessarily develop inside the concrete to satisfy equilibrium. These normal stresses cause the bond strength \( \tau_{\text{max}} \) to vary with the location from the bonded end. The above results show that the interaction of the various components of the stress along the bonded joint must be considered and that an adequate failure criterion for the FRP/concrete interface must be introduced before the appropriate bond-slip model can be accurately derived. The interaction between the normal and shear stress components for failure in concrete has been addressed previously in several experimental studies [Bresler
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and Pister, 1991; Moosavi and Bawden, 2003] and has led to the development of various failure criteria (e.g., Mohr-Coulomb and Drucker-Prager).

In summary, the above micromechanics-based finite element simulations show that the bond–slip profiles have the same overall shapes and the associated bond strength values vary along the bonded length and through the concrete depth. Moreover, an FRP/concrete joint subject to direct shear is not identical to a pure shear test. Consequently, it is not possible to simply select one $\tau$–$s$ relationship to represent the global behaviour of the FRP/concrete joint. To define a refined bond–slip model, a relation between $\tau_{\text{max}}$ and normal stresses at failure along the bonded length is necessary. Accordingly, we consider a relation of the form $\tau_{\text{max}} = g(\sigma_{ij}, f_t)$ in terms of the state of stress $\sigma_{ij}$ and concrete tensile strength $f_t$. This relation will be developed based on a suitable failure criterion for the concrete that considers the interaction between different stress components. Subsequently, in our analysis, a unified function will be proposed to define the bond–slip model in terms of the geometrical and material characteristics of the interface. This function will be developed to best fit the micromechanics-based finite element results. In the following sections, we will begin by defining $\tau_{\text{max}}$ as a function of the normal stresses along the bonded FRP length and then use this to propose a local bond–slip model.
4.4 Bond strength for FRP/concrete joints

Since the ascending branch of the bond-slip model is essentially elastic, while the nonlinearity mainly occurs in the descending part of the bond-slip curve (softening behaviour), we will adapt a linear elastic analysis in defining an explicit expression for the bond strength $\tau_{\text{max}}$. Subsequently, the results of the micromechanics-based FE models will be used to derive the local bond-slip model. In this section, we use the linear finite element analysis rather than the micromechanics-based results in order to eliminate the effect of concrete nonlinearity on the interfacial stress distribution. The relation $\tau_{\text{max}} = g(\sigma_{ij}, f_t)$ will be considered to define the bond strength of the FRP/concrete joint, therefore in the next sections we will begin by investigating the main interfacial stress components $\sigma_{ij}$ along the bonded joint that mostly affect the bond strength value. Then using these interfacial stresses, the $\tau_{\text{max}}$ of the bond-slip model will be defined by considering a suitable failure criterion for the concrete.

4.4.1 Interfacial stress distribution along the bonded joint

In defining $\tau_{\text{max}}$ of the FRP/concrete joint, a linear elastic FE analysis is adopted with a very refined FE mesh. This analysis is used to find the main interfacial normal stresses along the bonded length that have significant values compared to the shear stress. This FE analysis is carried out for the specimen BN5 tested by Bizindavyi and Neale [2001] to examine the distribution of various stress components along the interface. The element size is taken as 0.25 mm cube in the top 30 mm and 1.0 mm cube for the rest of the concrete. The average number of elements is $2.08 \times 10^6$. Four-node 3-D tetrahedral elements are employed to represent the adhesive, FRP laminate and concrete. An iterative solver with a free-form meshing format is used instead of the direct solver to solve the huge number of equations and to avoid exceeding the memory capacity (around one million equations). The finite element mesh is shown in Figure 4.8(a) and the boundary conditions are presented in Figure 4.8(b).

As a result of the fact that the debonding failure normally takes place a few millimeters inside the concrete, close attention is paid to the profiles of the normal and shear stress distributions at a distance 1.0 mm away from the adhesive/concrete interface line inside
4.4. BOND STRENGTH FOR FRP/CONCRETE JOINTS

(a) Finite element mesh at the FRP/concrete interface

(b) Typical direct shear test representation

Figure 4.8: Finite element simulation: Specimen BN5

The concrete. These profiles are presented in Figures 4.9(a) to 4.9(c). In these figures, we will refer to the normal stresses in the direction perpendicular to the interface (y-direction in Figure 4.8) as “peeling-off” stresses ($\sigma_y$) and to stresses in the horizontal direction (x-direction in Figure 4.8) as “normal stresses” ($\sigma_x$). We observe that the peeling-off stress ($\sigma_y$) and normal stress in the z-direction have negligible values along the bonded length except at the loaded end (Point 1). By contrast, the normal stresses $\sigma_x$ have significant values compared to the shear stress values.

In order to confirm the above conclusion regarding the influence of the normal stresses on $\tau_{\text{max}}$ of the bond–slip relationship, the Mohr-Coulomb criterion for concrete is used to study the states of stress at three different locations. These locations are at the loaded end, 5.0 mm, and 40.0 mm away from the loaded end and they are marked in Figure 4.8(b). The values of the normal and shear stresses at the three locations are summarized in Table 4.3.

Figure 4.10 shows the application of the Mohr-Coulomb criterion to the state of stress at Point “1” giving a concrete tensile strength of 3.5 MPa and pure shear strength of 2.4 MPa. In this figure, the solid Mohr’s circle represents the elastic state of stress at Point “1” (values in Table 4.3). The predicted Mohr’s circle corresponding to the state of stress of Point “1” at failure is the circle that is tangent to the failure envelope (solid line in Figure 4.10) and intersects the line of maximum shear stress of Point 1 (dashed line in Figure 4.10). The line of maximum shear stress is that which connects between the origin (Point “o”) and the point of the maximum shear stress at the current state of stress.
Figure 4.9: Stresses distribution along the interface (1.0 mm away from the interface inside the concrete)
4.4. BOND STRENGTH FOR FRP/CONCRETE JOINTS

(Point “a”). More details regarding the Mohr-Coulomb criterion and its applications can be found in Chen [1982].

The Mohr’s circle at failure for the state of stress at Point “1” is represented schematically by the dashed circle in Figure 4.10. Accordingly, the predicted $\tau_{\text{max}}$ at Point “1” is 12.51 MPa. This value is significantly higher than the pure shear strength of concrete (2.0 MPa). This confirms the conclusion that was drawn in the previous section. In the subsequent sections, only the normal stress in the $x$-direction ($\sigma_x$) is considered due to the negligible values of the other two normal stress components. Accordingly, the bond strength will be introduced in the form $\tau_{\text{max}} = g(\sigma_x, f_t)$. Then an equivalent expression in terms of geometrical and material parameters of the concrete, adhesive and FRPs will be proposed.

<table>
<thead>
<tr>
<th>Table 4.3: Components of state of stress at three different locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>Point 1</td>
</tr>
<tr>
<td>Point 2</td>
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<tr>
<td>Point 3</td>
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</table>

4.4.2 Local bond strength model, $\tau_{\text{max}}$

In order to build a mathematical bond strength model $\tau_{\text{max}}$ considering the interaction of the normal stress $\sigma_x$ along the bonded length, we will assume that brittle fracture of concrete takes place when the maximum principal stress, $\sigma_1$, at a point inside the concrete reaches a value equal to the tensile strength of concrete, $f'_t$ [Chen, 1982]. We will refer to the normal stress “$\sigma_x$” as “$\sigma_c$”. Hence the state of stress along the interface can be expressed as:

$$
\sigma_{ij} = \begin{bmatrix}
\sigma_c & \tau_c & 0 \\
\tau_c & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$
This leads to the maximum principal stress $\sigma_1$ as:

$$\sigma_1 = \frac{\sigma_c}{2} \pm \sqrt{\frac{\sigma_c^2}{4} + \tau_c^2}$$

(4.10)

According to the aforesaid assumption of failure, when $\sigma_1$ reaches a value equal to the concrete tensile strength $f'_t$, the corresponding shear stress $\tau_c$ is equal to the bond strength $\tau_{\text{max}}$. Therefore,

$$\tau_{\text{max}}^2 = \left(f'_t - \frac{\sigma_c}{2}\right)^2 - \frac{\sigma_c^2}{4}$$

(4.11)

or in another format,

$$\left(\frac{f'_t}{\tau_{\text{max}}}\right)^2 - \alpha \left(\frac{f'_t}{\tau_{\text{max}}}\right) - 1 = 0$$

(4.12)

Here $\alpha = \frac{\sigma_c}{\tau_{\text{max}}}$. Hence

$$\tau_{\text{max}} = \frac{2f'_t}{\alpha \pm \sqrt{\alpha^2 + 4}}$$

(4.13)

Based on Equation 4.13, the bond strength value $\tau_{\text{max}}$ of the bond–slip relation is a function of the normal/shear stress ratio $\alpha$ and the concrete tensile strength $f'_t$. Accordingly, the bond strength varies along the bonded length according to the value of $\alpha$. The bond strength relation in Equation 4.13 is represented schematically in Figure 4.11.
In spite of the applicability of using the previous mathematical expression of $\tau_{\text{max}}$ (Equation 4.13) as a user-defined subroutine in most of the commercial software packages, there is a need for a constant bond strength value along the interface. This value represents the average behaviour of the direct shear test and might be used in the other applications of FRP-strengthened concrete structures. Accordingly, the average value of $\alpha$ can be used in Equation 4.13. $\alpha_{\text{ave}}$ is expressed as a function of the average normal stress $\sigma_{\text{ave}}$ and the mean shear stress $\tau_m$ as:

$$\alpha_{\text{ave}} = \frac{\sigma_{\text{ave}}}{\tau_m}$$

(4.14)

where $\sigma_{\text{ave}}$ can be obtained by integrating the expression of $\sigma_c$ over the bonded length $L_i$ as:

$$\sigma_{\text{ave}} = \frac{1}{L_i} \int_0^{L_i} \sigma_c \, dx$$

(4.15)

Here $L_i$ is the length of the bonded FRP laminate. Details of the derivations of $\sigma_c$ and $\tau_m$ are presented in the Appendix A. Using the expression of $\sigma_c$ in Equation A.21 leads to:

$$\frac{\sigma_{\text{ave}}}{\tau_m} = \frac{1}{\lambda t_p \sinh(\lambda L_i)} \left[ 1 - \cosh(\lambda L_i) \right] \left[ \frac{E_c}{E_p} - \frac{E_c}{G_\alpha} \lambda^2 t_a t_p \right]$$

(4.16)

and

$$\alpha_{\text{ave}} = \frac{\sigma_{\text{ave}}}{\tau_m} = \frac{\rho}{\lambda t_p \sinh(\lambda L_i)} \left[ \cosh(\lambda L_i) - 1 \right]$$

(4.17)

The definitions of $\lambda$ and $\rho$, as given in Appendix A, are as follows: $\lambda^2 = \left[ \frac{G_\alpha}{E_p t_p t_a} \right] (1 + \eta \rho)$, $\eta = \frac{E_p}{E_c}$, and $\rho = \frac{A_p}{A_c}$. Here, $A_p$ and $A_c$ are, respectively, the FRP laminate cross section
and the concrete block cross section. Since the value of $L_i$ is relatively large, therefore:

$$\alpha_{ave} = \frac{\rho}{\lambda p \tanh(\lambda L_i)}$$  \hspace{1cm} (4.18)

In cases where $L_i$ is a relatively long, such as the cases of FRP-strengthened concrete beams, the value of $\alpha_{ave}$ can be simplified as:

$$\alpha_{ave} = \frac{\rho}{\lambda p}$$  \hspace{1cm} (4.19)

Substituting the value of $\alpha_{ave}$ from Equation 4.19 into Equation 4.13 leads to:

$$\tau_{max} = \frac{2\lambda p f'_t}{-\rho + \sqrt{\rho^2 + 4\lambda^2 t_p^2}}$$  \hspace{1cm} (4.20)

The variation of $\tau_{max}$ in Equation 4.20 indicates that the bond strength value depends on the mechanical characteristics of the FRP, adhesive, and concrete layers. Various experimental studies confirm this conclusion, where the bond strength depends on the tensile strength of the concrete $f'_t$ [Jones et al., 1980; Triantafillou and Plevris, 1992; Quantrill et al., 1996b; Bizindavyi, 2000], the axial stiffness of the FRP laminated section (represented by $t_p$ and $\lambda$ in Equation 4.20) [Triantafillou and Plevris, 1992; Dai et al., 2005], the strength and thickness of the adhesive layer ($\lambda$ in Equation 4.20) [Jones et al., 1980; Quantrill et al., 1996b; Dai et al., 2005], and on the geometrical dimensions of the bonded laminate ($\rho$ and $\lambda$ in Equation 4.20) [Jones et al., 1980; Quantrill et al., 1996b]. The average values of $\alpha$ in the most practical cases range from 1.5 to 2.5 that lead to the bond strength values varying from 2 to 3 times the concrete tensile strength. Since the bond-slip curves have the same overall relationship along the bonded length with different values of $\tau_{max}$, we will now derive a unified relation for the $\tau-s$ relationship as a function of $\tau_{max}$. This will be proposed based on the best fit technique with the micromechanics-based finite element results.

### 4.5 Proposed local bond-slip model ($\tau-s$ relationship)

In this section, we propose a new mathematical technique to build up the entire $\tau-s$ model. This mathematical approach depends on dividing the bond-slip model into three
4.5. PROPOSED LOCAL BOND–SLIP MODEL (τ–S RELATIONSHIP)

---

**Figure 4.12:** Slip components between the FRP laminate and the concrete

separate relations describing the shear–slip behaviour of the FRP laminate, the adhesive layer, and the concrete. The total slip value between the FRP laminate and the concrete block, accordingly, is divided into three different components; namely, the FRP ($S_p$), adhesive ($S_a$), and concrete ($S_c$) slip values, as shown in Figure 4.12. We will start by defining the three separate relations. In each relation, the values of the slip corresponding to $\tau_{max}$, initial slope, and interfacial fracture energy will be developed and then fed into a mathematical function to build up the entire bond–slip model. Based on the finite element results, the slip values between the two faces of the FRP, adhesive, and concrete are around, respectively, 1.0%, 60.0%, and 39.0%. In some particular cases, such as using a relatively thick FRP laminate with a low shear modulus, the slip value in the FRP laminate might be significant. In general, we aim to develop a general bond–slip model that considers all the parameters of the interface. In the following sections, a description of the three separate models is presented followed by the proposed entire local bond–slip model.

### 4.5.1 Shear–slip model for FRP laminates

In the finite element analyses, the output stress values are computed at the integration points instead of the nodal points. Accordingly, in our analysis the interfacial shear stresses at the FRP/adhesive interface are computed as the average of the values at the integrations points of the FRP and adhesive layer. This explains the nonlinearity observed in Figures 4.13(a) and 4.14(a). The shear stresses in these two figures are the average shear stress between the FRP and adhesive layer. The relationship between the applied shear
stress and the corresponding slip value between the two faces of the FRP is shown in Figure 4.13(a). Figure 4.13(b) shows our proposed shear–slip curve for the FRP laminate that matches the micromechanics-based finite element results.

In the proposed relation, the shear stress is a nonlinear third-order function until a value of 0.9\(\tau_{\text{max}}\), then it exhibits a yielding behaviour until the value of \(\tau_{\text{max}}\) with a tangent modulus that is found to be around one-fifth of the initial tangent modulus. Since the FRP laminate is an elastic material, after debonding the stresses in the FRP laminate unload. For the unloading part, in Figure 4.13(b), the laminate behaves linearly with a tangent modulus that equals the initial modulus until the shear stress reaches the residual shear stress, \(\tau_{\text{res}}\). Then the shear–slip relationship is decreased linearly with a tangent modulus, \(E_{pl}\). The ascending branch of the proposed model is defined as:

\[
\tau = \frac{G_p}{t_p} s - \frac{G_p}{5.47\tau_{\text{max}}^2 t_p^2} s^3 \quad \text{if} \quad s \leq s_1
\]

\[
\tau = 0.9\tau_{\text{max}} + E_{ps}(s - s_1) \quad \text{if} \quad s_p > s > s_1
\]

where \(\tau_{\text{max}}\) is previously defined in Equation 4.20, \(G_p\) (\(G_{23}\) in Equation 4.7) is the shear modulus of the FRP laminate, and

\[
s_1 = 1.35 \frac{\tau_{\text{max}}}{E_{po}} \quad \text{and} \quad S_p = 6.35 \frac{\tau_{\text{max}}}{G_p}
\]

The parameters in Figure 4.13(b) that have been developed based on our finite element analyses are:

\[
E_{po} = G_p/t_p, \quad E_{ps} = \frac{2G_p}{3}/t_p, \quad \text{and assume} \quad E_{pl} = E_{po}/50
\]

The interfacial fracture energy (\(G_{fp}^p\) in Figure 4.13(b)) is computed from the geometry of Figure 4.13(b) (neglecting the effect of residual stress) as:

\[
G_{fp}^p = 0.405 \frac{\tau_{\text{max}}^2}{E_{po}}
\]

This interfacial fracture energy will be used later in defining the descending part of the complete bond–slip model.
4.5. PROPOSED LOCAL BOND-SLIP MODEL (\(\tau-S\) RELATIONSHIP)

![Shear-slip behaviour between two faces of the FRP laminate](image)

Figure 4.13: Shear-slip behaviour between two faces of the FRP laminate

### 4.5.2 Shear-slip model for adhesive layer

In Figure 4.14(a), the bond-slip model of the adhesive layer based on the finite element analyses is presented and our proposed model is shown in Figure 4.14(b). This nonlinear response in Figure 4.14(a) arises from the fact that we use the average shear stress instead of the value in the adhesive layer. A third-order function is employed to describe the ascending branch of the shear-slip behaviour of the adhesive layer that match with the finite element results as:

\[
\tau = E_{ao}s - \frac{E_{ao}^3}{8\tau_{\text{max}}^2}s^3 \quad \text{if} \quad s \leq s_a
\]

where, \(E_{ao} = \frac{G_a}{4\tau_a}\) and \(G_a\) is the shear modulus of the adhesive layer. The parameters in Figure 4.14(b) are defined as follows:

\[
S_a = \frac{8\tau_{\text{max}}t_a}{G_a}, \quad \text{and} \quad G_{fa}^p = 0.405\frac{\tau_{\text{max}}^2}{E_{ao}}
\]

### 4.5.3 Shear-slip model for concrete

The major nonlinearity of the FRP/concrete joints subject to a direct shear arises from a thin layer inside the concrete referred to as the “interfacial layer”. The proposed shear model of concrete is depicted in Figure 4.15. The parameters of the model are found
indirectly using the displacement profile along the depth of the concrete (Figure 4.12) as:

\[ u = s \cdot \exp\left(-\frac{y}{h_{\text{eff}}}\right) \]  

(4.28)

where, \( h_{\text{eff}} \) is the thickness of the interfacial layer and \( s \) is the total slip. We use this indirect way to define the shear response of the concrete rather than using the finite element results to avoid the effect of the normal stresses on the shear stress values. Based on the first derivative of Equation 4.28, the elastic slip is expresses as \( s \) at \( y = 0 \):

\[ s = h_{\text{eff}} \frac{\tau}{G_c} \]  

(4.29)

Therefore the elastic slip value of the concrete, \( S_e^c \) (in Figure 4.15), equal to \( h_{\text{eff}} \frac{\tau_{\text{max}}}{G_c} \). From the micromechanics-based finite element results, we have found that \( S_e^c \) is around \( 3S_e^c \) and \( h_{\text{eff}} \) is equal 16.67 mm. In the literature, \( h_{\text{eff}} \) equals to 2.5–3 the maximum aggregate size (40-50 mm) in the experimental work of Brosens [2001], and 5.0 mm in the finite element simulations of Lu et al. [2005b]. In our analysis, we have observed that the value of \( h_{\text{eff}} \) changes from 16.67 mm in the elastic stage to 50.0 mm at failure.

The inelastic interfacial fracture energy, \( G_f^p \), shown in Figure 4.15, can be calculated assuming a quadratic function for the descending part of the bond–slip model shown in Figure 4.15 as:

\[ G_f^p = \frac{0.9\tau_{\text{max}}}{3}(S_{cf} - S_c) \]  

(4.30)
4.5. PROPOSED LOCAL BOND-SLIP MODEL ($\tau-S$ RELATIONSHIP)

![Shear-slip relationship of the concrete layer obtained by the finite element analysis](image)

**Figure 4.15:** Shear-slip relationship of the concrete layer obtained by the finite element analysis

Taking $S_c - S_{cf}$ equal to $10S_c$, leads to:

$$G_{fc}^p = 150\frac{\tau_{max}^2}{G_c}$$  \hspace{1cm} (4.31)

Here $G_c$ is the initial shear modulus of the concrete. This expression of $G_{fc}^p$ (Equation 4.31) will be used in the next section to define the descending part of the complete bond-slip model.

### 4.5.4 Total local bond-slip model

In order to define the entire bond-slip model, a nonlinear function with a rational initial slope value is used to describe the ascending branch of the bond-slip relationship depicted in Figure 4.16 as follows:

$$\tau = bs + as^\beta$$  \hspace{1cm} (4.32)

The constants $a$, $b$ and $\beta$ in Equation 4.32 are obtained by employing the following boundary conditions:

$$at \; s = 0 \quad \frac{\partial \tau}{\partial s} = E_o$$

$$at \; s = S_o \quad \tau = \tau_{max}, \quad \text{and} \quad \frac{\partial \tau}{\partial s} = 0$$

Here $S_o = S_c + S_a + S_p$, where $S_c$, $S_a$ and $S_p$ are the slip values in the interfacial concrete layer, adhesive and FRP laminates, respectively, and $\frac{1}{E_o} = \frac{t_p}{G_p} + \frac{t_a}{G_a} + \frac{16.6}{G_c}$. Combining
these boundary conditions with Equation 4.32, the constant \( b, \alpha \) and \( \beta \) are found to be:

\[
\begin{align*}
 b &= \frac{\tau_{\text{max}} - E_o S_o}{S_o^a}, \\
 \alpha &= 1 - \frac{\tau_{\text{max}}}{E_o S_o}, \\
 \beta &= 1 - \frac{\tau_{\text{max}}}{E_o S_o},
\end{align*}
\]

An exponential function is employed to describe the descending branch of the bond-slip model as:

\[
\tau = \tau_{\text{max}} \exp \left[ -\alpha_1 \left( \frac{s}{S_o} - 1 \right) \right] \tag{4.33}
\]

The value of the constant \( \alpha_1 \) in Equation 4.33 can be computed employing the inelastic interfacial fracture energy shown in Figure 4.16 (neglecting the effect of the residual stress, \( \tau_{\text{res}} \)) as:

\[
G_f^p = \int_{S_o}^{s_f} \tau ds \tag{4.34}
\]

where \( G_f^p = G_{f_c}^p - G_{f_d}^p - G_{f_p}^p \). Considering \( S_f = 4S_o \), then:

\[
G_f^p = \tau_{\text{max}} \frac{S_o}{\alpha_1} \left[ e^{\alpha_1} - 1 \right] = -0.9\tau_{\text{max}} \frac{S_o}{\alpha_1} \tag{4.35}
\]

Therefore

\[
\alpha_1 = -\frac{0.9\tau_{\text{max}}S_o}{G_f^p} \tag{4.36}
\]

### 4.6 Validation of the proposed local bond–slip model

This section includes comparisons among the characteristics of the proposed bond–slip relation and those of the Lu et al. [2005b], and Dai et al. [2005] models. It is out of interest to validate the latter two relationships; the comparisons are set to demonstrate
the characteristics of the proposed relation in reference to well established models. In addition, a comparison is presented in more detail in terms of the ultimate load capacities between the predictions using the proposed model and experimental data.

### 4.6.1 Comparison of the proposed bond-slip model with Lu et al. [2005b] and Dai et al. [2005] models

The Lu et al. [2005b] and Dai et al. [2005] relationships are selected from the large database based on the fact that they are proven to be effective to simulate interfacial bond-slip behaviour in several applications. In addition, they are derived based on two different techniques; using measured strain values along the bonded laminates resulting from 2-D finite element analysis in the case of the Lu et al. [2005b] model, or using the first derivative of the FRP laminate normal strain function computed from the experimental studies in the case of the Dai et al. [2005] relationship. Moreover, the two techniques employ a regression analysis in a certain stage of defining the bond-slip behaviour.

In Figures 4.17(a) to 4.17(c), the comparisons are depicted in terms of the effect of the concrete compressive strength, FRP Young's modulus and shear modulus of the epoxy layer, respectively, on the bond strength value. The concrete compressive strength has almost the same effect on \( \tau_{\text{max}} \) of the three models in the trend that the bond strength increases with increasing the concrete compressive strength (Figure 4.17(a)). However, the bond strength in the Dai et al. [2005] model is less sensitive to the concrete compressive strength than that of the proposed and the Lu et al. [2005b] relations. It is noticeable that the proposed and the Lu et al. [2005b] models have almost the same bond strength values. This might be a result of the fact that the two relationships are derived based on the finite element results whether from 2-D or 3-D analyses.

The Lu et al. [2005b] bond strength model does not intend to consider the effect of the FRP and adhesive characteristics on the bond strength values; however, it includes such an effect on the fracture energy (in defining the slope of the descending part of the bond-slip model). In Figure 4.17(b), the predictions using the Dai et al. [2005] relation and those using the proposed model have almost the same function with the Young's modulus of the FRP laminates. However, the values using the latter are little higher than those using the Dai et al. [2005] model. The relationship between the bond strength value and the FRP
CHAPTER 4. NONLINEAR MICROMECHANICS-BASED FEA OF FRP/CONCRETE JOINTS

(a) Effect of concrete compressive strength ($G_a = 717.28$ MPa, $E_p = 100,000$ MPa, $E_c = 4500 \sqrt{f'_c}$, $t_p = 0.33$ mm, $t_a = 1.00$ mm)

(b) Effect of the composite plate Young's modulus ($G_a = 717.28$ MPa, $f'_c = 35.0$ MPa, $t_p = 0.33$ mm, $t_a = 1.00$ mm)

(c) Effect of the epoxy shear modulus ($t_a = 1.0$ mm, $f'_c = 35.0$ MPa, $E_p = 100,000$ MPa, $t_p = 0.33$ mm)

(d) Bond-slip models

Figure 4.17: Comparison among the proposed bond-slip model and Lu et al. [2005b] and Dai et al. [2005] models
4.6. VALIDATION OF THE PROPOSED LOCAL BOND-SLIP MODEL

axial stiffness as that in Figure 4.17(b) (thick solid and dot lines) is reported in several studies [Bizindavyi and Neale, 2001; Nakaba et al., 2001]. In Figure 4.17(c), the proposed model in contrast to the Dai et al. [2005] model assumes that the bond strength increases when using a softer adhesive layer. This is the same conclusion reported experimentally [Dai et al., 2005]. This is attributed to the fact that the debonding occurs inside the concrete block rather than the adhesive layer. Increasing the the shear stiffness of the adhesive layer transfers more shear stresses to the concrete block and leads to more rapid debonding.

In Figure 4.17(d), a comparison is made regarding the shape of the three bond–slip relationships. Both the proposed and Dai et al. [2005] models are characterized by a rational initial slope rather than an infinite slope as in the case of the Lu et al. [2005b] bond–slip profile. The descending branch of the proposed and Lu et al. [2005b] relationships is concave downward (Figure 4.17(d)), while a convex upward or a concave downward based on the shear modulus of the adhesive layer characterizes the shape of the Dai et al. [2005] curve. In conclusion, no significant discrepancies have been observed between the three models, except what is observed in Figure 4.17(c). Notwithstanding, the proposed model is the only one among the three relations that is defined based on theoretically sound derivations rather than simple regression analyses.

4.6.2 Ultimate capacities

In this section, a nonlinear 2-D finite element analysis is carried out to validate the accuracy of the proposed bond–slip model to predict the ultimate capacities of FRP/concrete joints. The bond–slip relation is used to constitute the characteristics of the interface elements that are employed between the FRP and the concrete nodes. The comparisons are carried out for the predicted load values using the proposed and Lu et al. [2005b] models with the experimental data of the 40 specimens shown in Table 4.1. Another set of 118 specimens, including the previous 40 specimens, is considered to compare the ultimate capacities from the proposed and Lu et al. [2005b] relationships with the experimental data. The experimental data of the additional 78 specimens and their comparisons with the Lu et al. [2005b] model are taken from the analysis of Ebead and Neale [2007].
CHAPTER 4. NONLINEAR MICROMECHANICS-BASED FEA OF FRP/CONCRETE JOINTS

4.6.2.1 Finite element model

Four-node plane stress elements are used to represent the concrete. The element sizes of the concrete prism are 0.25 mm square. FRP laminates are modelled using 2-node truss elements with a length equal 0.25 mm. A series of discrete shear spring elements is used to connect the FRP nodes with the concrete nodes, as shown in Figures 4.18(a) and 4.18(b), to represent the interfacial behaviour. The proposed and Lu et al. [2005b] relations are used to constitute the interfacial behaviour of these elements. The constraint equations are enforced in the longitudinal direction of the FRP laminate between the interface and the concrete nodes, and between the interface and the FRP nodes, as detailed below. The length of each truss element (spring element) is taken as 0.25 mm representing the distance between two adjacent FRP nodes and its cross-sectional area represents the interfacial shear area between the FRP and the concrete within that length (area = 0.25\(b_{frp}\)). The boundary conditions are taken as depicted in Figure 4.18(a).

![Diagram of finite element model](image)

**Figure 4.18:** Finite element representation of the direct shear test

Let us denote the nodes on the FRP, the interface, and the concrete at the interface as \(NF(i)\), \(NI(i)\), and \(NC(i)\), respectively (Figure 4.18(b)). If we constrain the displacements in the horizontal direction at nodes \(NF(i)\) with those of \(NI(i + 1)\), and at nodes \(NI(i)\) with those at nodes \(NC(i)\), the constraint conditions can then be represented as follows:

\[
\Delta_{NF(i)} = \Delta_{NI(i+1)} \tag{4.37}
\]

\[
\Delta_{NI(i)} = \Delta_{NC(i)} \tag{4.38}
\]

The displacement in an interface truss element that joins nodes \(NI(i)\), and \(NI(i + 1)\) is:

\[
\Delta = \Delta_{NI(i+1)} - \Delta_{NI(i)} \tag{4.39}
\]
4.6. VALIDATION OF THE PROPOSED LOCAL BOND-SLIP MODEL

Hence, the differential displacements between nodes $NF(i)$ and $NC(i)$ are equal to the slip values in the interface elements joining nodes $NI(i+1)$ and $NI(i)$. Furthermore, the interfacial shear stresses causing these differential slips are equivalent to the stresses in the truss element joining the nodes $NI(i+1)$ and $NI(i)$, as computed from the above bond-slip models. Equation 4.37 and Equation 4.38 are attached as an input to the finite element package, ADINA, Version 8.4 to constitute the interfacial behaviour of the interface elements.

A hypoelastic model is employed to describe the nonlinear stress-strain relationship for the concrete and a linear elastic tensile model of the FRP materials is used. The details characteristics of the concrete constitutive law are presented in Appendix B. For the specimens CR2L1, to CR2L3, CR3L1 to CR3L3, AR2L3, AR3L3, GR2L3, and GR3L3, the bond strength, $T_{max}$, values in the predictions using the proposed model are taken as the minimum between the computed value from the proposed model and the yield shear stress of the adhesive layer. In the Lu et al. [2005b] model, the bond strength values of these particular specimens are always lower than the adhesive yield shear stress.

### 4.6.2.2 Prediction of the ultimate capacities

Figures 4.19(a) and 4.19(b) show a comparison between the numerical predictions using the proposed and Lu et al. [2005b] models, respectively, with the set of experimental results of the 40 specimens summarized in Table 4.1 in terms of the ultimate capacity. In Figures 4.19(c) and 4.19(d), the same comparisons are carried out for the entire specimens. The correlation coefficients for the comparisons are summarized in Table 4.4 in terms of the average, standard deviation (SD) and coefficient of variation (COV).

With regard to the set of the experimental specimens consisting of 40 FRP/concrete joints, the average ratio between the numerical predictions using the proposed model and experimental results is 0.978 (Table 4.4) with a standard deviation of 0.145 and coefficient of variation of 0.139. The correlation coefficients of the numerical predictions using the proposed model are better than those of the Lu et al. [2005b] model. However, the coefficients indicate that the latter model well represents the experimental data (average 0.812). The relatively low accuracy of these predictions when concerning the particular 40 specimens compared to that when concerning the entire group of data (118 specimens)
is due to the fact that the Lu et al. [2005b] model does not intend to simulate the effect of the softer adhesives. In other words, the model is not sensitive to the change of the adhesive characteristics as it is with the concrete characteristics.

The predictions using the proposed and Lu et al. [2005b] models are in the same range when concerning the entire group of data. It is of interest to mention that the predictions of the Lu et al. [2005b] relationship are generally underestimated and those of the proposed model are overestimated. The relatively high standard deviation of the results using the proposed and Lu et al. [2005b] relations (16.3 and 17.3% respectively) can be attributed to the fact that the thicknesses and the mechanical characteristics of the adhesive layer were not constantly reported in the original experimental work. In our FE analysis 1.0 mm is assumed for the thickness, 2410 MPa for Young’s modulus and 0.38 for Poisson’s ratio of
the adhesive layer. In the original reference of Ebead and Neale [2007], the function value describing the adhesive layer in the Lu et al. [2005b] relationship was assumed ranging from 1 to 2 to best fit with the experimental data.

### 4.7 Conclusion

In this chapter, nonlinear micromechanics-based finite element models were developed using the microplane concrete model to investigate the interfacial behaviour of FRP/concrete joints subjected to direct shear loadings. It was found that the state of stress components along the bonded length affected the bond strength value, $\tau_{\text{max}}$. Theoretical models for the normal and shear stress distribution at 1.0 mm beneath the adhesive/concrete interface line inside the concrete were derived (Appendix A). These models were used to define a refined and accurate bond strength relation in the form $\tau_{\text{max}} = g(\sigma_{ij}, f_i)$.

A new mathematical approach has been proposed to describe the entire $\tau-s$ relationship based on three separate models. The first model captures the shear behaviour of an orthotropic FRP laminate. The second model simulates the shear characteristics of an adhesive layer. The third model represents the shear nonlinearity of a thin layer inside the concrete referred to as the "interfacial layer". The proposed bond-slip model and the associated bond strength value reflect the characteristics of the FRP, concrete and adhesive layers. A large amount of experimental data was used to calibrate the proposed bond-slip model. The comparisons between the finite element predictions and the experimental results showed a very good agreement.

<table>
<thead>
<tr>
<th>No</th>
<th>Proposed model</th>
<th>Lu et al. [2005b] model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>SD</td>
</tr>
<tr>
<td>Selected 40 specimens</td>
<td>0.978</td>
<td>0.136</td>
</tr>
<tr>
<td>Entire 118 specimens</td>
<td>1.016</td>
<td>0.163</td>
</tr>
</tbody>
</table>
4.7.1 Summary of the model

The following is a summary of the proposed bond-slip model presented in the previous sections (see Figure 4.16). The interfacial shear stress $\tau$ is expressed as a function of the slip $s$ between the FRP and concrete as:

$$\tau = \begin{cases} 
E_0 s + \left( \frac{\tau_{\text{max}} - E_0 S_0}{S_0^\beta} \right) s^\beta & s \leq S_o \\
\tau_{\text{max}} e^{-\alpha \left( \frac{s}{S_0} - 1 \right)} & S_o \leq s \leq S_f \\
\tau_{\text{res}} & s > S_f 
\end{cases}$$

(4.40)

where

$$\frac{1}{\beta} = 1 - \frac{\tau_{\text{max}}}{E_0 S_o},$$

(4.42)

$$S_o = \tau_{\text{max}} \left[ 6.35 \frac{t_p}{G_p} + 8.5 \frac{t_a}{G_a} + \frac{3h_{\text{eff}}}{G_c} \right], \quad S_f = 4S_o$$

(4.43)

$$\tau_{\text{max}} = \frac{2f_c'}{-\alpha + \sqrt{\alpha^2 + 4}}$$

(4.44)

Here, $\alpha = \frac{\rho}{\lambda t_p}$, where $\lambda^2 = \left[ \frac{G_a}{E_p t_p t_a} (1 + \eta \rho) \right]$, $\eta = \frac{E_p}{E_c}$ and $\rho = \frac{A_p}{A_c}$. In the later, $A_c$ is the cross section area of the concrete (equal to $h_{\text{eff}} b_c$). In Equation 4.40, $\tau_{\text{res}}$ is computed as:

$$\tau_{\text{res}} = 0.1\tau_{\text{max}},$$

(4.45)

$$\alpha_1 = -0.9\frac{\tau_{\text{max}} S_o}{G_f^p}$$

(4.46)

and

$$G_f^p = \tau_{\text{max}}^2 \left[ 150 \frac{t_p}{G_c} - 0.405 \left( \frac{t_p}{G_p} + \frac{t_a}{4.25G_a} \right) \right]$$

(4.47)

In the above bond-slip model, $t_p$ and $t_a$ are the thicknesses of the FRP laminate and adhesive layer, respectively. $G_p$, $G_a$, and $G_c$ are the shear modulus of the FRP laminate, adhesive layer, and concrete respectively. In the next chapter, the proposed bond-slip model will be validated for the application of FRP-strengthened reinforced concrete beams.
Chapter 5

Nonlinear Finite Element Analysis of Reinforced Concrete Beams Strengthened in Flexure with Externally Bonded FRPs

As a contribution to bridge the gap between reliable simulations of debonding phenomena and experimental observations, two-dimensional and three-dimensional (2-D and 3-D) nonlinear displacement-controlled finite element models have been developed to investigate the flexural and FRP/concrete interfacial responses of FRP-strengthened reinforced concrete beams. The 3-D models have been created to accommodate cases of FRP-strengthened beams having FRP anchorage systems whether at a plate end, mid-span, or along the beam span (13 specimens are considered in the 3-D simulations). Discrete interface elements are used to simulate the FRP/concrete interfacial behaviour before and after cracking. The FE models are capable of simulating the various failure modes, including debonding of the FRP, either at the plate end or at intermediate cracks. In addition, the models successfully represent the actual interfacial behaviour in the vicinity of cracks including the stress/slip concentrations and fluctuations.

Results are presented in terms of the ultimate load carrying capacities, failure modes, strain distributions, and load-deflection relationships. Special emphasis is placed on the FRP/concrete interfacial and cracking behaviour of the strengthened beams. The numer-
ical results are compared to available experimental data for 35 specimens for both the 2-D and 3-D analyses, and they show a very good agreement. Of the 35 specimens, 25 specimens are considered in the 2-D and 13 specimens in the 3-D analyses (3 specimens are simulated twice using the 2-D and 3-D models for the sake of comparison between the two types of analysis).

Proper attention is paid to various numerical aspects that might affect the flexural responses of FRP-strengthened beams including the mesh size, discrete interface element length, interfacial fracture energy \( (G_f^b) \), and the concrete fracture energy \( (G_f) \). In addition, various finite element configurations are considered to represent the interface, including the full bond assumption, employing interface elements, and representing the epoxy layer using a refined mesh. Two distinct constitutive laws for the concrete are used in this study; namely, the phenomenological (hypoelastic concrete model available in the ADINA software, Version 8.4) and micromechanics-based model (microplane concrete model detailed in Chapter 3). Many of the results presented in this chapter can be found in Neale et al. [2006], Kotynia et al. [2008] and Abdel Baky et al. [2007].

5.1 Introduction

In the literature, three approaches have been proposed to simulate the debonding mechanisms in FRP-strengthened concrete beams and to predict the associated failure loads using finite element analysis. The first involves simulating cracking and failure of the concrete adjacent to the adhesive layer using a fine finite element mesh [Pham and Al-Mahaidi, 2005]. In this approach, as a result of using a smeared crack model that treats cracked concrete as a continuum, this analysis cannot represent the stress concentrations at the vicinity of cracks. In addition, using a small element size (less than the average aggregate size of concrete) might cause a spurious mesh sensitivity [Bážant and Planas, 1998]. A second approach employs a discrete crack model to represent the discontinuity due to major cracks at predefined locations, with a smeared crack model to represent the behaviour between these cracks [Yang et al., 2003; Kishi et al., 2005; Niu and Wu, 2006]. Despite the fact that this approach captures the debonding load and simulates the local behaviour at the predefined locations, the interfacial behaviour between the predefined cracks is not modelled. Furthermore, forcing the cracks to initiate at certain locations
and to propagate along specific paths may lead to an inaccurate response of the interfacial behaviour. In the third approach, interface elements having a predefined bond–slip relation are used to link the FRP and concrete nodes [Sand and Remlo, 2001; Wong and Vecchio, 2003; Abdel Baky et al., 2004; Lu et al., 2007]. Similar to the first two techniques, the third approach cannot be employed to predict the behaviour of stress concentrations along the FRP/concrete interface at the vicinity of cracks due to the use of continuous interface elements. However, this approach successfully captures the debonding load. When using such elements, a continuous interpolation function along the interface is used so that the stress fields are assumed constant, which contradicts the fact that there are stress concentrations at the crack tips [Lu et al., 2007].

In this chapter, we present an original technique to capture the stress/slip concentrations and fluctuations at the vicinity of cracks using a discrete interface element instead of a continuous interface element. In addition, the proposed FE models simulate the effect of microcracks, as well as large cracks, on the interfacial profile. We use two-node truss elements to simulate the interface. These elements are aligned in a discrete manner to overcome the problems arising from the use of a continuous interpolation function (as in the case with continuous interface elements), where each element connects the FRP and the concrete nodes without any interaction between the interface elements. In addition, a fixed smeared crack model with a suitable element size is used to represent the tensile behaviour of the concrete and to simulate the cracked concrete layer at the interface.

5.2 Objectives

The objective of this study is to investigate the flexural and FRP/concrete interfacial responses of FRP-strengthened reinforced concrete beams. We aim to simulate the various failure modes, including debonding of the FRP, either at the plate end or at intermediate cracks. Various numerical aspects are considered to address their influence on the flexural responses of strengthened beams. Three-dimensional finite element models are created to accommodate cases of FRP-strengthened concrete beams having FRP anchorage sheets. A new concept of discrete interface elements is proposed to simulate the FRP/concrete interfacial behaviour before and after cracking. This proposed discrete interface element successfully captures the stress concentrations and fluctuations at the vicinity of cracks.
The analysis intends to validate the bond–slip model proposed in Chapter 4 for applications of FRP-strengthened beams, and to assess the applicability of using the microplane concrete model to simulate the debonding mechanisms.

This chapter is organized as follows. First, a brief description of the beams that are analyzed is presented. Then, detailed descriptions of the finite element models are given. This involves a description of the various materials used and the proposed technique to represent the interface using discrete interface elements. The numerical results are displayed in terms of ultimate load carrying capacities, load–deflection relationships, strain distributions, modes of failure, interfacial shear behaviour, and cracking behaviour. Finally, particular sections investigating the influence of various numerical aspects on the predicted responses, and examining the applicability of the microplane model to capture the debonding phenomena are presented.

5.3 Description of the beams analyzed

Experimental results of 35 specimens, representing unstrengthened and FRP-strengthened reinforced concrete beams, are used to validate the numerical analysis of the 2-D and 3-D analyses. These beams have variable geometrical characteristics and material properties for the concrete, steel reinforcement, and FRP composites and are grouped in 8 sets. Of the 8 sets, 6 sets consisting of a total of 25 specimens are used to validate the 2-D analysis. The dimensions and different parameters for the beams used in the 2-D simulations are listed in Table 5.1, which correspond to specimens tested by Brena et al. [2003], Ross et al. [1999], Kaminska and Kotynia [2000], Ritchie et al. [1991], M'Bazaa [1995], and Chicoine [1997]. Ten specimens are considered in the 3-D analysis, which are grouped in 2 sets corresponding to the beams tested by Kotynia et al. [2008]. Figure 5.1 shows the dimensions and different parameters for the beams tested by Kotynia et al. [2008]. In addition, the beam set tested by Chicoine [1997] is re-analyzed using 3-D FE models in order to compare the 2-D and 3-D results for this particular set of specimens. The identifications of the beams (IDs) listed in Column 2 of Table 5.1 and in Figure 5.1 correspond to the notations employed in the original references.

The beam set tested by Brena et al. [2003] includes five specimens; namely, Control
<table>
<thead>
<tr>
<th>Beam set</th>
<th>Beam ID</th>
<th>Steel reinforcement</th>
<th>FRP sheets (mm)</th>
<th>Material properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beam ID</td>
<td>Longitudinal ratio</td>
<td>Transverse</td>
<td>Steel</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>FRP (mm)</td>
<td>f_m (MPa)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>ε_m (%)</td>
</tr>
<tr>
<td>Brena et al.</td>
<td>A1</td>
<td>0.46</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td>(2003)</td>
<td>A2</td>
<td>0.83</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>1.24</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>A4</td>
<td>1.83</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>2.50</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>B2</td>
<td>3.30</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>B3</td>
<td>4.00</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>B4</td>
<td>4.75</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>B5</td>
<td>5.50</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>B6</td>
<td>6.25</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td>Ross et al.</td>
<td>B-08/S</td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td>(1999)</td>
<td>BF-04/0.5S</td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>BF-06/S</td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td>Kaminska and Kotynia (2003)</td>
<td>B-08/S</td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
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<tr>
<td></td>
<td>BF-04/0.5S</td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>BF-06/S</td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
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<tr>
<td></td>
<td></td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
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<tr>
<td>Ritchie et al.</td>
<td>B-08/S</td>
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<td>0.75</td>
<td>440.0</td>
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<td>(1999)</td>
<td>BF-04/0.5S</td>
<td>0.66</td>
<td>0.75</td>
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<tr>
<td></td>
<td>BF-06/S</td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td>Chicoine (1997)</td>
<td>B-08/S</td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>BF-04/0.5S</td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td>BF-06/S</td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.66</td>
<td>0.75</td>
<td>440.0</td>
</tr>
</tbody>
</table>

Table 5.1: Geometrical parameters and material properties of the beam specimens that are analyzed in the 2-D simulations.

- L, W and H denote length, width and thickness respectively.
- Beam E is anchored at its ends and two different lengths of FRP laminates have been used for Beam F, the first length is 1981 mm and the second length is 1372 mm.
- Beam L has a fibre orientation 0-60°.

* L, W and H denote length, width and thickness respectively.
** Beam E is anchored at its ends and two different lengths of FRP laminates have been used for Beam F, the first length is 1981 mm and the second length is 1372 mm.
*** Beam L has a fibre orientation 0-60°.
and A1 through A4. These specimens investigate the influence of the length of the FRP laminate for a constant FRP laminate thickness. Six beams, B1 through B6, constitute the beam set of Ross et al. [1999], which have different steel reinforcement ratios. For the Kamiinska and Kotynia [2000] beam set, different combinations of compressive and tensile reinforcement ratios are used (Specimens B-08/S, BF-04/0.5S, and BF-06/S). The effects of changing the FRP laminate length, thickness and stiffness are assessed in the beam set tested by Ritchie et al. [1991]. These include six beams; namely, Control, E, F, G, L, and M. Two beams tested by M'Bazaa [1995], Control and P0, are included in the simulations. Finally, the beam set of Chicoine [1997] includes Specimens P0, P1, and P3. This set of beams investigated the effect of the anchorage sheets, where two configurations for the FRP anchorages were used. In the first configuration (Specimen P1), U-shaped anchorage sheets of dimensions 950.0 × 250.0 mm were used at the two FRP laminate ends. In the second configuration (Specimen P3), unidirectional transverse straps 100.0 mm in width and 300.0 mm in height and spaced at 100.0 mm were used along the laminate (dimensions of this configuration are shown later in Figure 5.5(a)). Specimen P0 did not have any anchorages.

The 10 specimens considered in the 3-D simulations have variable FRP anchorages; including anchorage sheets at the laminate end, separated sheets along the laminate length, and continuous sheets in the flexural zone. These specimens, tested by Kotynia et al. [2008], were strengthened with external CFRP strips or sheets bonded to the tension side. Continuous and spaced L-shaped CFRP reinforcement were used to delay the debonding of the bottom longitudinal CFRP laminates. The geometry and steel reinforcement for all the beams were the same. These beams are grouped in two series. Series I includes six specimens strengthened with strips of width 50.0 mm (with a Young's modulus of 172 GPa and an ultimate strength of 2915 MPa) or strips of width 120.0 mm (with a Young's modulus of 220 GPa and an ultimate strength of 274 MPa) bonded to the tension side of the beams. As shown in Figure 5.1, Specimens B-08S and B-08M were strengthened only with bottom strips. Specimens B-08Sm and B-08Mm were strengthened using additional continuous U-shaped laminates. Specimens B-08Sk and B-08Mk were strengthened using additional L-shaped laminates spaced at 180.0 and 200.0 mm, respectively (with a Young's modulus of 132.0 GPa, an ultimate strength of 2295 MPa, and width of 40.0 mm). These L-shaped laminates were applied on both sides of the beam web and overlapped at the bottom.
5.3. DESCRIPTION OF THE BEAMS ANALYZED

![Diagram of beam configurations](image)

**Figure 5.1:** Strengthening configurations for the beam specimens that are analyzed in the 3-D simulations (Kotynia et al. [2008])

In Series II of the Kotynia et al. [2008] specimens, Specimens B-08Smb and B0-08Smb were strengthened using strips bonded to the tension side of the beams and one layer of continuous U-shaped sheet. Three layers of continuous FRP sheet (with a Young’s modulus of 230 GPa and an ultimate strength of 3500 MPa) were used as the longitudinal strengthening material for the specimen B-083m. For the specimen B-083mb, two layers of continuous sheet were used for the longitudinal strengthening, and one layer of the same sheet was used as a U-shaped system. The direction of the fibres of the continuous U-shaped sheets used in Series I was perpendicular to the longitudinal axis of the beam. For Series II, the direction of the fibres of the U-shaped system was parallel to the longitudinal direction of the beam.
5.4 Finite element model

5.4.1 Material modelling

For some of the analyses, the formulations for the concrete, steel, and FRP of the ADINA, Version 8.4 software package are employed. They are described in detail in the ADINA theory and modelling guide [ADINA, 2004b], and are briefly summarized below. As described below, other simulations are based on the microplane model for concrete.

5.4.1.1 Concrete constitutive laws

As mentioned above, two different concrete models are employed. The first is the microplane concrete model detailed in Chapter 3, while the second is the phenomenological concrete model available in the ADINA software. The characteristics of the latter constitutive law are presented in more detail in Appendix B. A brief recapitulation of its main features is presented here.

A hypoelastic constitutive model is used to describe the nonlinear stress-strain relationship for the concrete. The general multiaxial stress-strain relations are derived from the nonlinear uniaxial stress-strain curve. A compressive uniaxial nonlinear relationship is used until the maximum concrete characteristic strength, $f'_c$, is reached beyond which the behaviour softens until the concrete crushes. The ultimate uniaxial compressive stress, $\sigma_u$, is taken as $0.85 f'_c$, and the ultimate uniaxial compressive strain, $\varepsilon_u$, is assumed to be 0.0035. In cases where the values of the concrete tensile strength, $f_t$, and the initial modulus of elasticity, $E_c$, are not reported, they are evaluated according to the CSA-A23.3 [2004] (Equation 2.3 and Equation 2.4 in Chapter 2). Poisson’s ratio, $\nu$, is taken as 0.18 for the concrete.

Failure envelopes are employed to establish the uniaxial stress-strain law accounting for multiaxial stress conditions, and to identify whether tensile or crushing failures of the concrete have occurred. The behaviour of the cracked concrete is described assuming a system of orthogonal cracks. Once a crack occurs in any direction, $\tau$, the material is considered orthotropic with the directions of orthotropy being defined by the principal
stress directions. Cracking of the concrete occurs when the principal tensile stress lies outside the tensile failure envelope. The elastic modulus of the concrete is reduced to zero in the direction parallel to the principal tensile stress direction and then a redistribution of stresses takes place. Once cracking occurs, the shear retention factor is decreased linearly from 1.0 for the uncracked section to 0.5 for cracked sections.

5.4.1.2 Constitutive models for reinforcement and FRP laminates

The steel reinforcement is modelled as a bilinear elastic-plastic material, with a tangent modulus in the strain-hardening regime taken to be one-hundredth of the elastic modulus. Although a bond-slip relation is considered for the FRP/concrete interfacial behaviour, as described below, a full-bond is assumed between the concrete and steel reinforcement. This is adopted since the tension stiffening model of the concrete can implicitly simulate the reduction of the structure stiffness arising from the relative slip between steel and concrete. A linear elastic relationship until rupture is assumed for the FRP composites. In the case of the 3-D analysis, the formulations of the orthotropic constitutive law presented in Subsubsection 4.3.3.3 are employed for the FRP laminates.

5.4.1.3 FRP/concrete interface models

The interfacial behaviour is modelled using bond-slip equations relating the local shear stress, \( \tau \), and the associated slip, \( s \), between the FRP laminate and concrete substrate. Three different relations are employed to simulate the interface; namely, the nonlinear and bilinear bond-slip models developed by Lu et al. [2005b] (Figure 5.2), and the new model proposed in Chapter 4. Lu et al. [2005b] derived the bond-slip relations for the most common types of adhesives having stiffnesses within ranges similar to those for the simulated beams. In the FE simulations, the characteristics of the adhesive layers are assumed to be within those of normal adhesive layers. Accordingly, the properties of the adhesive are implicitly considered in our modelling. The two models of Lu et al. [2005b] are detailed in Appendix C.

We summarize the values of \( \tau_{\text{max}} \), \( S_0 \), \( G^f_j \) and \( S_{\text{max}} \) in the Lu et al. [2005b] model and that proposed in the previous chapter in Table 5.2 and Table 5.3, respectively, for all the
specimens analyzed. The results obtained using the aforementioned three models will be compared to those based on the full-bond assumption.

### 5.4.2 Discrete interface model

Using continuous interface elements between the FRP and concrete nodes can successfully capture the debonding load. However, the continuity of the interpolation function of these elements prevents the simulation of discontinuities due to cracks. As a result, these elements cannot capture the stress concentrations and fluctuations at the vicinity of cracks. To overcome this problem, a discrete interface element in the form of a 2-node truss element is proposed to represent the interfacial behaviour. As a result of the discontinuities in a series of discrete elements, each truss element can carry positive or negative stresses according to its relative location from the nearest crack. The relative displacement in these truss elements represents the interfacial slip and the axial stress represents the interfacial shear stress, as shown in Figure 5.3. The length of each truss element, in general, is taken as 12.45 mm representing essentially the distance between two adjacent FRP nodes. The area of each truss interface element is $b_f \times l_f$, where $l_f$ is the distance between each two adjacent FRP nodes (Figure 5.3) (equal 12.5 mm), and $b_f$ is the width of the FRP. The constraint equation and all details concerning the implementation of this new technique was previously illustrated in Subsubsection 4.6.2.1 and can also be found in Abdel Baky et al. [2005, 2007].

The distribution of the discrete interface elements along the interface is shown in Figure 5.3. The sensitivity of the numerical results to the length of each truss element
### Table 5.2: Characteristic values in the bond–slip model of Lu et al. [2005b]

<table>
<thead>
<tr>
<th>Beam set</th>
<th>Beam ID</th>
<th>$f_c$ (MPa)</th>
<th>$\tau_{\text{max}}$ (MPa)</th>
<th>$S_0$ (mm)</th>
<th>$G_f$ (N/mm)</th>
<th>$S_{\text{max}}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brena et al. [2003]</td>
<td>A1, A2, A3</td>
<td>35.1</td>
<td>6.17</td>
<td>0.08</td>
<td>0.78</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>A4</td>
<td>37.2</td>
<td>5.51</td>
<td>0.07</td>
<td>0.59</td>
<td>0.22</td>
</tr>
<tr>
<td>Ross et al. [1999]</td>
<td>B1-B6</td>
<td>54.8</td>
<td>4.97</td>
<td>0.06</td>
<td>0.36</td>
<td>0.15</td>
</tr>
<tr>
<td>Kaminska and Kotynia [2000]</td>
<td>B-08/S</td>
<td>38.5</td>
<td>5.48</td>
<td>0.07</td>
<td>0.57</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>BF-04/0.5S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BF-06/S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ritchie et al. [1991]</td>
<td>E, G, M</td>
<td>35.9</td>
<td>4.02</td>
<td>0.05</td>
<td>0.32</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>F, L</td>
<td>33.1</td>
<td>3.86</td>
<td>0.05</td>
<td>0.32</td>
<td>0.16</td>
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<tr>
<td>M'Bazaa [1995]</td>
<td>$P_0$</td>
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<td>5.58</td>
<td>0.07</td>
<td>0.53</td>
<td>0.19</td>
</tr>
<tr>
<td>Chicoine [1997]</td>
<td>$P_0$, $P_1$, $P_3$</td>
<td>44.3</td>
<td>4.93</td>
<td>0.06</td>
<td>0.42</td>
<td>0.17</td>
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<tr>
<td>Kotynia et al. [2008] Series I</td>
<td>B-08S</td>
<td>32.3</td>
<td>4.62</td>
<td>0.060</td>
<td>0.62</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>B-08M</td>
<td>37.3</td>
<td>4.42</td>
<td>0.057</td>
<td>0.41</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>B-08Sm</td>
<td>33.5</td>
<td>5.61</td>
<td>0.073</td>
<td>0.69</td>
<td>0.24</td>
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<tr>
<td></td>
<td>B-08Mm</td>
<td>38.2</td>
<td>4.16</td>
<td>0.054</td>
<td>0.40</td>
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<td></td>
<td>B-08Sk</td>
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<td>5.28</td>
<td>0.069</td>
<td>0.67</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>B-08Mk</td>
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<td>3.91</td>
<td>0.051</td>
<td>0.38</td>
<td>0.20</td>
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<td>0.29</td>
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<tr>
<td></td>
<td>B-08Smb</td>
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<td>3.02</td>
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<td>0.19</td>
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<td>B-08Smb</td>
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<td>3.96</td>
<td>0.051</td>
<td>0.58</td>
<td>0.29</td>
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<tr>
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<td>B0-08Smb</td>
<td>27.4</td>
<td>4.46</td>
<td>0.058</td>
<td>0.61</td>
<td>0.28</td>
</tr>
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<td>Beam set</td>
<td>Beam ID</td>
<td>$\tau_{\text{max}}$ (MPa)</td>
<td>$E_0$ (MPa/mm)</td>
<td>$S_0$ (mm)</td>
<td>$C_f^b$ (N/mm)</td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----------</td>
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<td>------------</td>
<td>----------------</td>
<td></td>
</tr>
<tr>
<td>Brena et al. [2003]</td>
<td>A1, A2, A3</td>
<td>5.23</td>
<td>393.7</td>
<td>0.070</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
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<td>403.2</td>
<td>0.07</td>
<td>0.31</td>
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<td>Ross et al. [1999]</td>
<td>B1-B6</td>
<td>6.367</td>
<td>436.2</td>
<td>0.08</td>
<td>0.43</td>
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<tr>
<td>Kaminska and Kotynia [2000]</td>
<td>B-08/S</td>
<td>6.32</td>
<td>382.6</td>
<td>0.09</td>
<td>0.5</td>
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<tr>
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<td>BF-04/0.5S</td>
<td></td>
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<td>BF-06/S</td>
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<td></td>
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</tr>
<tr>
<td>Ritchie et al. [1991]</td>
<td>E</td>
<td>5.05</td>
<td>79.13</td>
<td>0.39</td>
<td>0.23</td>
<td></td>
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<tr>
<td></td>
<td>F</td>
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<td>43.7</td>
<td>0.76</td>
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<tr>
<td></td>
<td>G</td>
<td>4.88</td>
<td>78.8</td>
<td>0.38</td>
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<tr>
<td></td>
<td>L</td>
<td>4.95</td>
<td>319.4</td>
<td>0.09</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>M</td>
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<td>362.7</td>
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<tr>
<td>M'Bazaa [1995]</td>
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<td>382</td>
<td>0.08</td>
<td>0.4</td>
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<tr>
<td>Chicoine [1997]</td>
<td>$P_0$, $P_1$, $P_3$</td>
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<td>382</td>
<td>0.08</td>
<td>0.4</td>
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<tr>
<td>Kotynia et al. [2008] Series I</td>
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<td>B-08Sm</td>
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<td>368.49</td>
<td>0.083</td>
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<tr>
<td></td>
<td>B-08Mm</td>
<td>5.90</td>
<td>384.19</td>
<td>0.083</td>
<td>0.44</td>
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<tr>
<td></td>
<td>B-08Sk</td>
<td>5.40</td>
<td>369.40</td>
<td>0.078</td>
<td>0.39</td>
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</tr>
<tr>
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<td>B-08Mk</td>
<td>5.48</td>
<td>365.86</td>
<td>0.079</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>Kotynia et al. [2008] Series II</td>
<td>B-083m</td>
<td>4.35</td>
<td>390.33</td>
<td>0.059</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B-083mb</td>
<td>3.81</td>
<td>360.05</td>
<td>0.054</td>
<td>0.22</td>
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</tr>
<tr>
<td></td>
<td>B-08Smb</td>
<td>3.11 (12)</td>
<td>361.45</td>
<td>0.044</td>
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<td>B0-08Smb</td>
<td>3.50</td>
<td>368.66</td>
<td>0.049</td>
<td>0.18</td>
<td></td>
</tr>
</tbody>
</table>
5.4. FINITE ELEMENT MODEL

5.4.3 Structural modelling

In the 2-D finite element simulations, only one half of the beam is modelled due to the geometrical and loading symmetries, while for the 3-D models only one quarter of the beam is analyzed. Figure 5.4 and Figure 5.5 depict, respectively, typical 2-D and 3-D FE meshes.

For the 2-D simulations, the element sizes for the concrete are 50 mm square, except for the part of the concrete beam between the longitudinal tensile steel bars and the FRP laminates; here the element sizes are taken as small as 25 mm square to allow for finer meshing at the FRP/concrete interface. The concrete is modelled using 9-node plane stress elements with two translational degrees of freedom per node in two perpendicular directions in the plane of the beam. Accordingly, the distance between two adjacent concrete nodes is 12.5 mm, which closely corresponds to the average concrete aggregate size. The steel bars and FRP laminates are modelled using 3-node truss elements with two translational degrees of freedom. The FRP nodes are connected to the concrete nodes using separate 2-node truss elements as shown in Figure 5.4(b). The constraint equations are enforced in the longitudinal direction of the beam between the interface and the concrete nodes, and between the interface and the FRP nodes, as detailed in Subsection 5.4.2. The length of each truss element is taken as 12.5 mm representing the distance between two adjacent FRP nodes.

In the 3-D simulations, 8-node brick elements are used to model the concrete. The...
steel reinforcement is modelled using 2-node truss elements while 4-node thin membrane elements are used to represent the FRP sheets and laminates. Three translational degrees of freedom are considered at each node. The interface elements between the FRP and concrete nodes are aligned in the longitudinal direction of the beam (Detail A in Figures 5.5(a) and 5.5(b)), while a full strain compatibility is assumed in the other two directions. The element sizes of the concrete are 50 mm cube, except for the part of the concrete beam between the longitudinal tensile steel bars and the FRP laminates; here the element sizes are taken as small as 12.5 mm cube.

Three-dimensional modelling is adopted primarily to simulate FRP anchorages. A modified 2-D finite element model is also introduced to account for such anchorages in the case of the set of specimens of Chicoine [1997]. In the modified 2-D model, the behaviour of the anchorages is represented by modifying the area of the interface elements at the locations where the transverse sheets are used. This is done by adding the contact area of the side bonded sheets to the interface area. In addition, to account for the shear contribution of the FRP anchorage sheets, an equivalent area of these sheets is added to that of the steel stirrups after considering the modular ratio between the FRP and steel
5.4. FINITE ELEMENT MODEL

(a) Specimen P3 [Chicoine, 1997]: The case of separated sheet

(b) Specimen B-08 Sm [Kotynia et al., 2008]: The case of continuous sheet

(c) Type of elements used

Figure 5.5: Typical 3-D finite element model
reinforcements. In the 2-D FE model, the area of each interface element, in places where the U-wrap is placed, is $b_f l_f + 2 h_f l_f$ where $l_f$ is the distance between each adjacent FRP node (12.5 mm in the current analysis), $b_f$ is the bottom FRP width and $h_f$ is the U-wrap height.

The U-wrap is simulated using 4-node orthotropic thin membrane elements connected to the concrete elements through the discrete 2-node truss elements. The nodes of the FRP elements are connected to those of the concrete elements through interface elements. These elements are aligned in the direction of the fibre; i.e., in the longitudinal beam direction in the case of the bottom FRP laminates (Detail A in Figures 5.5(a) and 5.5(b)) and in the vertical direction of the beam for the case of spaced L-shaped laminates or continuous U-shaped sheets with a fibre orientation being parallel to the beam direction. When using continuous U-shaped sheets with the fibre orientation being perpendicular to the beam axis, the interface elements are aligned in both directions (Detail B in Figures 5.5(a) and 5.5(b)). At the intersection between the U-wrap and the bottom sheets, two layers of membrane elements are used; each layer has a different thickness and material characteristics. These elements simulate the U-wrap and bottom FRP sheets and they are connected together assuming full strain compatibility.

5.5 Numerical results and discussion

The numerical results in the subsequent sections are presented in terms of the load carrying capacities, modes of failure, axial strains in the tension reinforcement and FRP laminates, and load-deflection relationships for the beams. Predicted results are also shown for the shear stress profiles along the FRP/concrete interfaces at the various load levels in order to explain the debonding phenomena. At the end of this section, a particular focus is given to the effect of using FRPs for the flexural strengthening on the crack width and spacing.
5.5. NUMERICAL RESULTS AND DISCUSSION

5.5.1 Ultimate load carrying capacities

The comparisons between the 2-D FE numerical predictions and experimental results for all the specimens, in terms of the ultimate load carrying capacities and modes of failure, are summarized in Table 5.4. This table refers to the numerical results based on the nonlinear bond-slip model developed by Lu et al. [2007]. In this table, the ratio of the numerical-to-experimental load capacity is given for each beam.

As seen from Table 5.4, there is a very good agreement between the 2-D predicted load capacities and the experimental results for all the test specimens. The average numerical-to-experimental load ratios are 0.98, 1.04, 1.00, 1.1, 1.02, and 1.01 for the beam sets tested by Brena et al. [2003], Ross et al. [1999], Kaminska and Kotynia [2000], Ritchie et al. [1991], M'Bazaa [1995] and Chicoine [1997], respectively. This ratio is 0.99 when considering all of the 25 test specimens together; the corresponding standard deviation is 0.06, which indicates an excellent agreement between the numerical and the experimental results. This also indicates that the 2-D simulations provide simplicity and at the same time an acceptable level of accuracy. This is acceptable as long as the discrepancies between the model predictions and experimental results are not very noticeable, and when the numerical results fall within the range of the normal experimental scatter.

For the specimens tested by Chicoine [1997], we do not observe significant discrepancies, neither in terms of the ultimate load carrying capacities nor the load-deflection relationships, between the results predicted using the 2-D and 3-D FE models. However, differences occur in terms of the interfacial shear stresses, as will be presented later.

Regarding the 3-D predictions, Table 5.5 summarizes the predicted load capacities, \( P_{num} \), and FRP strains at ultimate loads, \( \varepsilon_{num} \), for the various specimens tested by Kotynia et al. [2008]. Comparisons between the numerical and experimental results are also given in this table. The average numerical-to-experimental load capacity ratio and its standard deviation are 0.998 and 0.028, respectively, indicating an excellent agreement.
### Table 5.4: Comparison between the numerical and experimental results in the 2-D simulations

<table>
<thead>
<tr>
<th>Beam set</th>
<th>Beam ID</th>
<th>Failure Load (kN)</th>
<th>Num./Exp.</th>
<th>Failure mode&lt;sup&gt;2&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Brena et al.</strong></td>
<td>Control</td>
<td>130.8</td>
<td>126.0</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>119.7</td>
<td>120.0</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>125.9</td>
<td>120.0</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>138.3</td>
<td>138.0</td>
<td>0.99</td>
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<tr>
<td></td>
<td>A4</td>
<td>129.0</td>
<td>132.0</td>
<td>1.02</td>
</tr>
<tr>
<td><strong>Ross et al.</strong></td>
<td>B1</td>
<td>80.1</td>
<td>80.0</td>
<td>1.00</td>
</tr>
<tr>
<td>[1999]</td>
<td>B2</td>
<td>97.9</td>
<td>100.8</td>
<td>1.03</td>
</tr>
<tr>
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<td>109.0</td>
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<td>107.6</td>
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<td>B6</td>
<td>169.1</td>
<td>162.0</td>
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<td>180.0</td>
<td>178.0</td>
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<td>G</td>
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<td>108.0</td>
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<td>M</td>
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<td>P&lt;sub&gt;3&lt;/sub&gt;</td>
<td>171.0</td>
<td>172.8</td>
<td>1.01</td>
</tr>
</tbody>
</table>

<sup>1</sup> Num./Exp. is the ratio of the value of the failure load calculated from numerical to that measured from experimental.

<sup>2</sup> ED indicates an end-plate debonding failure mode, IC indicates an intermediate crack debonding failure mode, C is a concrete crushing failure mode, and R indicate FRP rupture.
Table 5.5: Comparison between the numerical and experimental results in the 3-D simulations

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>$\tau_{\text{max}}$ (MPa)</th>
<th>$S_0$ (mm)</th>
<th>$P_{\text{num.}}$ (kN)</th>
<th>$P_{\text{num.}}/P_{\text{exp.}}$</th>
<th>$\varepsilon_{\text{num.}}$ (%)</th>
<th>$\varepsilon_{\text{num.}}/\varepsilon_{\text{exp.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-08S</td>
<td>4.62</td>
<td>0.060</td>
<td>93.7</td>
<td>0.98</td>
<td>0.665</td>
<td>1.08</td>
</tr>
<tr>
<td>B-08M</td>
<td>4.42</td>
<td>0.057</td>
<td>139.5</td>
<td>1.00</td>
<td>0.568</td>
<td>1.12</td>
</tr>
<tr>
<td>B-08Sm</td>
<td>5.61</td>
<td>0.073</td>
<td>105.4</td>
<td>1.03</td>
<td>0.703</td>
<td>1.07</td>
</tr>
<tr>
<td>B-08Mm</td>
<td>4.16</td>
<td>0.054</td>
<td>153.2</td>
<td>1.00</td>
<td>0.631</td>
<td>1.15</td>
</tr>
<tr>
<td>B-08Sk</td>
<td>5.28</td>
<td>0.069</td>
<td>100.3</td>
<td>0.98</td>
<td>0.809</td>
<td>0.94</td>
</tr>
<tr>
<td>B-08Mk</td>
<td>3.91</td>
<td>0.051</td>
<td>157.2</td>
<td>1.05</td>
<td>0.790</td>
<td>1.40</td>
</tr>
<tr>
<td>Series II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-083m</td>
<td>3.24</td>
<td>0.042</td>
<td>89.9</td>
<td>0.98</td>
<td>0.665</td>
<td>0.98</td>
</tr>
<tr>
<td>B-083mb</td>
<td>3.02</td>
<td>0.094</td>
<td>119.3</td>
<td>0.97</td>
<td>0.784</td>
<td>0.93</td>
</tr>
<tr>
<td>B-085mb</td>
<td>3.96</td>
<td>0.051</td>
<td>113.8</td>
<td>1.00</td>
<td>0.788</td>
<td>1.03</td>
</tr>
<tr>
<td>B0-085mb</td>
<td>4.46</td>
<td>0.058</td>
<td>106.0</td>
<td>0.96</td>
<td>0.792</td>
<td>1.26</td>
</tr>
</tbody>
</table>

$P_{\text{num.}}$ and $\varepsilon_{\text{num.}}$ are the numerical ultimate load and strain in the CFRP at ultimate load.

5.5.2 Load–deflection relationships

The results presented in Figures 5.6(a) to 5.6(d) refer to the comparisons between the 2-D predictions and experimental results for the beams tested by Ritchie et al. [1991], M'Bazaa [1995], Chicoine [1997] and Brena et al. [2003], respectively. The comparisons are made in terms of the load–deflection relationships for both the FRP-strengthened and the unstrengthened specimens (referred to in these figures as “Control”).

In the analysis, progressive debonding is modelled by deactivating the debonded FRP elements as the analysis continues, until a complete debonding occurs followed by the crushing of the concrete. This illustrates the ability of our FE models to capture not only the debonding load, but also the complete post-peak plateaus, as seen in Figure 5.6.

The numerical-to-experimental comparisons of the load–deflection relationships of selected specimens from both Series I and II of the Kotynia et al. [2008] set of specimens are plotted in Figures 5.7(a) and 5.7(b), respectively. It can be seen that the finite element models are capable of simulating the entire load–deflection relationships, including the descending parts. Very good agreements are obtained, as seen in Figures 5.7(a) and 5.7(b) between the results of the numerical predictions and those observed experimentally.
Figure 5.6: Load–deflection relationships
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Figure 5.7: Comparison between the 3-D numerical predictions and experimental load-deflection relationships of Kotynia et al. [2008]

5.5.3 Axial strains in the reinforcement steel bars and FRP laminates

The numerical-to-experimental comparisons, in terms of the load–strain relationships in the steel reinforcement, are shown in Figures 5.8(a) and 5.8(b) for two selected beams of Series I and II of the Kotynia et al. [2008] specimens, respectively. These show a very good agreement between the predictions and the experimental findings. The entire load–strain behaviour is captured in these figures, including the post-debonding behaviour as is the case for the specimen B-083mb.

Equation 5.1 below is a design rule to limit the ultimate strain in FRP laminates to mitigate debonding [ACI-440.2R-02, 2002]:

\[ \varepsilon_{ub} = k_m \varepsilon_{fu} \]  

(5.1)

Here \( \varepsilon_{fu} \) is the ultimate strain of the FRP and \( \varepsilon_{ub} \) represents the strain limitation to prevent debonding. The reduction factor, \( k_m \), is given in Equation 2.13 in Chapter 2.

A comparison between the reduction factors calculated according to Equation 5.1 and those predicted from our analysis is given in Table 5.6. Here we see that the ACI equation accurately predicts the strain limits for relatively long FRP laminates (greater than 0.75 of the beam span) and for steel reinforcement ratios that exceed 0.35%. The factor \( k_m \)
CHAPTER 5. NONLINEAR FEA OF FRP-STRENGTHENED REINFORCED CONCRETE BEAMS

Figure 5.8: Numerical and experimental the load-strain relationships

![Graph of load vs. strain for steel reinforcement and bottom FRP](image)

**Table 5.6:** Reduction factor for the strain in the FRP laminates to prevent debonding according to the ACI-440.2R-02 [2002] and FE analysis

<table>
<thead>
<tr>
<th>Beam set</th>
<th>Beam ID</th>
<th>Length ratio</th>
<th>Reinf. ratio</th>
<th>$E$ (GPa)</th>
<th>$t$ (mm)</th>
<th>strain at FRP laminates at debonding</th>
<th>Reduction factor, $k_m = \varepsilon_{ub}/\varepsilon_{ub}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brena</td>
<td>A1</td>
<td>0.40</td>
<td>0.56</td>
<td>230.0</td>
<td>0.33</td>
<td>0.82</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>A2</td>
<td>0.47</td>
<td>0.56</td>
<td>230.0</td>
<td>0.33</td>
<td>0.78</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>A3</td>
<td>0.78</td>
<td>0.56</td>
<td>230.0</td>
<td>0.33</td>
<td>1.20</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>A4</td>
<td>0.49</td>
<td>0.56</td>
<td>230.0</td>
<td>0.165</td>
<td>0.82</td>
<td>0.51</td>
</tr>
<tr>
<td>Ross</td>
<td>B1</td>
<td>1.00</td>
<td>0.46</td>
<td>138.0</td>
<td>0.45</td>
<td>1.14</td>
<td>0.72</td>
</tr>
<tr>
<td>Ritchie</td>
<td>F</td>
<td>0.80</td>
<td>0.67</td>
<td>11.7</td>
<td>9.50</td>
<td>0.99</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>1.00</td>
<td>0.67</td>
<td>118.0</td>
<td>1.27</td>
<td>1.11</td>
<td>0.88</td>
</tr>
<tr>
<td>Kamińska and Kotynia</td>
<td>B-08/S</td>
<td>0.90</td>
<td>0.35</td>
<td>171.5</td>
<td>1.20</td>
<td>0.80</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>BF-04/0.5S</td>
<td>0.90</td>
<td>0.35</td>
<td>171.5</td>
<td>1.20</td>
<td>0.85</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>BF-06/S</td>
<td>0.90</td>
<td>0.75</td>
<td>171.5</td>
<td>1.20</td>
<td>0.85</td>
<td>0.50</td>
</tr>
<tr>
<td>Chicoine</td>
<td>P9</td>
<td>0.97</td>
<td>0.26</td>
<td>82.0</td>
<td>0.90</td>
<td>0.67</td>
<td>0.40</td>
</tr>
</tbody>
</table>

$^1$ Length ratio defined as the ratio between FRP laminates length and beam span
in Equation 5.1 has been modified in the new ACI specifications [ACI-440.2R-07, 2007]; however, this design equation still ignores the effect of the steel reinforcement ratio and the length of the FRP laminates on the debonding strain.

5.5.4 Modes of failure

For all the test specimens (2-D and 3-D simulations), the finite element analyses accurately predict the mode of failure observed in the experiments, except for the specimens F and G in the Ritchie et al. [1991] set of specimens. Although the analysis does not simulate the failure mode in these particular specimens, it accurately predicts their ultimate load carrying capacities.

Figure 5.9: Failure modes of selected beam specimens Kotynia et al. [2008]

In the Ross beam specimens, it is observed that by increasing the steel reinforcement ratio, the mode of failure is altered from debonding for the small ratios (e.g., Specimen B1) to flexural failure for the high ratios (e.g., Specimen B6). The FE models are also able to capture the intermediate crack debonding failure modes, such as those observed for the specimens tested by Kaminska and Kotynia [2000]. In conclusion, using our finite element
models, the various modes of failure (FRP rupture, plate-end debonding, and intermediate crack debonding) can successfully be simulated. The 3-D predictions precisely captured the debonding mode of failure when using FRP anchorages as in the case of Kotynia et al. [2008] specimens.

In Figures 5.9(a) to 5.9(d), we show four beams of the set of Kotynia et al. [2008] as an example to explain the debonding mechanism by investigation of the interfacial shear stress distribution along the interface in the next section. The modes of failure are characterized by intermediate crack debonding for all these specimens.

### 5.5.5 Interfacial shear stresses distributions along the FRP/concrete interface

A great advantage of reliable finite element models is their ability to simulate effects that are virtually impossible to be measured experimentally, such as the stress/slip distributions and concentrations along the FRP/concrete interface. Such quantities are necessary for a better understanding of the debonding failure modes and the bond mechanisms involved. In addition, they can provide insight into the effects of micro and macro flexural cracks on the interfacial behaviour. The results of this section can be found in Abdel Baky et al. [2007] and Kotynia et al. [2008].

In Figure 5.10(a), the shear stress distributions along the interface for the specimen A3 tested by Brena et al. [2003] are shown for load levels prior to and at cracking. With an increase of the applied load up to the cracking load, the interfacial shear stress increases progressively along the bonded plate, with an abrupt increase at the plate end. At this stage of loading (prior to cracking) the interfacial shear stress distribution is similar to that obtained from direct shear tests [Ebead and Neale, 2007] or that predicted from the elastic analysis of FRP-strengthened beams [Tounsi and Benyoucef, 2007]. This interfacial behaviour before cracking is typical for all the FRP-strengthened beams. At the cracking load, higher fluctuations of the interfacial stress values are observed at the cracked sections, yet these are still lower than those at the plate end. However, as seen in Figure 5.10(b), with an increase of the applied load up to the yield load, the flexural cracks tend to open causing a shift of the maximum values of the bond stresses from the plate end to the mid-span. The fluctuations of the stress distribution become more noticeable when
5.5. NUMERICAL RESULTS AND DISCUSSION

approaching the yield load (Figure 5.10(b)).

![Diagram showing interfacial shear stress distributions](image)

**Figure 5.10**: Interfacial shear stress distributions for the specimen A3 Brena et al. [2003]

Figures 5.11(a) and 5.11(b) show the distinction between the interfacial shear stress distributions for short and long bonded lengths, respectively. For shorter FRP laminate lengths (Specimen A1 tested by Brena et al. [2003], Figure 5.11(a)) where the FRP laminates do not entirely cover the cracked sections, the interfacial shear stresses at the plate end merge with those at the shear cracks near the FRP laminate end. This causes the interfacial shear stresses to exceed the interfacial shear strength; as a result, the debonding occurs at the flexural cracks closest to the plate end. When the length of the FRP laminate is extended beyond the cracked sections (Specimen P_0 tested by M'Bazaa [1995], Figure 5.11(b)) the interfacial shear stresses dramatically increase at the flexural cracks near the concentrated load and at the shear cracks near the plate end. As a result, the maximum stress concentrations occur over the cracked sections causing intermediate crack debonding. The maximum shear stresses increase to 4.1 MPa at a point within the shear span. The associated slip value is 0.085 mm; this is slightly greater than the parameter S_0 (Table 5.2) in the bond–slip relationship for this specimen. Accordingly, the interfacial stress state at this point is at the beginning of the descending branch of the bond–slip model. All other stresses beyond this value are in the ascending branch, s < S_0. This means that there is no significant difference between using the bilinear or nonlinear bond–slip model as far as the prediction of the ultimate load carrying capacities is concerned. This explains the similarity of the predicted ultimate capacities of specimens in Subsection 5.7.1 when using various bond–slip models. The analysis also suggests that, in order
to prevent the end-plate debonding failure mode, the FRP laminates should be extended beyond the uncracked sections. We observed that plate-end debonding occurs due to the high interfacial shear stress concentrations arising from the wideness of the shear or flexural cracks around the plate end. This contradicts the common belief that the plate-end debonding results from an elastic interfacial shear stresses.

In the FE analysis, it is observed that the debonding initiates when the first interface element reaches $\tau_{\text{max}}$ at a certain time step. In the subsequent time step, the slip value in this particular interface element tends to a large value as a result of increasing the crack width passing through this element. When this occurs, the particular interface element is deactivated. Accordingly, the final debonding happens when all the interface elements cannot carry more slip due to the widening of the cracks. Thereon the entire FRP and interface elements are detached. In general, at the final debonding most interface elements are still in the ascending branch of the bond-slip model while few elements are in the beginning of the descending part. This explains the brittle nature of the debonding mode of failure in that the softening branch of the bond-slip model does not affect the results.

To address the effect of the steel reinforcement ratio, $\rho$, on the interfacial stress profiles, Table 5.7 gives a summary of the peak values of the interfacial shear stresses along the interface, $\tau_p$, at different load levels for the six specimens tested by Ross et al. [1999]. It is clear from Table 5.7 that as $\rho$ increases $\tau_p$ decreases. For example, at the load level of
### 5.5. NUMERICAL RESULTS AND DISCUSSION

#### Table 5.7: Peak values of interfacial shear stress along the interface for the beams tested by Ross et al. [1999]

<table>
<thead>
<tr>
<th>Beam ID</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>B4</th>
<th>B5</th>
<th>B6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reinf. Ratio, $\rho$, %</td>
<td>0.46</td>
<td>0.83</td>
<td>1.24</td>
<td>1.83</td>
<td>2.5</td>
<td>3.3</td>
</tr>
<tr>
<td>Failure mode</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Failure load (kN)</td>
<td>80.0</td>
<td>100.8</td>
<td>119.6</td>
<td>121.3</td>
<td>147.5</td>
<td>162</td>
</tr>
<tr>
<td>$\tau_p$, MPa (P= 20 kN)</td>
<td>0.10</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>$\tau_p$, MPa (P= 40 kN)</td>
<td>4.46</td>
<td>4.21</td>
<td>2.8</td>
<td>2.33</td>
<td>2.02</td>
<td>N/A</td>
</tr>
<tr>
<td>$\tau_p$, MPa (P= 60 kN)</td>
<td>4.83</td>
<td>4.25</td>
<td>3.75</td>
<td>3.5</td>
<td>3.01</td>
<td>2.83</td>
</tr>
<tr>
<td>$\tau_p$, MPa (P= 80 kN)</td>
<td>4.97</td>
<td>4.34</td>
<td>3.65</td>
<td>3.31</td>
<td>3.07</td>
<td>2.94</td>
</tr>
<tr>
<td>$\tau_p$, MPa (P= 100 kN)</td>
<td>N/A</td>
<td>4.92</td>
<td>4.15</td>
<td>3.93</td>
<td>3.44</td>
<td>3.12</td>
</tr>
<tr>
<td>$\tau_p$, MPa (P= 120 kN)</td>
<td>N/A</td>
<td>N/A</td>
<td>4.82</td>
<td>4.2</td>
<td>3.6</td>
<td>3.4</td>
</tr>
<tr>
<td>$\tau_p$, MPa (P= 140 kN)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>3.82</td>
<td>3.35</td>
</tr>
<tr>
<td>$\tau_p$, MPa (P= 160 kN)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>3.37</td>
</tr>
</tbody>
</table>

P denotes the applied load level.

At 80 kN, $\tau_p$ reduces from 4.97 MPa ($\rho = 0.46\%$) to 2.94 MPa ($\rho = 3.3\%$). This explains the change of the mode of failure from an intermediate crack debonding (Specimen B1) to a concrete crushing (Specimen B6) with an increase in the steel reinforcement ratio.

#### 5.5.5.1 Influence of the FRP anchorage sheets

The effect of using end-plate FRP anchorages on the interfacial stress distribution is depicted in Figure 5.12 to Figure 5.14. In Figures 5.12(a) and 5.12(b), we present the predicted interfacial shear stress profiles prior to cracking for the specimen $P_3$ tested by Chicoine [1997], from the 2-D and 3-D FE models, respectively. The shear stress distributions prior to cracking observed from the 2-D and 3-D models have some discrepancies at the locations of the concentrated load or around the centre of the beam. The interfacial stress profiles at failure for the specimen $P_3$ are depicted in Figures 5.13(a), and 5.13(b), respectively, for the 2-D and 3-D models. Although the two profiles are not identical in terms of shear stress distributions, the two profiles have almost the same maximum interfacial stresses along the interface (3.95 and 4.1 MPa for the 2-D and 3-D analyses,
respectively). Accordingly, the assumption used in the 2-D FE analysis to represent the FRP anchorage sheets is acceptable as long as the model predicts the actual mode of failure and approximately the same maximum interfacial stress as that predicted using the 3-D analysis. In the predicted interfacial shear stress profiles, either prior to cracking (Figure 5.12,) or at failure (Figure 5.13), we observe zones where the interfacial stresses decrease relative to those in the adjacent zones. This suggests that, in general, the addition of transverse FRP anchorage strips is quite effective for mitigating debonding failures in these regions.

Figure 5.12: Interfacial shear stress distributions along the interface prior to the cracking load for the specimen P3 Chicoine [1997]

The numerical results presented in Figure 5.14 refer to the stress distribution at failure for the specimen P1 investigated by Chicoine [1997]. It is obvious that the stress distribution is dramatically decreased at the anchorage zone, as one would expect, as a result of reducing the crack width at the location of the end-anchorage sheet. Along the interface and away from the end-anchorage sheet, the flexural cracks control the interfacial behaviour. At failure, as shown in Figure 5.14, the maximum interfacial shear stress occurs at the beginning of the FRP anchorage sheet, with a value of 4.39 MPa associated with a slip of 0.07 mm, causing the debonding to initiate and to propagate towards the mid-span. Points “a” and “b” in Figure 5.13 to Figure 5.14 refer to the horizontal locations of the load application point and the termination of the FRP flexural laminates, respectively; while point “c” in the same figures and in Figure 5.15 indicates the termination of the additional U-shaped strengthening system.
5.5. NUMERICAL RESULTS AND DISCUSSION

As far as the case of the FRP anchorage sheets applied away from the plate end is concerned, the interfacial shear stress profiles at failure are presented, in Figures 5.15(a)–5.15(d), for the different FRP-strengthening configurations of the Kotynia et al. [2008] set of specimens. Figure 5.15(a) depicts the results for the specimen B-08S (a specimen without any additional lateral system). Here we observe that the shear stresses increase from the mid-span to a value of 4.3 MPa at point “a”. The associated slip value is 0.067 mm; this is just slightly greater than the value of $S_0$ (Table 5.3) in the bond–slip relationship for this specimen. All other stresses beyond point “a” are in the ascending branch $s \leq S_0$. This profile suggests that debonding should initiate near the loading point “a”, and then propagate towards the end of the laminate where a high interfacial shear stress occurs at point “b” (3.5 MPa); i.e., that the failure mode is intermediate crack debonding. This is precisely what was observed experimentally (Figure 5.9(a)).

The predicted effect on the interfacial shear stress distribution of using a continuous U-shaped system with the fibres oriented perpendicular to the beam axis is shown in Figure 5.13(b) for the specimen B-08Sm. Here we observe that the interfacial shear stresses are diminished in the anchorage zone (from mid-span to point “c”). The maximum interfacial shear occurs at point “c”, indicating that debonding should initiate in the vicinity of this point. The interfacial shear stress profiles depicted in Figure 5.15(c) correspond to the specimen B-08Sk where intermediate L-shaped transverse sheets are used as an additional strengthening. In the predicted interfacial shear stress profiles, we observe oscillations.
Figure 5.14: Interfacial shear stress distributions along the interface at failure load for the specimen $P_1$ Chicoine [1997]

Figure 5.15: Predicted interfacial shear stress profiles at ultimate load for Kotynia et al. [2008] specimens
such that the shear values are minimum at each transverse FRP system and maximum in the intermediate zones between the L-shaped laminates. This result suggests that debonding should initiate locally in one of these intermediate zones, which is indeed what was observed experimentally (Figure 5.15(d)). Finally, in Figure 5.15(d) the predicted interfacial shear stress profile for the specimen B–083mb is given. Here the fibre orientation of the U-wrap was parallel to the beam axis. We observe that the interfacial shear stresses reach maximum values in the central region of the specimen, within the additional U-shaped strengthened distance. This result is consistent with the experimentally observed mode of failure (Figure 5.9(b)), where debonding initiated at the side bonded sheets and then propagated along the flexural reinforcement.

5.5.6 Cracking behaviour

In this section, a brief study is carried out to investigate the influence of attaching FRP sheets to the beam tension side on the values of crack width and spacing. Estimating the crack width of the FRP-strengthened beams can be advantageous for a better understanding of the flexural behaviour of such beams. In this section, we assess the capability of our FE models to predict the cracking behaviour of both unstrengthened and FRP-strengthened concrete beams.

The crack width value predicted in our simulations is twice the value of the interfacial slip drop at the locations of cracks, as shown in Figure 5.16. The vertical drop in the slip values in Figure 5.16 represents half the value of the crack width, $w/2$. Similarly, the horizontal distance between two adjacent peak slip or stress values represents the crack spacing, as shown in Figure 5.16 and also in Figure 5.10 to Figure 5.15.

The predicted values of the maximum crack width of the unstrengthened beam tested by M'Bazaa [1995] are compared to those computed using the Frosch [1999] model that is expressed as follows:

$$w = \frac{2f_s}{E_{sl}} \beta \sqrt{d_c^2 + \left(\frac{s}{2}\right)^2}$$

(5.2)

where $w$ is the crack width, $f_s$ is the steel reinforcement stress, $E_{sl}$ denotes the Young’s modulus of the steel reinforcement bars, $d_c$ is the concrete cover thickness, $s$ is the bar spacing, and $\beta = 1 + 0.0031d_c$. 

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Figure 5.17 shows a comparison between the predicted values for the maximum crack width using our predictions and those computed using the Frosch [1999] model for the unstrengthened beam specimen tested by M'Bazaa [1995]. The experimental axial stress values in the reinforcement steel bars, $f_a$, are used in the calculation of the maximum crack width using Equation 5.2. The Frosch [1999] model neglects the effect of concrete tensile strain; it is derived assuming that the axial strains in the steel reinforcement bars are uniformly distributed over the crack spacing (the same assumption used in the smeared crack approach). This explains the agreement observed in Figure 5.17(a) between our predictions and that of the Frosch [1999] model.

The comparison between the maximum crack width in the FRP-strengthened and control beams tested by M'Bazaa [1995] is presented in Figure 5.17(b). Generally, the crack widths appearing at the beam tension surface of the FRP-strengthened beam are narrower than those of the unstrengthened beam when subjected to the same load. In this prediction, the maximum crack widths at the yielding load for the unstrengthened and strengthened beams are approximately 0.06 mm and 0.039 mm, respectively. The crack widths at the load level just prior to the failure load are 0.52 mm and 0.33 mm for the unstrengthened and FRP-strengthened beams, respectively.

The crack spacing ranges from 75.0 mm to 125.0 mm for the unstrengthened beam, which corresponds to 1.7 to 2.8 times the concrete cover. However, for the strengthened beam, the spacing between the cracks ranges from 37.5 mm to 100.0 mm, which is equiv-
5.6. NUMERICAL ASPECTS OF FE SIMULATIONS

(a) Comparison between the FE predictions and that of the Frosch [1999] model

(b) Effect of the FRP-strengthening on the maximum crack width

Figure 5.17: Maximum crack width for beam specimens tested by M'Bazaa [1995]

alent to 0.85 to 2.2 times the concrete cover. Further research is still required to obtain a refined crack width model for FRP-strengthened beams. The steel reinforcement ratio, FRP and concrete characteristics should be considered in developing such the crack width model.

5.6 Numerical aspects of FE simulations

Various aspects concerning the numerical simulations of FRP-strengthened beams are discussed. Four of these are investigated in this study; namely, mesh size, discrete interface element length, interfacial fracture energy, $G^b$, of the bond–slip model, and the concrete fracture energy per unit area, $G_f$.

Mesh size is an essential aspect for the reliable numerical simulation of reinforced concrete structures. Various studies have revealed that, with a smeared crack model to simulate the cracking behaviour of concrete, convergence problems arise and the predicted results are not “objective” as they significantly depend on the mesh size [Shayanfar et al., 1997; Bažant and Planas, 1998]. The problem of mesh-dependency is related to the use of a constant unit fracture energy, $G_f$, for all discrete elements disregarding the fact that small elements require a higher fracture energy for crack initiation than do large elements.
In the ADINA software package, to obtain a mesh independent solution, the concrete fracture energy per unit area, \( G_f \), (area under the tensile stress-strain curve) is used to overwrite the value of \( \xi \) (tension stiffening factor) at each integration point, based on the size of the finite element as follows:

\[
\xi = \frac{2E_cG_f}{f_t^2h}
\]

where \( h \) is the equivalent width of the finite element perpendicular to the plane of tensile cracks. This concept has been adopted in several studies to overcome the effect of the mesh size on the predicted results [Shayanfar et al., 1997; Lu et al., 2007]. Bazant and Planas [1998] recommend the use of an element size within the range of 1 to 3 times the average aggregate size in their crack band width model. Accordingly, in all the previous FE simulations, the element sizes in the tension zones were taken approximately equal to the average aggregate size of the concrete.

Another numerical aspect to be considered is the length of each truss element representing the interface; in other words, the gap between two adjacent interface nodes (Figure 5.3). Two attempts have been investigated; one using a truss interface element of a length of 1.0 mm and another with a length of 12.5 mm (i.e., the distance between two adjacent nodes). No significant discrepancies were observed between the two cases in terms of the ultimate capacity. It was discovered that, with the discrete interface elements, the interfacial stiffness has a minimal influence on the overall structural stiffness.

The interfacial fracture energy of the FRP/concrete interface \( (G_{bf}) \) and the concrete fracture energy \( (G_f) \) are other aspects included in the investigation. They are addressed by considering the FRP-strengthened beam \( P_0 \) tested by M'Bazaa [1995]. The numerical aspect for the interfacial fracture energy is investigated by changing the order of the descending branch, \( \alpha \), of the bond-slip model of Lu et al. [2005b] from 0.1 to 10 (Figure 5.18(a)). The concrete fracture energy is also addressed by changing the associated tension stiffening factor, \( \xi \), from 1 to 20 (Figure 5.18(b)). This corresponds to a fracture energy of \( \frac{\xi hf_t^2}{2E_c} \) (Equation 5.3).

The influence of the interfacial fracture energy on the load-deflection relationships and ultimate capacities is depicted in Figures 5.19(a) and 5.19(b). Reducing the interfacial fracture energy has a modest effect on the ultimate capacities of the strengthened beams; however, the failure takes place at a slightly lower load. Decreasing the interfacial fracture
5.6. NUMERICAL ASPECTS OF FE SIMULATIONS

energy from 4.45 N/mm to 0.32 N/mm (corresponding to \( \alpha \) varying from 0.1 to 10), reduces the ultimate capacity by 17.0%. It is of interest to mention that the increase in the predicted debonding load when using the largest interfacial fracture energy (17.0%) is similar to the predictions assuming a full bond between the FRP and concrete nodes, as previously shown in Table 5.8. The value of \( \alpha \) in the Lu et al. [2005b] bond–slip model is 1.696 (corresponding to the interfacial fracture energy of 0.525 N/mm); it leads to a predicted ultimate load of 103.0 kN, which is a slightly higher than the experimental value (3.2 % over the experimental value). In general, a low interfacial fracture energy may result in early debonding; however, its effect is not greater than 17.0%.

Figure 5.18: Numerical aspects involved in the analysis

Figure 5.19: Effect of the order , \( \alpha \) of the descending branch of the bond–slip model on the response of unstrengthened and FRP-strengthened beams
The influence of the concrete fracture energy is illustrated in Figure 5.20. The concrete fracture energy rather than the interfacial fracture energy has a significant effect on the debonding load of the FRP-strengthened beams. Increasing the tension stiffening factor, $\xi$, within practical ranges from 4.0 to 20.0 reduces the debonding load by around 40%. This can be interpreted by observing that increasing the tension stiffening factor increases the cracking strain and the associated crack width, which accordingly increases the possibility of a more rapid debonding. From Figure 5.20(b), one can conclude that the use of the tension stiffening factor of 20.0 leads to a prediction of the debonding load similar to the failure load of the unstrengthened beam. The conclusions drawn from Figure 5.19 and Figure 5.20 highlight the fact that the flexural responses of FRP-strengthened reinforced concrete beams rely, in large part, on the characteristics of the cracked concrete sections more than that of the FRP/concrete interface. The only study found in the literature concerning the relation between the FRP characteristics and the concrete tension stiffening factor, $\xi$, is the numerical study of Ebead and Marzouk [2005]. However, as a result of adopting a full bond assumption between the FRP laminate and concrete, their study did not account for the effect of the tension stiffening model on the debonding mechanism. In all the previous simulations, the tension stiffening factor, $\xi$, was taken constant and equal to 8.0.
5.7 Finite element models for FRP/concrete interfaces

This section is set to discuss the accuracy of various models representing the interface. In addition, we aim to compare between the ability the microplane and hypoelastic concrete constitutive laws to capture the debonding phenomena.

Three different models are considered to represent the FRP/concrete interface in FRP-strengthened beams. In the first, the interface is modelled using three distinct layers for the concrete, adhesive and FRP assuming full interaction between each adjacent layer (Model I). Neglecting the adhesive layer and simulating the beams using two layers for the concrete and FRP with a full strain computability between points of the two layers is the second way of modelling the interface (Model II). The third approach is to introduce interface elements, where the constitutive law of these elements represents the interfacial behaviour between the FRP and concrete (Model III).

Two distinct concrete constitutive laws have been employed for the analyses in this section to represent the concrete characteristics in simulating the interface; one is based on the macroscopic concrete constitutive law (hypoelastic model in the ADINA software detailed in Appendix B), while the other model is the micromechanics-based law (microplane model presented in Chapter 3). The objective of this investigation is to assess the feasibility of using the microplane concrete model to represent the debonding behaviour of FRP-strengthened concrete beams without introducing an explicit interface element.

The specimen B-08S tested by Kotynia et al. [2008] has been considered in these simulations (3-D FE model). In Figure 5.21, the load–deflection relationships are depicted for the predictions of the three ways of modelling the interface using a hypoelastic concrete law. It is concluded that the phenomenological concrete model cannot capture the debonding load whether or not the epoxy layer is considered. This is because the hypoelastic constitutive law cannot accurately represent the interfacial shear stresses. Employing discrete interface elements with a predefined nonlinear bond–slip model overcomes this problem.

Figure 5.22 shows the predicted load–deflection relationships for the case where the
CHAPTER 5. NONLINEAR FEA OF FRP-STRENGTHENED REINFORCED CONCRETE BEAMS

Figure 5.21: Predicted load-deflection relationship of the specimen B-08S using hypoelastic concrete model

The microplane model is used to represent the concrete. Two mesh sizes are considered in this simulation. The first is a mesh of an element size of 12.5 mm along the interface. A comparison between the finite element prediction using this coarse mesh and experimental data in terms of the load-deflection relationship is shown in Figure 5.22(a). The second finite element model consists of a refined mesh (0.25 mm along the interface). The predicted load-deflection profile for the latter finite element mesh is depicted in Figure 5.22(b). The finite element model analysis based on the microplane constitutive law in the case of the coarse finite element mesh fails to simulate the debonding phenomena. Simulating the adhesive layer has a negligible effect on the accuracy of the simulations. The problem of the finite element model with the microplane concrete law, in the case of using a coarse mesh, arises from the fact that the debonding phenomenon leads to a strain discontinuity which cannot be modelled correctly with finite elements in which the strain varies continuously. The debonding causes an interfacial stress to soften with increasing cracking strain and causes a strain discontinuity due to the detachment of the laminates.

In Chapter 4, we showed that the finite element models with the microplane constitutive law were able to capture the debonding phenomena in the case of the FRP/concrete joint subjected to direct shear loadings. To assess why the finite element model based on microplane constitutive law could not simulate debonding in FRP-strengthened beams, a refined finite element mesh as was previously employed to simulate the FRP/concrete joints in Chapter 4 (0.25 mm cube with a total number of 762,012 elements) has been considered along the concrete layer between the FRP and reinforcement bars. In Figure
5.7. FINITE ELEMENT MODELS FOR FRP/CONCRETE INTERFACES

Figure 5.22: Predicted load-deflection relationship of the specimen B-08S using microplane concrete model

5.22(b), the load-deflection relationships are shown for the refined mesh along the interface. It is clearly observed that, when using a refined FE mesh, the finite element model with the microplane constitutive law successfully captures the debonding load. The use of a small element size allows capturing the debonding load because the strain discontinuity is accounted for by considering various elements in the vicinity of the cracks.

5.7.1 Effect of the bond-slip model

The 2-D finite element results in terms of the load capacities and the modes of failure are analyzed for six selected specimens to compare between the use of the nonlinear and bilinear bond-slip models of Lu et al. [2005b] and the new model proposed in Chapter 4. These specimens are chosen to cover a wide range of strain in the FRP sheets at failure and to represent the two experimentally observed mode of failure; i.e., debonding and rupture of the FRP sheets. Numerical results for these specimens based on the full-bond assumption are also included in the comparison.

As far as the type of the interface model is concerned, for all the specimens listed in Table 5.8, we see that very good predictions of the ultimate load carrying capacities are obtained when the three bond-slip laws are employed to model the interface. The
reasons for the similarities and any discrepancies between the results predicted using the two bond–slip models were previously discussed in the Subsection 5.5.5. These results draw attention to the fact that the interfacial stiffness, bond strength, and the bond–slip shapes have secondary effects on the flexural responses of the strengthened beams. This is because the debonding phenomenon is in large part due to the cracking behaviour of the concrete. A similar conclusion can be found in the numerical study of Niu and Wu [2006]. Despite the fact that the interfacial fracture energy of the nonlinear bond–slip model of Lu et al. [2005b] and that proposed in Chapter 4 are not identical (Table 5.2 and Table 5.3), the discrepancy in the predicted load capacities is not significant. This is illustrated in Figure 5.19(b). As one can observe from this figure, the interfacial fracture energy has only a significant effect on the ultimate capacities at very small values. However, for the values that fall within the practical ranges as those in the Lu et al. [2005b] and proposed models, the interfacial fracture energy has a small effect on the ultimate capacity.

Finally, as expected, the full–bond assumption leads to predicted load levels that are well above those observed experimentally. The average numerical-to-experimental loads ratio and the associated standard deviation when assuming full-bond are 1.17 and 0.19, respectively. This value (17%) is similar to that predicted when using a very low interfacial fracture energy (Figure 5.19).

5.8 Conclusion

In this chapter, numerical results from displacement-controlled finite element analyses have been presented and compared to the experimental data of 35 beams strengthened using externally bonded FRP laminates. In the analyses, discrete interface elements were proposed to simulate the FRP/concrete interfacial behaviour. The finite element models predicted the ultimate load carrying capacities with an average numerical-to-experimental ratio and standard deviation of 0.99 and 0.056, respectively.

In these analyses, three bond–slip laws (bilinear and nonlinear models of Lu et al. [2005b], and that proposed in Chapter 4) were used. The predictions of the ultimate load carrying capacities using the three laws were virtually the same. This is a result of the fact that the three laws were characterized by almost the same interfacial fracture energy.
Table 5.8: Comparison between different analyses

<table>
<thead>
<tr>
<th>Beam set</th>
<th>Beam ID</th>
<th>Load (kN)</th>
<th>Num./Exp.</th>
<th>Load (kN)</th>
<th>Num./Exp.</th>
<th>Load (kN)</th>
<th>Num./Exp.</th>
<th>Load (kN)</th>
<th>Num./Exp.</th>
<th>Load (kN)</th>
<th>Num./Exp.</th>
<th>Failure mode</th>
<th>Numerical</th>
<th>$\eta$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ross et al. [1999]</td>
<td>B1</td>
<td>80.0</td>
<td>1.00</td>
<td>80.0</td>
<td>1.00</td>
<td>82.6</td>
<td>1.00</td>
<td>105.0</td>
<td>1.03</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>72</td>
</tr>
<tr>
<td>Chicoine [1997]</td>
<td>P_3</td>
<td>172.8</td>
<td>1.01</td>
<td>172.8</td>
<td>1.01</td>
<td>170.1</td>
<td>0.99</td>
<td>175.2</td>
<td>1.02</td>
<td>R</td>
<td>R</td>
<td>IC</td>
<td>R</td>
<td>100</td>
</tr>
<tr>
<td>M'Bazaai [1995]</td>
<td>P_0</td>
<td>105.0</td>
<td>1.05</td>
<td>105.0</td>
<td>1.05</td>
<td>110.0</td>
<td>1.10</td>
<td>120.0</td>
<td>1.19</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>40.1</td>
</tr>
<tr>
<td>Kamińska and Kotynia [2000]</td>
<td>B-08/S</td>
<td>178.0</td>
<td>0.99</td>
<td>178.0</td>
<td>0.99</td>
<td>175.0</td>
<td>0.97</td>
<td>204.0</td>
<td>1.13</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>46.9</td>
</tr>
<tr>
<td>Brena et al. [2003]</td>
<td>A3</td>
<td>138.0</td>
<td>0.99</td>
<td>138.0</td>
<td>0.99</td>
<td>132.2</td>
<td>0.95</td>
<td>162.0</td>
<td>1.17</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>75.0</td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>120.0</td>
<td>1.00</td>
<td>120.0</td>
<td>1.00</td>
<td>118.3</td>
<td>0.98</td>
<td>138.0</td>
<td>1.15</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>IC</td>
<td>51.3</td>
</tr>
</tbody>
</table>

$\eta$ is the percentage of the maximum strains along the FRP laminates at failure of the beam calculated using the nonlinear model to the nominal rupture strain of FRP material, $\varepsilon_{frp}$. 
When assuming full-bond between the FRP and concrete, the numerical models over-predicted the experimental ultimate capacities of the strengthened beams, as expected.

In contrast to the common belief that plate-end debonding results from an elastic interfacial shear stress at the plate end, we observed that this failure mode occurs due to high interfacial shear stress concentrations arisen from the large width of the shear or flexural cracks around the plate end.

It was observed that the role of the adhesive layer in the FRP/concrete joints is different from that in the FRP-strengthened concrete beams. In the earlier application, the adhesive layer transfers shear stresses from the FRP laminates to the concrete block. However, in the latter case, the flexural behaviour and the associated widening of flexural and shear cracks govern the force transfer from the concrete beam to the FRP laminate. In general, the debonding of FRP laminates off the concrete surface arise from the flexure cracks in case of IC debonding and from flexure/shear cracks in case of end-plate debonding.

In our analyses, we found that the debonding initiated when the interfacial shear stress value reached the bond strength value at a point within the interface. Moreover, at the final debonding most interface elements were still in the ascending branch of the bond–slip model while few elements were in the beginning of the descending part.

A low interfacial fracture energy might result in early debonding, however the interfacial fracture energy has an insignificant effect on the ultimate capacity of the FRP-strengthened beams (not more than 17%). The main factor controlling the debonding mechanism of the strengthened beams was the cracking behaviour of the concrete including the steel reinforcement ratio and concrete fracture energy. Increasing the tension stiffening factor from 4.0 to 20.0 reduced the debonding load by 40%.

The maximum interfacial shear stress was seen to be dependent on the steel reinforcement ratio. With an increase of the steel reinforcement ratio from 0.46% to 3.3%, the maximum interfacial shear stress at the same load level was reduced by 60%. It was also found that it was very beneficial to use intermediate anchorage laminates along the FRP-strengthened beams to mitigate debonding, as they reduced the interfacial stresses at the locations where the anchoring laminates and FRP strengthening laminates overlapped.
The finite element model with the microplane constitutive law requires a very fine FE mesh for the interfacial layer to successfully capture the debonding load without using discrete interface elements. The use of the fine mesh successfully captures the debonding due to the fact that the strain discontinuities are represented. However, with a large element size along the interface, the microplane model fails to simulate the debonding behaviour.

Employing the reduction factor based on the ACI-440.2R-02 [2002] to account for possible FRP debonding might lead to a good prediction of the maximum strain along the FRP at failure as long as the length of the FRP laminate is greater than 0.75 of the beam length and the steel reinforcement ratio exceeds 0.35%.

Regarding the 3-D simulations, the finite element model predicted the ultimate load carrying capacities of the various FRP-strengthened beams with an average numerical-to-experimental ratio and standard deviation of 0.998 and 0.0276, respectively. As far as the FRP strains at the ultimate loads were concerned, the average numerical-to-experimental ratio and its corresponding standard deviation were 1.096 and 0.147, respectively. It was observed that not extending the length of the U-shaped distance to cover the ends of the laminates limited the effectiveness of the anchorage technique as far as the ultimate load capacities were concerned.
“Do not put your faith in what statistics say until you have carefully considered what they do not say”

(William W. Watt)
Chapter 6

Statistical Analyses and Parametric Study for FRP-Strengthened Reinforced Concrete Beams

In this chapter, statistical analyses and parametric studies are presented for reinforced concrete beams strengthened in flexure using FRP composites. Five variables are considered in this study; namely, the FRP axial stiffness, concrete strength, steel reinforcement ratio, beam depth, and beam span. These variables are selected based on the conclusions drawn in Chapter 5. The analyses begin by developing statistical models for the debonding load ($P_u$), the flexural capacity of the beam cross-section ($M_{u}/bd^2$), the maximum deflection at the debonding load ($\Delta_u$), the ductility index (i.e., the ratio between the maximum deflection at debonding load to that at yielding load), and the debonding strain level in the FRP laminate ($\varepsilon_{ub}$) based on the response surface methodology (RSM) technique. Proposed design equations are then developed by simplifying the RSM models. Of the five responses considered in the RSM analysis, three are considered in the design equations; namely, the debonding load, the associated maximum deflection and the debonding strain. (Note that, throughout this chapter the usual notations and conventions for statistical analysis are employed; i.e., for a model $y = ax + b$, we call "y" a response, "a" and "b" parameters and "x" a variable.) The data required for the statistical analyses were obtained from finite element models for beams having different combinations of the variables. The output of the FE study was fed into a statistical technique, the response surface methodology, to create statistical models.
Step One

Five variables
- FRP axial stiffness
- Concrete strength
- Steel reinforcement ratio
- Beam depth
- Beam span

Finite element results of 43 beams

RSM models for five responses
- Ductility index
- Flexural capacity
- Ultimate capacity
- Maximum deflection
- Debonding strain in FRP

Step Two

Monte Carlo simulator used to generate these random combinations

10,000 random combinations of five variables
- FRP axial stiffness
- Concrete strength
- Steel reinforcement ratio
- Beam depth
- Beam span

RSM models for only three responses
- Ultimate capacity
- Maximum deflection
- Debonding strain in FRP

10,000 random values of three response
- Ultimate capacity
- Maximum deflection
- Debonding strain in FRP

Step Three

Design equations
- Ultimate capacity
- Maximum deflection
- Debonding strain in FRP

Regression analysis

Figure 6.1: Steps of statistical analyses
The statistical analyses comprise three steps. These steps are shown schematically in Figure 6.1. The first step uses the response surface methodology technique to optimize the accuracy of the statistical models while minimizing the numbers of finite element runs [Montgomery and Runger, 1999]. This involves 43 finite element runs since we are considering five variables. The purpose of this analysis is to develop statistical models to be used as replacements for the finite element analyses. Among all the methods used in the RSM to fit a surface of response over the entire space, the face-centred central design (FCD) is applied for our statistical analysis to evaluate the influence of the above five critical variables on the five responses. The face-centred central composite response surface design is chosen because it has good design properties and because the region of interest and the region of operability are nearly the same. Comparisons between the statistical models and the finite element results are presented. The five variables, mentioned above, and the five responses are shown in Figure 6.1.

The ultimate goal of these statistical analyses is to develop robust design equations. Of the five responses, three are chosen to be represented in the design equations; namely, the debonding load, the associated maximum deflection and the debonding strain. The development of the design equations are carried out in the second and third steps of this study. With the second type of analysis, Monte Carlo simulations are used to generate large number of random combinations (10,000) of the five variables mentioned above. These combinations are then fed as input values into three RSM models to obtain the corresponding responses. A nonlinear regression analysis is finally applied to set up design equations that best fit these 10,000 random combinations.

The statistical analyses are followed by a parametric study, based on the RSM models, to investigate the effect of the above five variables and their interactions on the debonding load and the corresponding debonding strain level in the FRP laminate. This involves comparisons in terms of the debonding strain between the predictions of the proposed equation, on one hand, and those of the ACI, fib [fib, 2001], Chinese specifications and Australian standards.
6.1 Introduction

As demonstrated in the previous chapter, accurate numerical techniques can be reliably and effectively used to predict the load capacities and modes of failure for beams strengthened with FRP composites. Generally, conclusions based on the finite element technique are mostly reliable when a good, hence generally computationally demanding, numerical analysis is used. This emphasizes the need for a substitute that is as good as a finite element package, but less costly in time and computer resources. A simple statistical model or a mathematical equation that can be used in lieu of a finite element analysis to correlate certain variables and responses of interest would thus have tremendous advantages. One of the tools that can be used to develop such a model is the response surface methodology (RSM) [Montgomery and Runger, 1999]. The RSM is built up by simulating solutions at systematic points in the design space of the various variables, and then setting up a model of the response at these points. This model can then be employed in subsequent calculations as a substitute for the original finite element simulator. This advanced statistical technique is a powerful tool to optimize the necessary number of finite element simulations required to develop a refined statistical model. Ebead et al. [2002] have presented an application of this methodology for two-way slabs. A brief summary of this method is presented in the next sections.

In determining permissible strains in FRPs to avoid debonding, most of the code provisions and design guidelines consider only the axial stiffness of the FRP and the concrete compressive strength. From the finite element results of Chapter 5, we find that the flexural response of FRP-strengthened concrete beams and the associated debonding strain levels depend on various variables such as the steel reinforcement ratio, concrete compressive strength, and to a lesser extent on the geometric characteristics of the unstrengthened beams. It is also found that the debonding phenomena for FRP-strengthened concrete beams does not simply depend on only one or two variables (the axial stiffness of the FRP laminate and the concrete compressive strength). Accordingly, five variables are considered in this study; namely, the FRP axial stiffness, concrete strength, steel reinforcement ratio, and beam depth and beam span. Prior to the end of this chapter, we will assess the accuracy of the predicted debonding strain in various code specifications; namely, ACI-440.2R-07 [2007], fib [fib, 2001], Chinese FRP code and Australian standard considering random combinations of the omitted variables.
Despite the fact that the RSM models mentioned above can substitute for finite element analyses to simulate the flexural response of FRP-strengthened concrete beams and to predict the debonding strain level, there is nonetheless a need for relatively more simple design equations. A relatively simple and accurate approach is applied in this study to develop such design equations based on nonlinear regression analyses. The accuracy of a nonlinear regression model is generally dominated by the number of points used to fit the model. In our applications, the limited numbers of finite element results (43) are insufficient to produce a regression model that adequately represents all other possible cases. Accordingly, we use 10,000 points in this regression analysis, which are actually the output values of the RSM model. First, the Monte Carlo simulation method is used to generate 10,000 stochastic combinations of the five variables, then these combinations are fed as input values into the RSM model to compute their output responses. Finally, a nonlinear statistical regression analysis is used to determine the values of the parameters defining the proposed model that best fits these 10,000 stochastic responses. Of the five responses analyzed using the RSM technique, three are of interest for design purposes; namely, the debonding load, debonding strain level in the FRP laminate, and the maximum deflection at the debonding load.

Finally, we present a parametric study, based on the RSM models, to investigate the effect of the above five variables and their interactions on the debonding load and the corresponding debonding strain level in the FRP laminate. Most of the parametric studies have been set to only one variable (at a time), ignoring the interaction between the other variables on the debonding mechanisms [Thomsen et al., 2001; Faella et al., 2006]. A parametric study using one variable at a time to analyze the debonding phenomena of the FRP-strengthened concrete beams needs to be reconsidered to include the interaction among the various variables controlling the debonding behaviour. This is necessary to improve our understanding of the debonding phenomena.

6.2 Objectives

The objectives of this chapter can be summarized as follows:

- To provide powerful statistical models, based on the RSM technique, for various
responses to be used as substitutes for finite element analysis. These models are thought of as useful practical tools for researchers and engineers to pre-design their experimental programs and/or to carry out parametric studies;

- To develop practical design guidelines as well as governing equations by simplifying the RSM models using nonlinear regression analysis. These equations can be used in the design of FRP-strengthened reinforced concrete beams; and

- To investigate the relative importance of various variables and their interactions on the behaviour of FRP-strengthened beams through a parametric study.

This chapter is organized as follows. First, a brief recapitulation of the main features of the statistical methods used in our study is presented. These methods include the response surface methodology, Monte Carlo simulations, and nonlinear regression analysis. The five variables that have been chosen to develop the statistical models are then introduced. Based on the RSM procedures, we present the statistical models for the debonding load ($P_u$), flexural capacity of the beam cross-section ($M_u/bd^2$), the maximum deflection at the debonding load ($\Delta_u$), the ductility index, and the debonding strain level in the FRP laminate ($\varepsilon_{ub}$). Proposed design equations are then developed by simplifying the RSM models. Of the five responses, three are chosen to be represented in the design equations; namely, the debonding load, the associated maximum deflection and the debonding strain. This involves comparisons in terms of the debonding strain between the predictions of the proposed equation, on the one hand, and those of the ACI, *fib* [fib, 2001], Chinese specifications and Australian standards. Ten thousand random combinations of the five variables mentioned above are used for this purpose. Prior to the end of this chapter, a particular attention is given to the previously conducted finite element analyses (43 runs) in terms of modes of failure and load-deflection characteristics. A parametric study is carried out, based on the RSM models, to address the influence of the five variables and their interactions on the debonding load and the debonding strain level in the FRP laminates. Some of the results of this chapter can be found in Ebead et al. [2006, 2007].
6.3 Statistical methods

The following subsections include a brief description of the statistical methods used in the response surface methodology, Monte Carlo simulation, and nonlinear regression analysis.

6.3.1 Response surface methodology (RSM)

The RSM technique as defined by Montgomery and Runger [1999] is a collection of mathematical and statistical techniques that are useful for the modelling and analysis of problems in which a response of interest is influenced by several variables, and where the objective is to optimise this response. RSM also quantifies relationships among one or more measured responses and the vital input variables [Montgomery and Runger, 1999].

The most popular response surface method is the face centred design (FCD), which is generally designed to estimate the parameters of a quadratic or cubic model. Version 6.0.1 of the Design-Expert software is used in our study to develop the response surfaces [Design-Expert, 2000].

The response surface methodology [Montgomery and Runger, 1999] is very efficient since only minimal numbers of finite-element runs are required. Based on the responses obtained from finite element runs, simple regression models using the RSM technique are developed. The advantage of this statistical approach is its ability to account for the interaction between several variables on a response surface instead of analyzing only one
variable at a time. Models that are developed to describe the outputs of the FE simulations are approximated, based on functional relationships between the responses $y_{\text{est}}$ and one or more of the input variables ($F_1, F_2, ..., F_k$). Five responses are considered: the debonding load ($P_{\text{d}}$), flexural capacity of the beam cross-section ($M_u/bd^2$), the maximum deflection at the debonding load ($\Delta_u$), the ductility index and the debonding strain level in FRP laminates ($\varepsilon_{\text{ub}}$). Five input variables have been chosen in this application; namely, the steel reinforcement ratio ($F_1$), concrete compressive strength ($F_2$), FRP axial stiffness ($F_3$), beam depth ($F_4$) and beam span ($F_5$). In our analysis, where a nonlinear relationship exists between a particular response and the input variables, a third-order polynomial (cubic equation) was selected as an approximation for the response. The typical fitted third-order polynomial for each of the responses is given as:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i F_i + \sum_{i=1}^{k} \beta_{ii} F_i^2 + \sum_{i<j} \beta_{ij} F_i F_j + \sum_{i=1}^{k} \beta_{iii} F_i^3 + \sum_{i<j<k} \beta_{ijk} F_i F_j F_k \quad (6.1)$$

Here $\beta_0, \beta_1, ...$ etc. denote the parameters of the model and $k$ is the total number of variables ($k = 5$). The fitted response can be represented graphically as shown schematically in Figure 6.2, where the response $y$ is plotted against the variables $F_i$ and $F_j$, in three-dimensional space. In our application, however, the response is a surface in a six-dimensional space. In Section 6.5, the third-order polynomial model for each response (the debonding loads, flexural capacity of the beam cross section, the maximum deflection at the debonding load, the ductility index and the debonding strain in the FRP laminate) is determined. The least squares technique is used to fit a model containing the input variables by minimizing the residual error; i.e., measured by the sum of the squares of the differences between the actual and the estimated responses. This involves the calculation of the statistical parameters ($\beta$ coefficients in Equation 6.1).

Among all the methods used in the RSM to fit a surface over the entire space, the face-centred central design is considered for our statistical analysis. The latter is chosen due to its satisfactory design properties and because its region of interest and region of operability are approximately the same. Each variable is set at three levels in a predefined manner (minimum, maximum and average) [Myers and Montgomery, 1995]. Due to the nonlinearity of the predicted responses, a third-order model is selected to fit each response. The minimum number of finite element runs required for this fitting is 43, that is $(2^k + 2k + 1)$ where $k = 5$ is the total number of variables. Section 6.4 describes these variables in detail, as well as their range of interest.
6.3.2 Monte Carlo simulation

After creating the RSM regression models, the Monte Carlo simulation method is applied to generate random combinations (10,000 combinations) of the five variables. These stochastic numbers are fed as input variables into each RSM regression model to determine the corresponding responses. The eventual objective is to set up a simple design equation to best fit these random responses using nonlinear regression analyses.

Monte Carlo simulation is generally used to evaluate a deterministic model using sets of random numbers as input variables. This method is often used when the model is complex, nonlinear, or involves more than just a few uncertainty variables. In our analyses, using this method purports to generate immense outputs for a particular RSM model rather than simply evaluating it. The circular loop option within Microsoft Office Excel 2003 is used to perform these simulations.

6.3.3 Nonlinear regression analysis

The Monte Carlo methods are used only to create random combinations of the five input variables to evaluate their corresponding output responses of the RSM statistical models. Subsequently, nonlinear regression analysis is used to set up a simplified model that best fits these responses. We aim to use such models as design equations similar to those in the currently available specifications and guidelines. The DataFit software, Version 8.1.69 is used for the calculations associated with the nonlinear regression analysis [DataFit, 2005]. Generally, the goal of the nonlinear regression analysis is to determine the best-fit parameters for a model by minimizing the chosen test function. This function defines the difference between the actual and predicted responses and is given by:

\[ \chi^2 = \sum_{i=1}^{i=N} [y_i - y(F_i)]^2 \]  

(6.2)

where \( N \) is the total number of random input data, \( y_i \) is the actual response value (outputs of a particular RSM model) and \( y(F_i) \) is the response value computed from the proposed nonlinear regression model.

Nonlinear regression analysis is essential when the response model exhibits a nonlinear
dependence on the unknown variables (the five variables in this study). Hence, the process of test function minimization is an iterative procedure. This iterative process begins with some initial estimates and incorporates algorithms to improve the estimates in an iterative manner. The new estimates then become a starting point for the next iteration. These iterations continue until the values of the test function converge [Oakdale-Engineering, 2005]. In Section 6.6, the nonlinear regression models (referred to as the proposed design equations) are presented.

In the following sections, the five variables used to perform the statistical analyses are elaborated upon. Then the RSM models for the five different responses are presented. Of the five responses considered in the RSM models, only three are selected in this subsection to develop design equations.

### 6.4 Variables of interest

Five different variables of interest are altered and combined based on the face-central composite technique using the response surface methodology; namely, the steel reinforcement ratio, concrete compressive strength, FRP axial stiffness, beam depth, and the beam span. The foregoing variables have been chosen based on the conclusions drawn from the finite element simulations in Chapter 5. Table 6.1 details these variables and their ranges of interest. It is intended that the numerical values of all the variables be of the same scale. Variable $F_1$ represents the steel reinforcement ratio multiplied by 1000; variable $F_2$ denotes the concrete compressive strength $f'_c$ in MPa; variable $F_3$ represents the axial stiffness of the FRP composites, given as the multiplication of the elastic modulus of the FRP $E_{frp}$ in $kN/mm^2$ by the area of FRP section that is $t_{frp}b_{frp}$ in $mm^2$ divided by 1000, where $t_{frp}$ and $b_{frp}$ are the thickness and width of the FRP laminates, respectively. Variables $F_4$ and $F_5$ represent the beam depth and loaded span, respectively. The beam depth $d$ (variable $F_4$) is computed as the total height of the beam from which a cover of 40.0 mm is subtracted. The concrete compressive strength (variable $F_2$) controls the concrete stress–strain relationship, tensile strength of the concrete, concrete fracture energy, and the bond–slip law of the interface.
Table 6.1: Variables and their ranges of interest

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Low</th>
<th>Mid</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
<td>$= 1000 A_f/ bd$</td>
<td>3.0</td>
<td>9.0</td>
<td>15.0</td>
</tr>
<tr>
<td>$F_2$</td>
<td>= Concrete compressive strength, $f'_c$ (MPa)</td>
<td>25.0</td>
<td>40.0</td>
<td>55.0</td>
</tr>
<tr>
<td>$F_3$</td>
<td>$= E_{pp} t_{pp} b_{pp} / 1000000$ (MN)</td>
<td>11.70</td>
<td>30.24</td>
<td>48.78</td>
</tr>
<tr>
<td>$F_4$</td>
<td>= Depth of the unstrengthened section, $d$ (cm)</td>
<td>21.0</td>
<td>26.0</td>
<td>31.0</td>
</tr>
<tr>
<td>$F_5$</td>
<td>= Loaded span, $l/10$ (cm)</td>
<td>25.0</td>
<td>30.0</td>
<td>35.0</td>
</tr>
</tbody>
</table>

6.5 RSM models

In Table 6.2, the responses predicted using the FE analysis are presented in terms of the debonding load, ductility index, and axial strains in the reinforcement steel bars and the FRP laminate.

Using the input variables and responses in Table 6.2, the third-order polynomial models are fitted. The debonding load model ($P_u$) is expressed as:

$$P_u(kN) = 2.554 + \sum_{i=1}^{5} \sum_{j=1,j>i}^{5} \left[ \alpha_i F_i + \beta_i F_i^2 + \eta_{ij} F_i F_j \right] + \mathbf{F}^T \mathbf{A}$$

where

$$\mathbf{A} = \begin{bmatrix} -63.354 \\ 3.097 \\ 0.436 \\ 18.429 \\ -6.774 \end{bmatrix}, \quad \alpha = \begin{bmatrix} 3.224 \\ -0.033 \\ -0.356 \\ 0 \end{bmatrix}, \quad \beta = \begin{bmatrix} -0.022 \\ -0.022 \\ -0.033 \end{bmatrix}, \quad \eta = \begin{bmatrix} 0.509 & -0.188 & 0 \\ 0.043 & 0.055 & -0.105 \\ 0 & 0 & -0.033 \end{bmatrix}$$

The RSM model for the flexural capacity of the beam cross-section ($M_u/bd^2$) in terms
### Table 6.2: Responses for different combinations of variables

<table>
<thead>
<tr>
<th>Run</th>
<th>Variables</th>
<th>Responses</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_1$</td>
<td>$F_2$</td>
<td>$F_3$</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>40</td>
<td>30.24</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>55</td>
<td>48.78</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>40</td>
<td>30.24</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>55</td>
<td>48.78</td>
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<tr>
<td>5</td>
<td>3</td>
<td>25</td>
<td>48.78</td>
</tr>
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<td>40</td>
<td>30.24</td>
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<tr>
<td>7</td>
<td>15</td>
<td>55</td>
<td>11.7</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>25</td>
<td>11.7</td>
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<tr>
<td>9</td>
<td>15</td>
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<td>11.7</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>25</td>
<td>48.78</td>
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<td>55</td>
<td>11.7</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>40</td>
<td>48.78</td>
</tr>
<tr>
<td>13</td>
<td>9</td>
<td>40</td>
<td>30.24</td>
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<td>14</td>
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<td>30.24</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>55</td>
<td>11.7</td>
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<tr>
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<td>15</td>
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<td>11.7</td>
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<td>3</td>
<td>25</td>
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<td>3</td>
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<td>11.7</td>
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<td>3</td>
<td>55</td>
<td>48.78</td>
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<tr>
<td>34</td>
<td>3</td>
<td>25</td>
<td>48.78</td>
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<td>3</td>
<td>55</td>
<td>11.7</td>
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<tr>
<td>36</td>
<td>3</td>
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</tr>
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<td>42</td>
<td>15</td>
<td>25</td>
<td>11.7</td>
</tr>
<tr>
<td>43</td>
<td>9</td>
<td>55</td>
<td>30.24</td>
</tr>
</tbody>
</table>

Finite element results are based on numerical model shown in Figure 5.4
of the five variables is:

\[
\frac{M_u}{bd^2} \text{(MPa)} = 37.20369 + \sum_{i=1}^{5} \sum_{j=1, j>i}^{5} \left[ \alpha_i F_i + \beta_i F_i^2 + \eta_{ij} F_i F_j \right] + \\
\begin{bmatrix}
0.001698 & -0.01234 \\
-0.00016 & -0.00063
\end{bmatrix}
\begin{bmatrix}
F_2 \\
F_5
\end{bmatrix} F_1^2 + \\
\begin{bmatrix}
F_3 \\
F_4
\end{bmatrix} F_1 F_2 + \\
\begin{bmatrix}
0.000493 & -0.00028
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2
\end{bmatrix} F_3 F_4
\]

where

\[
\alpha = \begin{bmatrix}
-7.34154 \\
0.10257 \\
0.00627 \\
-0.47922 \\
-0.75076
\end{bmatrix}, \quad \beta = \begin{bmatrix}
0.386128 \\
-0.00344 \\
-0.00226
\end{bmatrix}
\]

and

\[
\eta = \begin{bmatrix}
0 & 0.067987 & -0.04241 & -0.01159 & 0.24257 \\
0 & 0 & 0.011756 & 0.007496 & -0.00559 \\
0 & 0 & 0 & -0.00312 & 0.008339 \\
0 & 0 & 0 & 0 & -0.00219
\end{bmatrix}
\]

For the maximum deflection at the debonding load, we have:

\[
\Delta_u \text{(mm)} = 4.93889 + \sum_{i=1}^{5} \sum_{j=1, j>i}^{5} \left[ \alpha_i F_i + \beta_i F_i^2 + \eta_{ij} F_i F_j \right] + 0.11063 F_4 F_5 + \\
\begin{bmatrix}
0.00759 & 0.05051
\end{bmatrix}
\begin{bmatrix}
F_3 \\
F_4
\end{bmatrix} F_1^2 - 0.0008 F_1 F_2 F_3
\]

where

\[
\alpha = \begin{bmatrix}
25.215 \\
-0.2532 \\
-1.2075 \\
-0.3241 \\
-0.5941
\end{bmatrix}, \quad \beta = \begin{bmatrix}
1.5261 \\
0 \\
0.01354
\end{bmatrix}
\]

and

\[
\eta = \begin{bmatrix}
0 & 0.0423 & -0.0908 & -0.9306 \\
0 & 0 & 0.0101 & 0
\end{bmatrix}
\]
The RSM model for the ductility index is:

Ductility index = \(-89.8627 + \sum_{i=1}^{5} \sum_{j=1, j>i}^{5} \left[ \alpha_i F_i + \beta_i F_i^2 + \eta_{ij} F_i F_j \right] + 0.0187 F_1^2 F_4 + 0.0022 F_1^2 + \begin{bmatrix} F_3 \\ F_4 \\ F_5 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \begin{bmatrix} F_3 \\ F_4 \end{bmatrix} + \begin{bmatrix} 0.0013 \\ 0.0005 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} + \begin{bmatrix} 0.0003 \\ 0.0011 \end{bmatrix} \begin{bmatrix} F_3 \\ F_4 \end{bmatrix} \begin{bmatrix} F_5 \end{bmatrix} \) \quad (6.6)

where

\[ \alpha = \begin{bmatrix} 12.6577 \\ 4.5640 \\ 1.2357 \\ 1.6273 \\ 0.3935 \end{bmatrix}, \quad \beta = \begin{bmatrix} -0.4056 \\ -0.0278 \\ -0.0062 \end{bmatrix} \text{ and } \eta = \begin{bmatrix} 0 & -0.2440 & -0.0174 & -0.4211 & -0.0266 \\ 0 & 0 & -0.0183 & -0.0614 & -0.0559 \\ 0 & 0 & 0 & -0.0294 & -0.0113 \\ 0 & 0 & 0 & 0 & 0.0166 \end{bmatrix} \]

For the debonding strain level in the FRP laminate, the RSM model is:

\[ \ln (\varepsilon_{ub}) = 14.55633 + \sum_{i=1}^{5} \sum_{j=1, j>i}^{5} [\alpha_i F_i + \eta_{ij} F_i F_j] \] \quad (6.7)

where

\[ \alpha_i = \begin{bmatrix} -0.13719 \\ 0.018547 \\ -0.0501 \\ -0.21829 \\ -0.15509 \end{bmatrix} \text{ and } \eta_{ij} = \begin{bmatrix} 0 & 0.001078 & 0.000662 & -0.0029 & 0.003054 \\ 0 & 0 & 0.000071 & 0.00017 & -0.00057 \\ 0 & 0 & 0 & 0.000189 & 0.000649 \\ 0 & 0 & 0 & 0 & 0.006472 \end{bmatrix} \]

For convenience, the RSM models given by Equation 6.3 to Equation 6.7 are represented in table-form in Table 6.3. For example, to evaluate the debonding load for a beam,
each value in the first column under the heading "$P_u (kN) =$" should be multiplied by the associated variable in the adjacent column. The summation of the resulting values is the required value of the debonding load. The application of this technique is very suitable for use with spreadsheets. From the RSM models of Equation 6.3 to Equation 6.7, one can notice that there are second order effects in all the variables. Moreover, there is a clear interaction between the variables and, in some cases, between a variable and the second order of another indicating the significance of the associated parameters in obtaining the better fit. This interaction between variables is missing in existing design guidelines.

The parameters of all the RSM models are statistically significant and all the assumptions of regression, such as the normality of residuals ($R^2$), standard deviation (SD) and coefficient of variance (C.O.V) are fulfilled, as summarized in Table 6.4. (Note that, the definitions of these terms are stated below Table 6.4.) The results show that in each RSM model the actual $R^2$ value is very close to the predicted $R^2$ value, thus showing a very good fit and good prediction accuracy. This accuracy is evident in Figures 6.3(a) to 6.3(e) that show comparisons between the predicted results (using the RSM models) and FE results for the values of $P_u$, the maximum deflection at the debonding load ($\Delta_u$), ductility index, debonding strain in the FRP laminate and the flexural capacity of the beam cross-section ($M_u/bd^2$), respectively. Since the fitted surfaces (RSM models) are adequate approximations of the finite element results, using the fitted surfaces will be essentially equivalent to conducting finite element analyses.

6.6 Proposed design guideline equations

The RSM models developed in the preceding sections provide powerful numerical techniques for predicting the various responses for FRP-strengthened reinforced concrete beams. Despite the fact that these statistical models are compatible with spreadsheets, there is nonetheless a need for relatively more simple design equations. A relatively simple and accurate approach is applied in this section to develop such design equations based on a nonlinear regression analysis. Of the five responses analyzed using the RSM technique, three are of interest for design purposes; namely, the debonding load, debonding strain level in the FRP laminate, and the maximum deflection at the debonding load.
Table 6.3: Regression models for different responses

<table>
<thead>
<tr>
<th>$F_u$ (kN)</th>
<th>$M_u/bd^2$ (MPa)</th>
<th>$\Delta_u$ (mm)</th>
<th>Ductility index</th>
<th>$\ln(\varepsilon_{fp})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.554</td>
<td>37.20369</td>
<td>14.93889</td>
<td>12.6577</td>
<td>-9.8627</td>
</tr>
<tr>
<td>-63.354</td>
<td>$F_1$</td>
<td>$-0.734154$</td>
<td>25.215</td>
<td>$14.55633$</td>
</tr>
<tr>
<td>+3.097</td>
<td>$F_2$</td>
<td>0.10257</td>
<td>-2.532</td>
<td>$F_3$</td>
</tr>
<tr>
<td>+0.436</td>
<td>$F_3$</td>
<td>0.008267</td>
<td>-1.2075</td>
<td>$F_4$</td>
</tr>
<tr>
<td>+18.429</td>
<td>$F_4$</td>
<td>-0.47922</td>
<td>-1.321</td>
<td>$F_5$</td>
</tr>
<tr>
<td>-6.774</td>
<td>$F_5$</td>
<td>-0.75076</td>
<td>-0.5941</td>
<td>$F_6$</td>
</tr>
<tr>
<td>+3.224</td>
<td>$F_6$</td>
<td>0.386128</td>
<td>-1.5261</td>
<td>$F_7$</td>
</tr>
<tr>
<td>-0.033</td>
<td>$F_7$</td>
<td>-0.00344</td>
<td>-0.0456</td>
<td>$F_8$</td>
</tr>
<tr>
<td>-0.022</td>
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Figure 6.3: Comparisons between the predictions from the regression models and FE analysis
Table 6.4: Regression coefficients based on RSM for different responses

<table>
<thead>
<tr>
<th></th>
<th>Load</th>
<th>Flexural capacity, $M_u/\text{bd}^2$</th>
<th>Deflection</th>
<th>Ductility index</th>
<th>FRP strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.9791</td>
<td>0.989</td>
<td>0.9605</td>
<td>0.9532</td>
<td>0.939</td>
</tr>
<tr>
<td>$SD$</td>
<td>6.85</td>
<td>0.70</td>
<td>2.96</td>
<td>1.00</td>
<td>1.99</td>
</tr>
<tr>
<td>$C.O.V$</td>
<td>5.47</td>
<td>5.42</td>
<td>13.06</td>
<td>10.76</td>
<td>0.16</td>
</tr>
</tbody>
</table>

$R^2$ is the proportion of variability in the response data that is accounted for by a statistical model and given as:

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}$$

where $y_i$, $\hat{y}_i$ and $\bar{y}$ are the original response values (FE results), values computed from the statistical model, and the mean value of the response values, respectively.

$SD$ is the standard deviation associated with the estimation error and expressed as: 

$$SD = \frac{\sum (y_i - \bar{y})^2}{N}$$

where $N$ is the total number of responses (43 in case of the RSM models).

$C.O.V$ is the error expressed as a percentage of the mean and computed as: 

$$C.O.V = \frac{SD}{\bar{y}} \times 100$$

The accuracy of a nonlinear regression model is generally dominated by the number of points used to fit this model. In our applications, the limited numbers of finite element results (43) are insufficient to produce a regression model that adequately represents all other possible cases. Accordingly, we use 10,000 points in this regression analysis, which are actually the output values of the RSM model. First, the Monte Carlo simulation method is used to generate 10,000 stochastic combinations of the five variables, then these combinations are fed as input values into a RSM model to compute their output responses. Finally, a nonlinear statistical regression analysis is used to determine the values of the parameters defining the proposed model that best fits these 10,000 stochastic responses.

A power-law expression is found to be the most suitable model to predict the debonding loads of the FRP-strengthened beams. This law is given by:

$$P_u = 10.82 \rho^{0.12} f_c^{0.62} (EA)_{frp}^{0.11} d^{0.24}$$  \(6.8\)

This equation defines the debonding load level as a function of the steel reinforcement ratio ($\rho \times 1000$), concrete compressive strength, ($f_c$ in MPa), FRP axial stiffness ($(EA)_{frp}$ in...
6.6. PROPOSED DESIGN GUIDELINE EQUATIONS

MN), depth of the unstrengthened section \((d \text{ in cm})\), and the beam span \((l \text{ in cm } \times 10)\).

It is obvious that the debonding load increases with the increase of the steel reinforcement ratio \(\rho\), the concrete compressive strength \(f'_c\), FRP axial stiffness \((EA)_{frp}\), and concrete depth \(d\). However, with an increase in beam span \(l\), the debonding load decreases. The variable most affecting the rate of debonding load increase is the concrete compressive strength \(P_u \propto f'_c^{0.62}\). On the other hand, the steel reinforcement ratio and the axial stiffness of the FRP laminate have almost the same effect \(P_u \propto (EA)_{frp}^{0.11}\) and \(\rho^{0.12}\). The accuracy of the proposed design equation is demonstrated in Figure 6.4(a). The abscissa represents the debonding load predicted using the RSM model (Equation 6.3), while the ordinate shows the values determined from the proposed design equation (Equation 6.8).

The two debonding strain levels in the FRP laminate are given below in Equation 6.9 and Equation 6.10 as a function of the five variables shown in Table 6.1:

\[
\varepsilon_{ub} = 0.75\varepsilon_{fu} + 70.9f'_c + 106.8l - 225.7\rho - 113.1(EA)_{frp} - 283.4d \quad (6.9)
\]

\[
\varepsilon_{ub} = 0.83\varepsilon_{fu} - 544.5\frac{(EA)_{frp}^{0.38}d^{0.96}}{f'_c^{0.28}\rho^{0.33}} \quad (6.10)
\]

These proposed design equations have high \(R^2\) value (0.9) which indicates a good fit with the RSM model of Equation 6.7. In Figure 6.4(b), a comparison is presented between the debonding strain values computed using the RSM model and those predicted from the design equation (Equation 6.10). Equation 6.10 is proposed in order to introduce a threshold for the debonding strain limit and to ensure that the predicted value is less than \(\varepsilon_{fu}\). The ultimate debonding strain level is 0.83\(\varepsilon_{fu}\) where \(\varepsilon_{fu}\) is the ultimate strain of the FRP laminate. This value decreases with an increase of the steel reinforcement ratio, FRP axial stiffness, and beam span. The ACI-440.2R-07 [2007] assumes a slightly higher prediction for this ultimate value (0.9\(\varepsilon_{fu}\)).

In the ACI-440.2R-07 [2007] design equation, the debonding strain is given in terms of the concrete compressive strength and the axial stiffness of the FRP laminates. However, one can observe from Equation 6.9 and Equation 6.10 that the debonding strain strongly depends on the steel reinforcement ratio, beam depth and beam span. Figure 6.5(a) shows a comparison between the predicted debonding strain limits according to the ACI-440.2R-07 [2007] design equation and that of the finite element results comprising of 43 runs (Table 6.2). From the figure, it is clear that the ACI equation has unsafe predictions for certain combinations of the five variables. As a result of neglecting the influence of
Figure 6.4: Comparison between responses of derived models based on RSM and that using the proposed design equations
the steel reinforcement ratio, and both the beam span and depth, the ACI specifications give the same predictions for various combinations of input variables. For example, points “1-8” in Figure 6.5(a) have the same predicted debonding strain according to the ACI (10887 με). However, their actual values from the finite element simulations range from 9967 to 2053 με. Point “1” has a combination of steel reinforcement ratio, beam depth and beam span of 3.0%, 210 mm and 2500 mm, respectively. Similarly, point “8” has the values of 1.5%, 310 mm and 2500 mm, respectively. The two points have the same concrete compressive strength and FRP axial stiffness of 55 MPa and 11.7 MN, respectively.

Another comparison is presented in Figure 6.5(b) between the ACI predictions and those using the proposed design equation. In this figure, we consider 10,000 stochastic combinations of the above five variables. It is obvious that the ACI-440.2R-07 [2007] has poor predictions when considering random combinations of the five variables. This highlights the need to incorporate the three omitted variables into the ACI design code equation. Figures 6.5(b) to 6.5(h) illustrate similar comparisons between the predictions of various code specifications; namely, fib [fib, 2001], the Chinese FRP code [Ye et al., 2005] and the Australian standard [Oehlers et al., 2006] and those of the proposed design equation. The fib and Chinese equations give more conservative predictions than those of the ACI, however the results at certain combinations are unsafe.

A polynomial and a power-law design equation defining the maximum deflection at the debonding load of the FRP-strengthened reinforced concrete beams are given as follows in Equation 6.11 and Equation 6.12, respectively:

$$\Delta_u = 1.91 - 0.7p + 0.2f'_c - 0.38EA_{frp} - 1.6d + 2.3l$$  \hspace{1cm} (6.11)

$$\Delta_u = 0.28 \frac{f_{c0.33}}{\rho^{0.2} EA_{frp}^{0.4} d^{0.15}}$$  \hspace{1cm} (6.12)

These two proposed design equations fit very well with the values of the RSM model of Equation 6.5 with an $R^2$ value of 0.93 (Figure 6.4(c)). The design equations for the debonding load ($P_u$), debonding strain level in the FRP laminate ($\epsilon_{ub}$) and the maximum deflection at the debonding load ($\Delta_u$) have $R^2$ values of 0.9, 0.9, and 0.93, respectively, indicating a good fit with the RSM models.
CHAPTER 6. STATISTICAL ANALYSES FOR FRP-STRENGTHENED RC BEAMS

(a) Comparison with the finite element results - ACI

(b) Comparison with the proposed design equation - ACI

(c) Comparison with the finite element results - fib

(d) Comparison with the proposed design equation - fib

(e) Comparison with the finite element results - Chinese code

(f) Comparison with the proposed design equation - Chinese code

(g) Comparison with the finite element results - Australian standards

(h) Comparison with the proposed design equation - Australian standards

Figure 6.5: Predicted debonding strain according to various code specifications
6.7 Numerical results and parametric studies

The following sections discuss the numerical results of the finite element runs of 43 specimens in terms of the modes of failure and load-deflection relationships. In addition, a parametric study is carried out to investigate the influence of the five variables and their interactions on the debonding load and strain level predicted using the RSM models.

6.7.1 Modes of failure of the beams

In Table 6.2, the last column lists the type of failure, where “IC” indicates intermediate crack debonding and “EP” indicates plate-end debonding. As seen in this table only these two types of failure are predicted. Intermediate crack debonding is associated with higher FRP strains than those associated with plate-end debonding. The debonding strain in the case of IC debonding ranges from 2143 to 9968 με and from 792 to 5368 με for EP debonding. Higher strain values generally indicate an effective utilization of the strengthening system. Hence, the undesirable plate-end debonding mode limits the utilization of the FRP laminate. We also observe that IC debonding is likely to occur for a longer beam span (3500.0 mm in our study). In the case of a beam span of 2500.0 mm, the mode of failure is generally plate-end debonding, except for the cases that combine a low axial stiffness of the FRP laminate (11.7 MN) and a low steel reinforcement ratio (0.03 %). For the beam span of 3000.0 mm, IC debonding characterizes the mode of failure for the cases of a low reinforcement ratio or small beam depth. Otherwise, EP debonding governs the mode of failure. Generally, intermediate crack debonding is likely to occur for combinations of long loaded spans, small beam depths and low reinforcement ratios. Otherwise, plate-end debonding is the dominant failure mode.
Figure 6.6: Load-deflection relationship predicted from FE runs No. 1 - 15
Figure 6.7: Load–deflection relationship predicted from FE runs No. 16 – 30
Figure 6.8: Load-deflection relationship predicted from FE runs No. 31 - 43.
6.7.2 Load–deflection characteristics

The results presented in Figure 6.6 to Figure 6.8 refer to predicted load–deflection relationships from the finite element analysis of 43 specimens. From the analysis of these relationships, it can be concluded that the load–deflection behaviour of an FRP-strengthened beam is almost bilinear characterized by two points describing the cracking and debonding loads. The cracking load can easily be computed from the tensile strength of the concrete and elastic analysis can be used to calculate the corresponding deflection. The point describing the debonding load and the corresponding deflection can be determined using Equation 6.3 and Equation 6.5, respectively.

6.7.3 Debonding load and strain level in the FRP laminate

In this study, the debonding load actually refers to the least value of load associated with either the debonding mode of failure at the mid-span (intermediate crack debonding), or at the plate end (plate-end debonding). In some cases, debonding of the FRP laminates occurs at a load level less than the failure load of the unstrengthened beams. In this particular case, the load is taken to be the failure load of the unstrengthened beam. Figure 6.9 and Figure 6.10 show the effect of the five variables on the debonding load and debonding strain level in the FRP laminate, respectively. The study of this section is presented based on the RSM equations (Equation 6.3 and Equation 6.7).

With an increase of the steel reinforcement ratio, the debonding load increases (Figure 6.9(a)), and the associated debonding strain reduces (Figure 6.10(a)). This indicates that the utilization ratio of the FRP laminate (i.e., the ratio of the strain in the FRP at debonding to its ultimate strain) reduces as the steel reinforcement ratio increases. From Figure 6.9(a), the rate of increase of the debonding load depends on the steel reinforcement ratio; i.e., the slopes of the curves in this figure reduce as the steel reinforcement ratios increase. This confirms the fact that the enhancement of the ultimate capacity due to FRP strengthening is a function of the steel reinforcement ratio. In other words, the improvements in the ultimate capacities of the strengthened beams over those of the unstrengthened beams are higher for lightly reinforced beams than for heavily reinforced beams. The same conclusion has been drawn from the experimental work of Ross et al.
CHAPTER 6. STATISTICAL ANALYSES FOR FRP-STRENGTHENED RC BEAMS

Figure 6.9: Effect of various variables on the debonding load
The axial stiffness of the FRP laminate has a modest influence on the relationship between the steel reinforcement ratio and the debonding strain levels; i.e., the three curves in Figure 6.10(a) have almost the same shape.

Increasing the concrete compressive strength increases both the debonding load and debonding strain level in the FRP laminate, as shown in Figures 6.9(b) and 6.10(b), respectively. The higher the concrete compressive strength, the smaller the crack widths become as well as the associated slip values along the interface. In turn, the debonding strain increases. The relationship between the concrete compressive strength is concave downward for the debonding load and convex upward for the debonding strain.

The effect of the FRP axial stiffness on the debonding load and debonding strain is depicted in Figures 6.9(c) and 6.10(c), respectively. In the ACI-440.2R-07 [2007] design equation, the debonding strain reduces with the square root of the FRP axial stiffness. This is approximately the same relation in Figure 6.10(c). However, unlike the ACI equation, the relationship between the debonding strain and FRP axial stiffness depends on the steel reinforcement ratio. Should the latter ratio decrease, the relationship between the FRP axial stiffness and debonding strain becomes better represented by a steeper curve.

The influence of the depth and the loaded span of the unstrengthened section on the debonding load is shown in Figures 6.9(d) and 6.9(e), respectively. The higher the flexural stiffness (increasing the depth or decreasing the loaded span), the more debonding load will be carried by the beam. However, the debonding strain reduces with an increase of the flexural stiffness as a result of increasing the applied load (Figures 6.10(d) and 6.10(e)). The steel reinforcement ratio has a negligible effect on the relationship between the flexural stiffness and the debonding strain or debonding load. In general, reducing the flexural stiffness of the unstrengthened beam cross-section increases the utilization ratio of the FRP laminate. This may increase the probability of rupture failure at the FRP laminate.
(a) Reinforcement ratio ($f'_c = 35$ MPa, $l = 3500$ mm, $d = 300$ mm)

(b) Concrete compressive strength ($EA_{frp} = 10$ GPa, $l = 3500$ mm, $d = 300$ mm)

(c) FRP axial stiffness ($EA_{frp}$ = 10 MN, $l = 3500$ mm)

(d) Height of the unstrengthened section, $t$ (mm, $d = 300$ mm)

(e) Loaded span ($f'_c = 35$ MPa, $d = 35$ mm, $EA_{frp} = 10$ GPa)

Figure 6.10: Effect of various variables on the debonding strain
6.8 Conclusion

In the above statistical analyses, we considered the effects of five variables (steel reinforcement ratio, concrete compressive strength, FRP axial stiffness, beam depth, and beam span) on five responses; namely, the debonding load \( (P_u) \), flexural capacity of the beam cross-section \( (M_u/bd^2) \), the maximum deflection at the debonding load \( (\Delta_u) \), ductility index, and the debonding strain level in the FRP laminate \( (\varepsilon_{ub}) \). The data required for the statistical analyses were obtained from finite element models for beams having different combinations of the variables. The output of the FE study was fed into a statistical technique, the response surface methodology, to create statistical models.

The statistical analyses presented in this chapter were comprised of three steps. The application using the RSM technique to develop five models defining the above five responses was the first step. These RSM models are thought of as a useful practical tool for researchers and engineers to be used as a replacement for the more complicated finite element analyses. In the second step, Monte Carlo simulations were conducted to generate a large combination (10,000) of the five variables that were, in turn, fed as input values into the RSM models. The corresponding response values were the outcome. Finally, a nonlinear regression analysis was applied to set up simple design equations that best fit the large response data set. Of the five responses considered in the RSM models, only three responses are of interest in the particular nonlinear regression analysis; namely, the debonding load, debonding strain level in the FRP laminate, and the maximum beam deflection at the debonding load.

Comparing the results of the RSM models to those of the finite element analysis, in terms of the various responses, indicated that the RSM models were as good as the finite element analysis within the ranges and limitations of the strengthened beam specifications considered. Using these RSM models may provide a helpful guide to pre-design beams strengthened with FRPs so that undesirable premature debonding failures can be mitigated.

In the third step of this statistical study, design equations describing the debonding load, debonding strain level in the FRP laminate, and the maximum deflection at the debonding load were proposed. The regression and statistical coefficients of the proposed
design equations indicated that the predictions were in good agreement with the RSM models. The proposed equations hold an advantage over those available in most code specifications due to the fact that they account for the interaction between various variables on the predicted debonding strain. The comparisons between the proposed design equation and those of the ACI-440.2R-07 [2007], fib [fib, 2001], Chinese specifications and Australian standards, in terms of the debonding strain, showed that these specifications were sometimes unsafe predictors for certain combinations of the five variables.
Chapter 7

Conclusions and Recommendations

To conclude this thesis work, some remarks should be made on micromechanics-based FE analysis of the interfacial behaviour of FRP-strengthened reinforced concrete beams. This dissertation attempted to build up powerful and refined numerical tools that might enhance our comprehension of the bond performance of such beams. The research work here was divided into two phases.

The first phase started with reformulating the microplane concrete model by proposing an alternative incremental formulation of the M4 version originally proposed by Bažant and co-workers. This was done to overcome the computational complexities associated with the original formulation. Through a number of numerical applications, this incremental formulation was shown to be equivalent to the original M4 model. Then, the microplane model was coded in FORTRAN and implemented as a user-defined subroutine into the commercial finite element package ADINA, Version 8.4. The first phase of this dissertation includes as well a nonlinear micromechanics-based finite element analysis using the software package ADINA with the implemented microplane subroutine. This aimed to investigate the interfacial behaviour of FRP/concrete joints subjected to direct shear loadings. Based on the results of this study, a new bond-slip ($\tau$–$s$) model for the FRP/concrete interface was proposed. The bond-slip relation was developed considering the interaction between the interfacial normal and shear stress components along the bonded length as a failure criterion for the FRP/concrete joint. The proposed mathematical relation was used to describe the entire $\tau$–$s$ curve based on three separate models. The
first model captured the shear response of an orthotropic FRP laminate. The second sim­
ulated the shear characteristics of an adhesive layer, while the third model represented the 
shear nonlinearity of a thin layer inside the concrete referred to as an “interfacial layer”.
The proposed bond–slip model reflects the geometrical and material characteristics of the 
FRP, concrete, and adhesive layers.

The second phase of this thesis work was devoted to simulate the debonding phe­
nomena and flexural response of FRP-strengthened reinforced concrete beams. Nonlinear 
two-dimensional and three-dimensional displacement-controlled finite element models were 
developed to achieve this objective. The three-dimensional simulations were conducted to 
accommodate cases of FRP-strengthened beams having FRP anchorage systems. A new 
technique was proposed to simulate the interfacial behaviour using discrete interface ele­
ments instead of using continuous elements. The proposed approach successfully captured 
the FRP/concrete interfacial behaviour before and after cracking and the stress concen­
ctrations and fluctuations involved. The analyses were capable of simulating the various 
failure modes, including debonding of the FRP, either at the plate end or at intermediate 
creacks. Finally, statistical analyses and parametric studies were performed to generalize 
the finite element results and to propose new design guidelines. The variables under inves­
tigation included two material characteristics; namely, the concrete compressive strength 
and axial stiffness of the FRP laminates, as well as three geometric properties: the steel re­
inforcement ratio, beam span length, and beam depth. The parametric study was followed 
by a statistical analysis for 43 strengthened beams incorporating the five aforementioned 
variables. The response surface methodology (RSM) was employed to optimize the ac­
ccuracy of the statistical models while minimizing the numbers of finite element runs. In 
particular, a face-centred design (FCD) was applied to evaluate the influence of the crit­
cical variables on the debonding load and debonding strain limit in the FRP laminates. 
Based on the results of these statistical models, design guidelines were recommended for 
the FRP flexural strengthening of reinforced concrete beams using nonlinear statistical 
regression analysis.

Based on the main findings of this thesis, observations concerning the microplane 
concrete model, shear response of FRP/concrete joints, interfacial behaviour of FRP-
strengthened reinforced concrete beams, and design guidelines can be made.
7.1 Microplane concrete model

An incremental formulation of the M4 version of the microplane model of Bažant et al. [2000] was developed. With this formulation, a rate form of the constitutive law relating the macroscopic stress increments and strain increments was proposed in which explicit expressions were obtained for the tensor of incremental moduli. The proposed incremental approach was applied to the various cases previously analyzed by Caner and Bažant [2000]. The incremental scheme was seen to lead to essentially the same predictions as those reported by Caner and Bažant [2000].

The softening branch of the uniaxial compressive stress–strain relationship as well as the uniaxial compressive strength predicted using microplane mode was observed to be dependent on the increment size. However, the sensitivity of the predicted uniaxial compressive strength to the increment size was less than 10%. The increment size sensitivity might be ascribed to the localization of microscopic stress components at particular microplanes. A low increment size allowed microcracks to localize at more planes.

7.2 Shear response of FRP/concrete joints

Based on the results of the micromechanics-based finite element analyses of FRP/concrete joints subjected to direct shear loadings, it was concluded that direct shear tests did not correspond to cases of pure shear stress. The state of stress along the bonded length affected the bond strength value, $\tau_{\text{max}}$. Accordingly, a new bond–slip model was developed accounting for the normal stress components along the interface. A large amount of experimental data was used to calibrate this bond–slip model. The comparisons between the finite element predictions and the experimental results showed a very good agreement.
7.3 Interfacial behaviour of reinforced concrete beams strengthened in flexure with externally bonded FRPs

It was found that using various functions for the bond-slip model did not significantly affect the predictions of the debonding load carrying capacities since they were characterized by almost the same interfacial fracture energy. When assuming full-bond between the FRP and concrete, the numerical models over-predicted the experimental ultimate capacities of the strengthened beams, as one might expect.

In contrast to the common belief that plate-end debonding results from an elastic interfacial shear stress at the plate end, we observed that this failure occurred due to high interfacial shear stress concentrations arising from the large crack width of the shear or flexural cracks around the cut-off point. In general, the debonding failure mode in strengthened beams was different than that in the case of FRP/concrete joints. In the latter, failure occurred due to elastic local interfacial shear stresses, however in the former it occurred because of stress concentration at vicinities of flexural or shear cracks.

In the analysis, we found that debonding initiated when the interfacial shear stress value reached the bond strength value at a point within the interface. Moreover, at the final debonding most interface elements were still in the ascending branch of the bond-slip model while few elements were in the beginning of the descending part. This explained the brittle nature of the debonding mode of failure in that the softening branch of the bond-slip model did not affect the results.

A low interfacial fracture energy might result in early debonding; however, the interfacial fracture energy had a relatively small effect on the ultimate capacities of FRP-strengthened beams. The main factor controlling the debonding mechanism of such beams was the cracking behaviour of the concrete; i.e., the steel reinforcement ratio and the concrete fracture energy. Increasing the tension stiffening factor of the concrete from 4.0 to 20.0 reduced the debonding load by 40%.

The maximum interfacial shear stress was seen to be dependent on the steel reinforcement ratio. With an increase of the steel reinforcement ratio from 0.46% to 3.3%, the
maximum interfacial shear stress at the same load level was reduced by 60%. It was also observed that using intermediate anchorage laminates along an FRP-strengthened beam was very useful to mitigate debonding, as they reduced the interfacial stresses at the locations where the anchoring laminates and FRP strengthening laminates overlapped. Not extending the length of the U-shaped distance to cover the ends of the laminates limited the effectiveness of the anchorage technique as far as the ultimate load capacities were concerned.

When using a refined FE mesh with the microplane constitutive law for the interfacial layer the debonding load was successfully captured without using an explicit interface element. However, with a large element size along the interface, the microplane model failed to represent the debonding behaviour.

Employing the reduction factor based on the ACI-440.2R-02 [2002] to mitigate FRP debonding might lead to a good prediction of the maximum strain along the FRP at failure as long as the length of the FRP laminate was greater than 75.0% of the beam length and the steel reinforcement ratio exceeded 0.35%.

### 7.4 Design guidelines

Statistical models have been developed to evaluate the debonding failure loads of FRP-strengthened reinforced concrete beams, as well as the associated strain values in the FRP composites, the ductility index, and the deflection at the debonding load. These models were based on the statistical response surface methodology (RSM) technique. The data required to develop the models was obtained from finite element analyses of beams having different combinations of five variables; namely, the steel reinforcement ratio, the concrete compressive strength, the FRP axial stiffness, the depth of the beam, and the span length. The comparison between the results of the RSM models and the finite element predictions indicated that these models were as good as using the finite element analysis within the ranges and limitations of the strengthened beam parameters considered.

Design equations describing the debonding load, debonding strain in the FRP laminates, and the deflection at the debonding load were presented. These design equations
were developed based on a nonlinear regression analysis to simplify the RSM models. The regression and statistical coefficients of the proposed design equations indicated that the predictions using the design equations were as good as those based on the RSM models. The proposed equations hold an advantage over those available in current code specifications due to the fact that they account for the interaction between the various variables on the predicted debonding strain. The comparisons between the proposed design equation and those of the ACI-440.2R-07 [2007], fib [fib, 2001], Chinese specifications and Australian standards, in terms of the debonding strain, showed that these specifications were sometimes unsafe predictors for certain combinations of the five variables.

7.5 Recommendations for further research

The author believes that a comprehensive numerical study is required to investigate the applicability of using the microplane concrete model to study the debonding phenomena in various other cases having different material and geometrical characteristics for the concrete, steel reinforcement and the FRP laminate. Further numerical studies using the microplane concrete model are necessary in other applications of FRP-strengthened reinforced concrete structures.

It is recommended for future studies that a cracking model that evaluates the crack width and the spacing between flexural cracks of FRP-strengthened beams be developed based on the finite element results presented in this study. Moreover, there is a need for comprehensive studies to relate the concrete tension stiffening and fracture energy with the geometric and material characteristics of the FRP laminates.

Most of the experimental studies and finite element analyses have been restricted to small scale beams (less than 4.0 m long). Accordingly, there is a need for finite element simulations of large beams of spans greater than 12.0 m. The results of these simulations will be useful to validate the code guidelines.

Finally, it should be emphasized here that, although the numerical models presented in this dissertation were restricted to the interfacial and flexural responses of FRP-strengthened reinforced concrete beams, it in not difficult in principle to adopt these
7.5. RECOMMENDATIONS FOR FURTHER RESEARCH

models to other types of FRP-strengthened concrete structures.
"There is no greater mistake than the hasty conclusion that opinions are worthless because they are badly argued"

(Thomas Huxley, 1825 - 1895)
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Appendix A

Continuum mechanics models for the shear and normal stress profiles along FRP/concrete joints

In this appendix, continuum-mechanics models based on the theory of elasticity are developed for the shear and normal stress profiles along the FRP/adhesive, adhesive/concrete interface and at a certain depth inside the concrete (1.0 mm, in particular). These models are used in Chapter 4 to derive an expression for $\tau_{\text{max}}$ as a function of the normal stress components along the interface. This study generalizes the analysis of Bizindavyi and Neale [2001] for the shear stress distribution along an FRP/adhesive interface.

A.1 Interfacial shear stress distributions

Bizindavyi and Neale [2001] developed Equation A.1 to Equation A.4 based on the equilibrium of a discrete element $dx$ of an FRP laminate to describe the shear stress $\tau_{pa}(x)$ profile along FRP/adhesive interface as a function of axial stress, $f_p(x)$, in the FRP laminate as

$$
\tau_{pa}(x) = \frac{df_p}{dx} t_p
$$

(A.1)
A.1. INTERFACIAL SHEAR STRESS DISTRIBUTIONS

The governing differential equation for the shear stress distribution along the bonded joint was derived in the Bizindavyi and Neale [2001] studies as

\[
\frac{d^2 \tau_{pa}(x)}{dx^2} - \lambda^2 \tau_{pa}(x) = 0
\]  

(A.2)

where, \( \lambda^2 = \left[ \frac{G_a}{E_p t_p / t_a} \right] (1 + \eta \rho) \), \( \eta = \frac{E_p}{E_c} \), and \( \rho = \frac{A_p}{A_c} \). Here, \( A_p \) and \( A_c \) are, respectively, the FRP laminate cross section and the concrete block cross section. In these equations, \( G_a \), \( E_p \), and \( E_c \) are, respectively, the shear modulus of the adhesive layer, and Young’s modulus of the FRP laminate and the concrete. The thicknesses of the FRP laminates and adhesive layer are denoted by \( t_p \) and \( t_a \), respectively. Solving Equation A.2 leads to the interfacial shear stress distribution, \( \tau_{pa}(x) \), along the FRP/adhesive interface as [Bizindavyi and Neale, 2001]

\[
\tau_{pa}(x) = \frac{\tau_m \lambda L_i}{\tanh(\lambda L_i)} \left[ \cosh(\lambda x) - \sinh(\lambda x) \right]
\]  

(A.3)

and for axial stress distribution, \( f_p(x) \), along the bonded length \( L_i \)

\[
f_p(x) = \frac{\tau_m L_i}{t_p \sinh(\lambda L_i)} \left[ \sinh(\lambda(x - L_i)) \right]
\]  

(A.4)

where \( \tau_m \) is the mean shear stress on the bonded joint (equal \( F/L_i \)).

![Figure A.1: Infinitesimal element of the FRP/concrete joint](image)

(a) Equilibrium of a discrete element  
(b) Displacement of bonded joint

In our analysis, based on the equilibrium of a discrete element \( dy \) inside an adhesive layer (see Figure A.1(a)), one can write

\[
\frac{d \tau_{ca}}{dy} + \frac{d \sigma_a}{dx} = 0
\]  

(A.5)
APPENDIX A. CONTINUUM MECHANICS MODELS FOR INTERFACIAL STRESS PROFILES

Hence

$$d\tau_{ca} = -\frac{d\sigma_a}{dx} dy$$

(A.6)

By integrating Equation A.6 over the depth of the adhesive layer, we get

$$\tau_{ca} - \tau_{pa} = -\int_0^{t_a} \frac{d\sigma_a}{dx} dy$$

(A.7)

For simplicity, the normal stress in the adhesive layer \(\sigma_a\) can be assumed to be constant along the thickness of the adhesive layer, then:

$$\tau_{ca} = \tau_{pa} + \frac{d\sigma_a}{dx} t_a = \frac{df_p}{dx} t_p + \frac{d\sigma_a}{dx} t_a$$

(A.8)

However based on the finite element results presented in Section 4.4.1 and the closed-form solution of Teng et al. [2002], the values of \(\sigma_a\) vary along the depth of the adhesive layer. Assuming that \(\sigma_a\) is a linear function as

$$\sigma_a(x,y) = \sigma_a(x) y$$

(A.9)

we obtain the following

$$\tau_{ca}(x) = \tau_{pa}(x) + \left. \frac{d\sigma_a(x)}{dx} \right|_0^y y \frac{1}{2} = \frac{df_p(x)}{dx} t_p + \frac{d\sigma_a(x)}{dx} t_a$$

(A.10)

The same procedures can be applied to extract the shear stress distribution \(\tau_c\) at a certain depth in the concrete, \(t_c\), assuming a constant function for the normal stress, \(\sigma_c\) along the depth (Figure A.1(a)):

$$\tau_c(x) = \frac{df_p(x)}{dx} t_p + \frac{d\sigma_a(x)}{dx} t_a + \frac{d\sigma_c(x)}{dx} t_c$$

(A.11)

### A.2 Normal stress distributions

From the strain compatibility of the bonded joint in Figure A.1(b), one can conclude that

$$\epsilon_a dx = \frac{1}{2} (u_1 + du_1 + u_2 + du_2 - u_2 - u_1)$$

(A.12)
Hence, the axial strain $\varepsilon_a$ and shear strain $\gamma_a$ in the adhesive layer can be calculated using

$$\varepsilon_a = \frac{1}{2} \left( \frac{du_1 + du_2}{dx} \right) = \frac{\varepsilon_c + \varepsilon_p}{2} \quad (A.13)$$

and

$$d\gamma_a = \frac{(du_2 - du_1)}{t_a} \quad (A.14)$$

Reformulating Equation A.14 to obtain:

$$\frac{d\gamma_a}{dx} t_a = \frac{(du_2 - du_1)}{dx} = \varepsilon_p - \varepsilon_c \quad (A.15)$$

and solving Equation A.13 with Equation A.15 yields

$$\varepsilon_a(x) = \varepsilon_p(x) - \frac{1}{2} t_a \frac{d\gamma(x)}{dx} \quad (A.16)$$

$$\varepsilon_c(x) = \varepsilon_p(x) - t_a \frac{d\gamma(x)}{dx} \quad (A.17)$$

Since the adhesive layer is a linear elastic material, then

$$\varepsilon_a(x) = \varepsilon_p(x) - \frac{t_a}{2G_a} \frac{d\tau(x)}{dx} \quad (A.18)$$

$$\varepsilon_c(x) = \varepsilon_p(x) - \frac{t_a}{G_a} \frac{d\tau(x)}{dx} \quad (A.19)$$

Assuming the concrete shear behaviour is linear elastic until the shear stress reaches the value of the bond strength, gives

$$\sigma_a(x) = \frac{E_a}{E_p} f_p(x) - \frac{t_a}{2G_a} \frac{d^2 f_p(x)}{dx^2} E_a \quad (A.20)$$

$$\sigma_c(x) = \frac{E_c}{E_p} f_p(x) - \frac{t_a}{G_a} \frac{d^2 f_p(x)}{dx^2} E_c \quad (A.21)$$

where, $\sigma_a(x)$, and $\sigma_c(x)$ are the normal stresses in the adhesive layer and at 1.0 mm inside the concrete, respectively. Substituting $\sigma_a(x)$, and $\sigma_c(x)$ from Equation A.20 and Equation A.21 in Equation A.11, leads to:

$$\tau_c = \frac{df_p(x)}{dx} t_p + \frac{E_a}{E_p} \frac{df_p(x)}{dx} t_a - \frac{t_a^2}{2G_a} \frac{d^2 f_p(x)}{dx^2} E_a + \frac{E_c}{E_p} \frac{df_p(x)}{dx} t_a - \frac{t_a}{G_a} \frac{d^2 f_p(x)}{dx^2} E_a \quad (A.22)$$
where \( \tau_c \) is the shear stress distribution along certain depth \( t_c \) inside the concrete (1 mm in this applications). Solving Equation A.1 and Equation A.2 together and then substituting in Equation A.22, one can write

\[
\tau_c = \frac{df_p(x)}{dx} t_p + \frac{E_a}{E_p} \frac{df_p(x)}{dx} t_a - \frac{E_a t_a^2}{2G_a} \lambda^2 \tau_{pa}(x) + \frac{E_c}{E_p} \frac{df_p(x)}{dx} t_a - \frac{E_a t_a t_c}{G_a} \lambda^2 \tau_{pa}(x) \tag{A.23}
\]

Simplifying Equation A.23 and using Equation A.1 we get

\[
\tau_c = \frac{\tau_{pa}(x)}{t_p} \left[ t_p + \frac{E_a}{E_p} t_a + \frac{E_c}{E_p} t_a - \frac{E_a t_a \lambda^2}{G_a} \left( \frac{t_a}{2} + t_c \right) \right] \tag{A.24}
\]
Appendix B

Concrete constitutive law: ADINA, Version 8.4

In this appendix, a brief recapitulation of the main features of the concrete model that is available in ADINA, Version 8.4 is presented. This concrete constitutive law was used in the FE simulations detailed in Chapter 4 and Chapter 5. The concrete model is a hypoelastic phenomenological concrete model. It combines three features to simulate the basic characteristics of the concrete. These features are a nonlinear stress–strain relation, a failure criterion, and a fixed smeared crack model. The nonlinear stress–strain relation allows for the weakening of the material under increasing compressive stresses and considers the strain softening behaviour. A failure criterion for concrete is used to define the cracking stress in tension and the failure envelope in compression. A fixed smeared crack model is used to describe the post-cracking behaviour of the concrete. A brief recapitulation of the main features of the nonlinear stress–strain relation, failure criterion, and fixed smeared crack model is presented below.

B.1 Nonlinear stress–strain relations: Concrete in compression

A hypoelastic (incremental) model is utilized to describe the nonlinear stress–strain relationship of the concrete. It is assumed that the strain increment vector \( \{ \varepsilon \} \) is linearly
related to the stress increment vector \( \{ \sigma \} \) through the material response moduli, \( [C] \), which can be written in a matrix form as

\[
\{ \sigma \} = [C] \{ \epsilon \}
\]

(B.1)

The matrix of tangential moduli \( C \) assumes an orthotropic material with the directions of orthotropy being defined by the principal stress directions. It has the following form:

\[
C = \frac{1}{(1+2\nu)} \times \begin{bmatrix}
E_{p1} & \nu E_{12} & \nu E_{13} & 0 & 0 & 0 \\
\nu E_{12} & E_{p2} & 0 & 0 & 0 & 0 \\
\nu E_{13} & 0 & E_{p3} & 0 & 0 & 0 \\
0 & 0.5(1-2\nu) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{0.5(1-2\nu)}{1+\nu} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{0.5(1-2\nu)}{1+\nu} & E_{13} \\
\end{bmatrix}
\]

(B.2)

where \( E_{p1}, E_{p2} \) and \( E_{p3} \) are the equivalent multiaxial Young's moduli in the principal directions, computed according to the value of the principal strain, \( \varepsilon_{p1} \). When \( \varepsilon_{u} \leq \varepsilon_{p1} \leq 0 \), where \( \varepsilon_{u} \) is computed in Equation B.4, the Young's moduli in the principal directions are evaluated as

\[
E_{p1} = \frac{E_c}{1 + \frac{2\nu}{(1+\nu)\varepsilon_{c}^{'} + B \left( \frac{\varepsilon_{p1}}{\varepsilon_{c}^{'}} \right)^{2} + C \left( \frac{\varepsilon_{p1}}{\varepsilon_{c}^{'}} \right)^{3}}} \]

where,

\[
A = \frac{E_c}{E_u} + \frac{(P^2 - 2P^2) E_c}{P^2 - 2P + 1}, \quad B = \left[ \left( \frac{2E_c}{E_s} - 3 \right) - 2A \right], \quad C = \left[ \left( \frac{2 - E_c}{E_s} \right) + A \right]
\]

and the values of \( E_s = \frac{\sigma_{c}^{'}}{\varepsilon_{c}^{'}} \), \( P = \frac{\varepsilon_{u}^{'}}{\varepsilon_{c}^{'}} \) and the parameters \( \varepsilon_{u}^{'}, \varepsilon_{c}^{'}, \sigma_{u}^{'} \)

\[
\varepsilon_{u}^{'}, \varepsilon_{c}^{'}, \sigma_{u}^{'} = \varepsilon_{u} \gamma_{1} + \sigma_{c} \gamma_{2}
\]

(B.4)

and

\[
\sigma_{c}^{'} = \gamma_{1} \sigma_{c}
\]

(B.5)
B.2. MATERIAL FAILURE ENVELOPES

Figure B.1: Concrete constitutive model [ADINA, 2004b]

where $C_1$ and $C_2$ are taken as 1.4 and -0.4, respectively. The value of $\gamma_1$ is computed from the biaxial failure envelope of the concrete according to the ratio $\sigma_{p2}/\sigma_c$, as shown in Figure B.1(b). The equivalent multiaxial Young's moduli required to compute the off-diagonal components of the material response matrix, $C$ (Equation B.2), are computed as:

$$E_{ij} = \frac{\sigma_{pi} |E_{pi}| + |\sigma_{pj}| |E_{pj}|}{|\sigma_{pi}| + |\sigma_{pj}|} \quad (i \neq j) \quad i, j = 1 \text{ to } 3 \quad (B.6)$$

where $\sigma_{pi}$ is the principal stress value.

When the principal stress state lies on the failure envelope, it is assumed that the material strain softens isotropically in all directions, which corresponds to the case of $\varepsilon_{pi} \leq \varepsilon_u$. The stresses, in all principal directions, are assumed to linearly reduce to zero using the following modulus:

$$E_{pi} = \frac{\sigma_u - \sigma_c}{\varepsilon_u - \varepsilon_c} \quad (B.7)$$

B.2 Material failure envelopes

To identify whether the material has failed or cracked, the principal stresses are used to locate the current stress state. Having established the principal stresses $\sigma_{pi}$ with $\sigma_{p1} \geq \sigma_{p2} \geq \sigma_{p3}$, the stresses $\sigma_{p1}$ and $\sigma_{p2}$ are held constant and the minimum stress value that
would have to be reached in the third direction to cause crushing is calculated using the failure envelopes.

Let \( \sigma'_c \) be the third principal stress causing crushing. First the value of \( \sigma_{p1}/\sigma_c \) is computed, then the two-dimensional failure envelope function for the stresses \( \sigma_{p2} \) and \( \sigma_{p3} \) are evaluated based on the ratio \( \sigma_{p1}/\sigma_c \). If the stress state corresponding to \( \sigma_{p2} \) and \( \sigma_{p3} \) lies on or outside the biaxial failure envelope, then the material failure has occurred. In Figure B.1(b), we depict the biaxial failure envelope of the concrete. For the tensile failure envelope, it is assumed that the tensile strength of the concrete in a principal direction does not depend on the tensile stresses in the other principal stress directions.

B.3 Post-cracking model: Fixed smeared crack model

The concrete is assumed to behave as an isotropic linear material for tensile stresses less than the tensile strength value. When the principal tensile stress exceeds its limiting value, a crack is assumed to occur in a plane normal to the direction of the corresponding principal strain and this crack direction is then fixed for all subsequent loading (fixed-smeared crack model). The effect of the concrete cracking is that the normal and shear stiffness, \( E_c \) and \( G_c \), respectively, across the plane of cracks are reduced using reduction factors \( \eta_n \) and \( \eta_s \), respectively. The plane stress condition is assumed to exist at the plane of tensile cracks. After concrete cracking, the material response matrix, \( C \) in Equation B.2 becomes:

\[
C = \begin{bmatrix}
E_c\eta_n & 0 & 0 & 0 & 0 \\
0 & \frac{1}{(1-\nu^2)} \begin{bmatrix} E_{p2} & \nu E_{23} \ \nu E_{23} & E_{p3} \end{bmatrix} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\frac{E_c\eta_s}{2(1+\nu)} & 0 & 0 & \frac{E_{23}}{2(1+\nu)} & 0 \\
symmetric & 0 & 0 & 0 & \frac{E_{23}}{2(1+\nu)}
\end{bmatrix}
\]  

(B.8)

The factors \( \eta_n \) and \( \eta_s \) in Equation B.8 imply a sudden drop of the initial stiffness \( E_c \) in
the direction of the crack and the shear stiffness \( G_c \) in the plane of tensile cracks, respectively. When the principal tensile strain, \( \varepsilon_t \leq \varepsilon \leq \varepsilon_m \), where \( \varepsilon_m \) and \( \varepsilon_t \) are represented in Figure B.1(a), the secant Young's modulus \( E_t \) replaces the norm \( E_c \eta_n \) in the material response matrix. Beyond the strain level, \( \varepsilon_m \), the factor \( \eta_n \) is taken 0.0001 (in order to avoid the possibility of a singular stiffness matrix). The factor \( \eta_s \), known as the shear reduction factor, is assumed 1.0 in the case of \( \varepsilon \leq \varepsilon_t \). Then \( \eta_s \) is reduced linearly to be 0.5 at a strain level \( \varepsilon_m \) and remains constant to consider several physical factors such as aggregate interlock, reinforcement dowel action and friction between cracks. In Figure B.1(a), the factor \( \xi \) defines the amount of tension stiffening computed at each integration point based on the size of the finite element as

\[
\xi = \frac{2E_c G_f}{f^2 h}
\]  

(B.9)

where \( h \) is the width of the finite element perpendicular to the plane of tensile cracks. \( G_f \) in Equation B.9 is the concrete fracture energy released per unit area. If the normal strain across the existing crack becomes greater than that just before crack formation, the crack is assumed to open; otherwise, it is closed. For cracked concrete in which all sets of cracks are closed, the concrete is assumed to behave as an uncracked elements.
Appendix C

Bond–slip models of Lu et al. [2005b]

In this appendix, the details of the nonlinear and bilinear bond–slip models of Lu et al. [2005b] are presented. These models are employed in the FE simulations in Chapter 4 and Chapter 5.

The first model for the interface is the nonlinear shear stress–slip relation defined for the ascending and descending parts as follows:

\[ \tau = \tau_{\text{max}} \sqrt{s/S_0} \quad \text{if } s \leq S_0 \]  
\[ \tau = \tau_{\text{max}} \exp \left[ -\alpha \left( s/S_0 - 1 \right) \right] \quad \text{if } s \geq S_0 \]  

The maximum shear stress \( \tau_{\text{max}} \) is governed by the concrete tensile strength, \( f_t \), and the FRP width ratio factor, \( \beta_w \), and is taken as

\[ \tau_{\text{max}} = 1.5\beta_w f_t \]  

where \( \beta_w \) is given by

\[ \beta_w = \sqrt{\left( 2.25 - \frac{b_f}{b_c} \right) / \left( 1.25 + \frac{b_f}{b_c} \right)} \]  

in which \( b_f \) and \( b_c \) are the widths of the FRP and concrete beam, respectively. The slip \( S_0 \) is also dependent on \( f_t \), and \( \beta_w \) as follows:

\[ S_0 = 0.0195\beta_w f_t \]
The factor $\alpha$ in Equation C.2 is related to the interfacial fracture energy, $G_f^b$, (i.e., the energy per unit bond area required for complete debonding) according to the following equation:

$$\alpha = 1/ \left[ G_f^b / (\tau_{\text{max}} S_0) - 2/3 \right] \quad \text{(C.6)}$$

where

$$G_f^b = 0.308 \beta_0 \sqrt{f_t} \quad \text{(C.7)}$$

For the formulations for the second interface relation, the bilinear model, we have:

$$\tau = \frac{\tau_{\text{max}} S}{S_0} \quad \text{if } S \leq S_0 \quad \text{(C.8)}$$

$$\tau = \frac{\tau_{\text{max}} (S_{\text{max}} - S)}{(S_{\text{max}} - S_0)} \quad \text{if } S_0 < S \leq S_{\text{max}} \quad \text{(C.9)}$$

where

$$S_{\text{max}} = \frac{2G_f^b}{\tau_{\text{max}}}, \quad \text{and} \quad S_0 = 0.0195 \beta_u f_t \quad \text{(C.10)}$$
“Men do not follow titles, they follow courage, and if you are willing to show courage, they will be willing to follow you...”

From the movie “Braveheart”