OPTIMIZATION OF ACTUATOR CONFIGURATION FOR
THE REDUCTION OF STRUCTURE-BORNE NOISE IN
AUTOMOBILES

OPTIMISATION DE LA CONFIGURATION
D’ACTIONNEURS POUR LA RÉDUCTION DU BRUIT DE
ROULEMENT À L’INTÉRIEUR DES AUTOMOBILES

Master’s thesis
Speciality: mechanical engineering

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ABSTRACT

In this thesis, a strategy is proposed for the optimization of actuator configuration in the implementation of Active Structural Acoustic Control (ASAC) of road noise in an automobile suspension.

First, a laboratory test bench consisting of a quarter-car suspension consisting of a wheel/suspension/lower A-arm assembly is modeled. A 12 degrees-of-freedom discrete element model of the rigid parts of the suspension is first used to produce global suspension resonances. Equivalent rigidity models of flexible components are then measured experimentally or identified with the help of genetic algorithms. The discrete element model and the equivalent rigidity models are combined to reproduce test bench Frequency Response Functions (FRFs) measured on the test bench.

Second, the impedance of the test bench tire table is corrected using experimental measurements and analytical road profiles to reproduce a realistic road excitation. Afterward, a Chevrolet EPICA LS automobile is instrumented and operated on a concrete test track to observe the relative importance of road-induced vibrations compared to other noise sources in a moving car at 50 km/h. FRFs between the road excitation and the car interior pressure level at the driver’s head are measured and compared to a complete car transmission path tool (composed of the quarter-car test bench and a car frame finite element model). The FRF analysis between reference sensors and error microphones reveals the difficulty of obtaining sufficient experimental coherence to realise the active control of a suspension.

Finally, an algorithm is implemented to find optimal actuator locations and orientations in the ASAC, using the suspension model and the filtered road excitation. Genetic algorithm tools are used with the suspension model for actuator positioning: the tire, the coil spring & the car panels control volumes are included into the model to constrain the evolution of the algorithm. For a given actuator configuration, the optimal control command is obtained by quadratic minimization of specific cost function (displacement at suspension links, force transmissibility & sound pressure level). Optimal actuator configurations are then suggested for future studies.
RÉSUMÉ

Cette thèse a pour objectif d’optimiser la position des actionneurs dans la mise en œuvre d’un contrôle vibratoire du rayonnement acoustique d’une suspension d’automobile.

Premièrement, un banc de test constitué d’un quart de suspension d’automobile (roue, tige de réaction, cardan et table de suspension) est modélisé. Les composantes flexibles démontrant un grand nombre de résonances sont modélisées à l’aide de rigidités équivalentes mesurées expérimentalement ou par des rigidités identifiées par algorithme génétique. Les composantes démontrant peu de résonances (moyeu, cardant, jante,…) sont modélisées par des éléments discrets et rigides qui relient les différents modèles dynamiques. La combinaison des éléments flexibles et des éléments rigides forme un modèle analytique de suspension qui est validé par la mesure des fonctions de transfert de transmissibilité en force sur le banc de test.

Deuxièmement, l’impédance de la table supportant le pneu du banc de test est corrigée à l’aide de mesures expérimentales et de profil de route analytique. Cette procédure perpe rapprochant d’une vraie excitation routière. Ensuite, une voiture complète (une Chevrolet EPICA LS) est instrumentée et testée sur piste pour étudier le bruit de roulement dans une voiture roulant à 50 km/h. La mesure des vibrations du moyeu de la roue avant côté conducteur et des niveaux de pression près de la tête du conducteur est effectuée. Ces mesures servent à valider l’excitation du banc de test et comparer les niveaux de pression prédits par le banc de test. Les fonctions de transfert permettent de constater qu’un des défis important du contrôle vibro-acoustique de structures est l’obtention d’une bonne cohérence entre le signal de référence et le signal d’erreur.

Finalement, un algorithme d’optimisation de positionnement d’actionneur est mis en oeuvre. Le modèle analytique de suspension sert de plate-forme pour l’analyse de différentes configurations d’actionneurs : l’algorithme combine control optimal et algorithme génétique pour optimiser la position, l’orientation, et la commande de chaque actionneur. Différentes fonctions-coût sont minimisées par control optimal (déplacement de points d’ancrage, transmissibilité de forces, et niveau de bruit) et leur capacité à réduire le bruit à l’intérieur d’un habitacle d’automobile est étudiée.
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CHAPTER 1

1) INTRODUCTION

Since its invention by Karl Benz in 1885 (Fig. 1-1), the automobile has lived a never-ending evolution. Today, this invention is the driver of an important international market in full effervescence. Car manufacturers are constantly in search for new ways to gain advantages over their competitors. AUTO21 is a Network of Centres of Excellence funded by the government and industry in Canada. It is specifically dedicated to automobile research. It regroups over 24 universities within Canada and supports more than 500 researchers in various disciplines such as sociology, Medicine and Engineering.

As technologies and manufacturing methods evolved, cars have become lighter and more powerful. Car frames are consequently subjected to stronger forces while being less stiff. This inevitably leads to vibration and noise generation. The acoustic environment of a vehicle has become a key factor in sales because of the perception of quality it gives to the product. Also, information technologies are beginning to be slowly implemented into cars (cellular telephones, global positioning systems, etc.). Car manufacturers are now considering acoustic quality in their design processes.

Many improvements have been achieved concerning acoustic comfort inside automobiles. For example, sound packages (acoustic and damping treatment) are now included inside floors, ceilings and other structural parts to reduce high frequency noise. Also, the evolution of computer science has allowed implementing active noise control technologies for a relatively low cost: loudspeakers are used with Active Noise Cancellation (ANC) to reduce low frequency noise. However, these techniques have their limitations: foams are limited to high frequency because a thicker sheet of foam would take too much space and results in excessive weight; ANC is limited to low frequency noise because too many speakers are required if high frequencies are to be controlled.
At first, suspensions were included on cars to filter out obstacles and improve ride comfort. More recently, researchers realized that suspensions could also reduce road-induced vibrations & noise: road vibrations have been found as the main interior noise source in the 80 and 125 Hz octave bands [3]. The AUTO21 FIN-03 project was launched in 2001 to study and implement Active Structural Acoustic Control on a car suspension to reduce road-induced interior noise. [1] The rest of this chapter presents the structure of this document.

1.1) Literature review

The literature review will help to identify the main contemporary automotive main noise problems and characterize road-induced noise and its transmission path(s). Also, a general overview of existing noise reduction solutions will be done to observe the primary trends of sound reduction research:

- Passive noise reduction techniques;
- Semi-active noise reduction technologies:
  - Control algorithms;
  - Actuator(s)/Sensor(s) positions and type.
- Active Noise Cancellation (ANC) and Active Structural Acoustic Control (ASAC);
- Active suspension systems for vibratory and acoustic control:
  - Control algorithms;
  - Actuator(s)/Sensor(s) positions and type.

The review presented in Chapter 2 reveals a common problem in most ASAC problems: the positioning of actuators and sensors is usually done by trial and error. With the help of tools previously designed within the FIN-03 project group, this thesis will describe the different steps taken to further develop an analytic active suspension optimization tool that reduces the time needed to locate actuator(s) and optimize their efficiency.

1.2) Elaboration of a analytic quarter-car suspension model:

The first step to create an active suspension optimization tool is the development of a suspension model. In previous studies within the FIN-03 project, Douville & al. have designed a quarter-car suspension test bench that could be used as a basis for the creation of the model. [28] Therefore, the aim of Chapter 3 is thus to:
Define the system to be modeled:

- Review the specifics of a 1998, V6, Ford Contour «McPherson » front suspension, which is the suspension actually under study on the test bench;
- Obtain the quarter-car suspension test bench force transmissibility frequency responses functions (FRFs) and the car vibroacoustic numerical FRFs of the car frame/air cavity from a FEM model. The combinations of the suspension FRFs with the vibroacoustic FRFs gives the complete transmission path, which contributes to the perceived road noise.

Create a simplified quarter-car suspension vibroacoustic model:

- Validate the different steps and assumptions used to implement a model incorporating discrete model of certain suspension components and equivalent continuous models of other components;
- Describe and validate the algorithm, the genetic operators and the evolution laws used by a genetic algorithm to create a quarter-car suspension model that matches the test bench force transmissibility FRFs;
- Validate the three-dimensional vibroacoustic model of the suspension over the 0–250 Hz frequency band with the testing of the implementation of an experimental control actuator.

This model, which matches the vibro-acoustic behaviour of the suspension test bench, will be exploited within the optimization tool to test different actuator configurations without experimental burden.

1.3) Experimental study of road excitation

Up to now, the tools designed for the FIN-03 project have been entirely based on literature review or on laboratory measurements. The study of the vibroacoustic behaviour of a complete Chevrolet Epica LS automobile provides precious data to:

- Validate and/or adapt the test bench excitation mechanism to reproduce realistic road excitation;
- Compare the frequency response functions obtained with the test bench against those obtained on a complete car;
- Study the experimental feasibility of the ASAC strategy over a conventional suspension.
Simultaneously, it will be possible to observe different vehicle interior noise sources of a car powered by an internal combustion engine. This fourth chapter will therefore analyse when possible the relative importance of road-induced interior noise compared to other interior noise contributors under specific conditions.

1.4) Elaboration of an optimization tool for active suspension

The fifth chapter will use the suspension model of Chapter 3 and the filtered excitation of Chapter 4 to study a variety of actuator configurations. This chapter aims to:

- Elaborate, validate & study an active suspension actuator positioning algorithm capable of optimizing the position and orientation of one or many actuators depending on the actuator characteristics;

- Study the potential of one or several control actuators placed in a parallel configuration with a conventional suspension under different configurations:
  - A single actuator with one suspension linkage;
  - Two independent actuators with one suspension linkage each;
  - A single actuator with two-suspension linkages.

- Study the effect of minimizing different cost functions on interior noise:
  - Minimization of suspension link displacement;
  - Minimization of suspension link force transmissibility;
  - Minimization of cabin sound pressure level at driver’s ears.

The results of the optimization algorithm will suggest specific actuator configurations to test experimentally on a quarter-car test bench in future studies.
CHAPTER 2

2) **LITERATURE REVIEW:**

There is no trivial solution when trying to implement active control of a noisy environment. Any given system will have intrinsic properties that must be taken into account for sufficient control to be achieved. In the case of an active car suspension, the understanding of vibratory and acoustic mechanisms for road noise generation and transmission is crucial for designing an adapted control system.

Then there is noise control itself. The subject is vast, but noise reduction techniques can be divided into three main categories: passive control, semi-active control and active control. All have advantages and limitations. Each noise reduction method must respect certain requirements and principles to be implemented adequately: actuator(s) and sensor(s) types, actuator(s) and sensor(s) positions, control algorithms, etc.

An analysis of these different subjects provides a global picture of the actual challenges in road noise reduction technologies. It will also help to find existing methodologies to implement ASAC control solutions and analyse the results achieved in previous studies.

2.1) **Interior noise in an automobile**

All automotive interior noise problems can be divided into three elements: the receiver(s) the noise source(s) and the transmission path(s) between them.

2.1.1) Receivers (passengers)

First of all, one must distinguish vibration comfort from acoustic comfort: for example, large vertical wheel displacements have been measured around 15 Hz. This “wheel hopping” mode can generate vibrations perceptible by some human organs and is known to generate long-term fatigue to the car passengers. However, they are not heard since humans only perceive sounds between the frequencies of 20 Hz and 20 kHz. This thesis is only concerned about interior acoustic comfort or vibrations that are heard by passengers.

The human ear has a logarithmic sensitivity; Sound Pressure Levels (SPL) are consequently measured in decibels (dB). [20] By definition, the power of a sound goes as the average of the squared pressure $P$ during a certain period $T$ (Eq. 2.1 & 2.2).
\[ P_{\text{eff}} = \frac{1}{T} \int_0^T p(t)^2 \, dt \]  
(2.1)

\[ L_p = 10 \log_{10} \left[ \frac{P_{\text{eff}}}{P_0^2} \right] \]  
(2.2)

Where \( P_0 \) is the standard threshold of audibility: \( 20 \times 10^{-6} Pa \).

The outer part of the human ear is known to influence the perception of incident noise (Fig. 2-1): its geometry is able to amplify certain pressure waves depending on the position of the noise source. This is one of the human body mechanisms that can localize noise sources in a three-dimensional environment. [31]

![Image of pressure distribution at resonances in outer part of the human ear](image)

**Fig. 2-1: Pressure distribution at resonances in outer part of the human ear [31]**

The *phon* is a unit that is related to \( dB \) by the psychophysically measured frequency response of the ear (Fig. 2-2). At 1 kHz, readings in phon and \( dB \) are, by definition, the same. The phon scale is a function of both the frequency and the sound level.

![Image of phon scale](image)

**Fig. 2-2: The phon scale (from Fletcher, H. & Munson, W.A.; 1933) [20]**

6
Standard *phon* correction curves have been created: the contour A presented in Fig. 2-3 more accurately approximates the human hearing response for sounds of medium loudness. Contours B and C are more appropriate for monitoring loud sounds (C is used for traffic noise surveys). For most purposes, a survey of $dB(\text{lin})$ and $dB(A)$ measurements constitutes a practical assessment of the sound field (Eq. 2.3).

$$L_P(dB(A)) = L_P(dB(\text{lin})) - 11.15 \left( \log_{10} f \right)^2 + 75.2 \left( \log_{10} f \right) - 125.25$$  \hspace{1cm} (2.3)

Where $f$ is the considered frequency. The present work will present sound pressure levels in $dB(A)$ in the chapters to follow.

2.1.2) Sources of noise

Fig. 2-4 shows a list of typical noise sources in an automobile with an internal combustion engine. Literature distinguishes two main categories of noise sources: structural sources and airborne sources.

Fig. 2-4: Noise sources on an internal combustion engine automobile
For example, typical car acoustic sources are:

- Aerodynamic noise of airflow over the exterior surfaces (turbulence);
- The tire horn effect caused by air compression between the tire and the road;
- Exhaust noise created by the evacuation of the combustion gases;
- Noise generated by the Heat, Ventilation and Air Conditioning systems (HVAC).

Noise sources can be categorized into two types of excitations: deterministic or random. Deterministic noise sources can be anticipated by measurements. For example, a HVAC system generates deterministic pressure waves because of the rotation of the fan blades. Random noise due to turbulent airflow in ventilation ducts can not be anticipated.

![Blade passing frequencies](image1.png)

![Turbulent air flow](image2.png)

Fig. 2-5: HVAC noise spectral analysis [18]

The frequency content of a blade noise is dependent of the fan speed and the number of blades. Then, if a tachometer is placed on the fan, the speed information could be used to help cancel the blade passing frequency noise by active control. Random noise, on the other hand, cannot be predicted exactly, and because of this, is much harder to actively control. One example of random acoustic sources is the turbulence generated by the expelled air from a fan (Fig. 2-5). Its energy is principally contained inside the 0-2 kHz frequency band. [18]

Disturbances created by air turbulence around a moving car are also a random noise source with its energy below 2 kHz. [19] However, this noise becomes an important contributor to interior noise if the car is travelling faster than 100 km/h.

Structural noise originates from structural vibrations of the car components. Examples of structural noise sources include:

- Internal combustion engine (Rotating shafts, gas combustion, etc.);
- Road irregularities (bumps or road profiles);
- Vibration induced by brake callipers when contacting the brake disks;
- Interaction between transmission gears.

Like acoustic sources, structural sources can be deterministic or random. A deterministic structural source example would be the vibration induced by transmission gears (occurring at the gear meshing frequency) while a random excitation would be the
vibration generated by bumps and cracks on the pavement. Many articles confirm that most of the structural excitation energy is located within the 0-500 Hz frequency band. [2, 3, 5, 6 & 8]

Kinoshita & al. [16] have shown that the internal combustion engines are an important source of vibration in the 50-400 Hz range. More specifically, vibration created by the rotation of the engine, combined with a floor resonance and a dashboard resonance, creates an important “rumble noise” between 100 & 200 Hz during acceleration.

Gu & al. [17] have studied the effect of road/suspension interaction on car interior noise. They have detected important correlation between road excitation and interior noise below 250 Hz. More specifically, this excitation is maximum when the car speed is 50 km/h (13.9 m/s), which is the standard maximum speed in North American cities.

Cebon & al. [21] have studied road-ageing mechanisms. Their analytic road profiles were based on a Draft ISO formulation (Eq. 2.4). The spectral density \( S_u(\kappa) \) of road irregularity \( u \) were categorised in different road classes.

\[
S_u(\kappa) = \begin{cases} 
S_u(\kappa_0) \left( \frac{\kappa}{\kappa_0} \right)^{-n_1} & \text{if } \kappa \leq 1 \\
S_u(\kappa_0) \left( \frac{\kappa}{\kappa_0} \right)^{-n_2} & \text{if } \kappa > 1 
\end{cases}
\]  

(2.4)

Where \( \kappa \) is the wavenumber & \( \kappa_0 \) is the reference wavenumber;
\( S_u(\kappa) \) is the displacement spectral density;
\( S_u(\kappa_0) \) is the spectral density at \( \kappa_0 \).

The constants are: \( n_1 = 3, n_2 = 2.25, \kappa_0 = 1 \) and \( S_u(\kappa_0) \) values are described in Table 2-1.

<table>
<thead>
<tr>
<th>Spectral densities of typical European roads</th>
<th>10^-6 m^2/cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Road Class</td>
<td></td>
</tr>
<tr>
<td>Very Good</td>
<td>2-8</td>
</tr>
<tr>
<td>Good</td>
<td>8-32</td>
</tr>
<tr>
<td>Average</td>
<td>32-128</td>
</tr>
<tr>
<td>Poor</td>
<td>128-512</td>
</tr>
<tr>
<td>Very Poor</td>
<td>512-2048</td>
</tr>
</tbody>
</table>

Table 2-1: Road class power spectral density values [21]

Fig. 2-6: Road profile spectral density curve [21]
Fig. 2-6 shows the standard road class spectral densities, but also experimental results obtained near the wheel paths (full line) and away from the wheel paths (dotted line). If the car speed is known, it is possible to change Eq. 1.4 from a wavenumber scale to a frequency scale with Eq. 2.5.

\[ f = \left( \frac{V_{car}}{2\pi} \right) \kappa \]  \hspace{1cm} (2.5)

Where \( V_{car} \) is the car speed \((m/s)\), \( f \) is the frequency corresponding to the wavenumber \( \kappa \).

2.1.3) Noise transmission paths

There are two types of transmission paths in car vibroacoustic excitations: structure-borne transmission and airborne transmission. Noise generated by an HVAC fan is a good example of airborne transmission: the air transmits the pressure waves generated by the fan to the ears of the passengers.

For structure-borne transmission, Constant & al. [3] demonstrated that defects or texture of the road generate forces on the suspension and wheels. The suspension, being attached to the car frame, deforms allowing vibrations to be transmitted from the road to the car. This creates a structure-borne path. Finally, the vibration of the car panels and frame generates pressure waves inside the car allowing passengers to hear road-induced noise.

Park & al. [22] have studied the correlation of road/suspension with car interior SPL. Accelerometers have been placed on different suspension components and good coherence was observed (Fig. 2-7). They have detected important participation of suspension vibrations on interior noise bellow 800 Hz.
One must also know how a given car component (e.g. wheel, suspension, frame) participates to the transmission. For example, when a hammer hits a bell, you can hear a distinctive sound. However an impact is supposed to excite all frequencies at the same level. This is the proof that the dynamic response of the structural components on the transmission path has an impact on received noise. As seen on Fig. 2-8, the global sound of a bell is tainted by a combination of resonances.

![Fig. 2-8: Tire and bell modal analysis comparison](image)

In the physical domain, each resonance represents a preferred geometric deformation of the bell. As long as the bell integrity or geometry is unchanged, the sound emanating from the bell will be the same. Just like a bell, car components have a specific geometry and are linked between each other in a way that will lead them to resonate at specific frequencies.

The interest of modal analysis for road-induced vibrations is that if the suspension vibration behaviour can be modeled, it can also be predicted and cancelled. Then, even if road excitation is a random excitation source, the prediction of the vibroacoustic transmission path will help to cancel suspension’s resonances and noise heard by the driver can be reduced. The inspection of specific suspension components or the analysis of car subsystems should increase our understanding of the transmission paths and their effects on road-induced interior noise.
2.2) Analysis of road-induced noise transmission paths sub-systems

The road noise transmission paths will be studied in three sections: the wheel, the suspension and the car frame/cabin. Since most road induced structure-borne interior vibrations and noise problems appear below 250 & 500 Hz respectively [5], the modal analysis will concentrate on this frequency band.

2.2.1) Wheel modal analysis

Wheels are made of three components: the tire, the rim and the air chamber. Wheels are very complex to analyse because of the numerous materials employed in their fabrication and also because rubber is non-linear. Also, the reproduction of exact operating conditions represents a challenge in lab environments: wheel rotation creates an asymmetry between the left and right section of the tire proportional to its rotation speed while the weight of the car creates another asymmetry between the top and lower parts of the tire.

![Fig. 2-9: Wheel Finite element model [4]](image)

![Fig. 2-10: FEM first rim resonance at 290 Hz [4]](image)

Ni, & al. [4] have detected that interior SPL around 300 Hz is about 5-7 dB(A) higher in a vehicle equipped with aluminium wheels rather than steel design. They modeled an AMERI*TECH P205/65R15 tire on a standard aluminium rim (Fig. 2-9) with MSC/NASTRAN®. No rotation effect of the wheel is taken into account. Ni’s wheel model was virtually inflated to 35 psi and loaded with 900 lbs to simulate the car’s weight. Experimental Frequency Response Functions (FRFs) simulated the rest of the transmission paths, starting at the wheel hub and ending at the driver’s ears.
Ni’s model showed a first “potato-chip” rim resonance at 290 Hz (Fig. 2-10) and two air cavity resonances at 223 & 227 Hz. A Design Sensitivity Analysis (DSA) was performed to find that a slight increase of the spoke thickness and rim area closest to the spoke would raise the “potato-ship” rim resonance from 288.6 to 302.2 Hz while adding only 1.9 % of additional mass to the original concept. Results showed that the modified rim decreases by 5 $dB(A)$ the Sound Pressure Level (SPL) in the 280-320 Hz frequency band.

A study supported by *BMW & Goodyear* [3] aimed to identify methods to reduce road-induced structure-borne interior noise. FRFs were measured to identify the main suspension parts affecting the interior noise at target frequencies. The results showed that structure-borne noise was mainly in the 125 Hz octave band when using a 15” tire. However, when using a 16” tire, the 80 Hz octave band became more important.

![Fig. 2-11: Suspension transmissibility FRF: 15” VS 16” tire [3]](image)

Tire force transmissibility FRF measurements allowed the identification of vertical tire resonances around 20, 85, 120 and 150 Hz (Fig. 2-11). A DSA was executed on a tire/air chamber FEM model with a fixed rigid rim, by testing different air chamber pressures, a 2 bar pressure decrease (35 psi to 14 psi) shifted the second tire resonance down 25 Hz and reduced noise level by 5 $dB(lin)$ between 300 & 350 Hz.

Douville & al. [2] performed a Doppler Laser Vibrometry scan (DLV) of a static wheel placed on a quarter-car suspension to observe tire/air chamber resonance of a non-rotating tire between 0 and 250 Hz: first, the spatially averaged quadratic transverse velocity of the tire surface was measured to locate resonance. Second, mode shapes were obtained at these specific frequencies. All transverse tire mode shapes detected by this study are presented in Table 2-2.
<table>
<thead>
<tr>
<th>Circumferential mode order and its description</th>
<th>Mode type</th>
<th>Natural frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - Breathing mode of the tire</td>
<td></td>
<td>56.5 Hz</td>
</tr>
<tr>
<td>2\textsuperscript{nd} - Bending mode of the circumference of the tire</td>
<td></td>
<td>87.8 Hz</td>
</tr>
<tr>
<td>3\textsuperscript{rd} - Bending mode of the circumference of the tire mixed with the 6\textsuperscript{th} of the entire suspension assembly</td>
<td></td>
<td>146.0 Hz</td>
</tr>
<tr>
<td>4\textsuperscript{th} - Bending mode of the circumference of the tire mixed with the 5\textsuperscript{th} of the entire suspension assembly</td>
<td></td>
<td>194.3 Hz</td>
</tr>
<tr>
<td>5\textsuperscript{th} - Bending mode of the circumference of the tire</td>
<td></td>
<td>216.6 Hz</td>
</tr>
<tr>
<td>7\textsuperscript{th} - Bending mode of the circumference of the tire mixed with the 10\textsuperscript{th} of the entire suspension assembly</td>
<td></td>
<td>238.4 Hz</td>
</tr>
</tbody>
</table>

Table 2-2: Tire X-axis resonance description – 14’’ tire, 35 psi with a 3.34 kN load. [2]

Similarities are observable in resonance frequencies between \textit{BMW/Goodyear} and Douville’s results: tire modes are detected at 88 and 146 Hz. On the other hand, some modes are not detected in both studies (e.g. transversal mode at 56 Hz): Douville observed transversal resonances with a vertical spindle excitation while \textit{BMW} observed vertical resonances with a vertical lower A-arm excitation. The frequency response of system being dependent on the positioning and the orientation of the excitation force, certain modes could have been more or less excited in one case or another.
2.2.2) Car suspension vibration analysis

Kido & al. [8], in collaboration with Toyota Motor Co., wanted to reduce the vibration transmission of two noise peaks (around 140 & 250 Hz) typically seen in front wheel drive cars equipped with McPherson suspensions (Fig. 2-12).

![Fig. 2-12: Interior SPL of an automobile equipped with McPherson suspension [8]](image)

![Fig. 2-13: Finite element model of a quarter-car suspension [8]](image)

To better understand the suspension vibroacoustic behaviour, each component was modeled by shell or solid elements (Fig. 2-13). To save calculation time, some suspension components, like the lower A-arm and the tire, have been transformed into analytical models. Rubber bushings and axle bearings were modeled with dynamic spring constants derived from experiment. An excitation force was applied in the lateral direction on the lower end of the wheel rim.

![Fig. 2-14: Deformed suspension FEM under a lateral effort at 135 Hz [8]](image)

![Fig. 2-15: FRF validation of the FEM suspension model [8]](image)
By measuring the model’s inertance FRF between the excitation and the lateral displacement of at the strut tower linkage (Fig. 2-15), Kido identified the origin of the 150 Hz noise peak to a suspension transversal resonance (Fig. 2-14) and the 250 Hz noise peak to a tire air cavity resonance.

Kido also explains that most suspension modes below 80 Hz are dominated by rigid body motion. Above this frequency, the contribution of component modes is more important. To reduce lateral force transmissibility, it was proposed to increase the stiffness of lower A-arm bushings. An increase of approximately 200% in rigidity (6.5x10^3 N/mm to 11x10^3 N/mm) effectively reduced by 5 dB(lin) the lateral force transmissibility of the suspension in the 110-150 Hz frequency band.

Douville & al. [2] has conducted a force transmissibility study of a quarter-car suspension combined with an Operational Vibratory Shape Analysis (OVSA) to identify resonance frequencies. The transmissibility between road excitation and each direct suspension frame linkage (linkage 1 to 5 on Fig. 2-16) were measured on a quarter-car suspension test bench. His study identifies important vertical suspension resonances around 25 Hz, which corresponds to the wheel-hopping mode, but also at harmonic frequencies. Harmonics have also been associated with multiple degrees of freedom global suspension resonance. For example, at 75 Hz, the suspension rotates on the Z-axis while moving up and down.

The lower A-arm shows four clear resonances in the 0-250 Hz band (Table 3). His study surprised that it is an important transmission path for lateral (X) and vertical (Z) efforts. As seen in Fig. 2-16, lower A-arm links are the second most important energy transmitter to lateral effort (link 3, X axis) and the principal contributor for vertical effort (link 4, Z axis). This energy transmission distribution can be explained by the fact that the wheel, strut tower and lower A-arm centre of mass are not aligned: it results in moment generation in the suspension. The angular efforts generate forces in all three axes, but in the X and Z-axis particularly.

Finally, coil spring resonances have also been detected. Its first mode (39 Hz) is a vertical motion of the coils themselves. This displacement was sometimes so important that coils were impacting with the base plates of the suspension, thus creating non-linear response. Other coil resonances, generated by torsion and translation of the coils, were also detected in the transversal FRFs (X & Y).
<table>
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<tr>
<th>Frequency</th>
<th>Test bench</th>
<th>Scanning vibrometer</th>
<th>Relative error</th>
<th>A-arm</th>
<th>Rapid rotation</th>
<th>Entire suspension</th>
<th>Coil spring</th>
<th>Time</th>
<th>Relative contribution</th>
<th>Predominant linkages</th>
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</thead>
<tbody>
<tr>
<td>Hz</td>
<td>Hz</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
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<tr>
<td>237.00</td>
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<td></td>
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<td></td>
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</tbody>
</table>
2.2.3) Car frame/Air cavity modal analysis

Douville & al. [2] created a geometric model of a 2000 Honda Civic car frame using CATIA® (Fig. 2-17). The model was converted into the NASTRAN/PATRAN® software, in a Finite Element Model (FEM) to calculate the structural response of the car frame. Car panel stiffeners located inside doors or floors are not present in the model. The NASTRAN velocities are then imposed as the boundary conditions of a cabin air cavity BEM under COMET/ACOUSTICS® (Fig. 2-18). The damping caused by acoustic materials and passengers inside the air cavity is not considered.

Fig. 2-17: Honda Civic 2000 FEM frame [2]  
Fig. 2-18: Honda Civic 2000 BEM air cavity [2]

To obtain the car numerical FRFs from the FEM/BEM model, structural excitation was applied at each front left suspension links separately. Each link was excited with 1N in the X, Y & Z-axis. Fig. 2-19 shows two driver’s SPL/Suspension link force FRFs measured with the model.

Sound pressure level measured at the driver’s head location for an input force of 1N, along the X-axis direction, at respective suspension/chassis linkages

Fig. 2-19: X-axis FRFs of a Honda Civic body FEM/BEM [2]
Douville’s coupled models identified a total of 622 modes in the 0-250 Hz frequency band. The most important air cavity modes are seen around 75, 140, 165, 190, 210, 240 & 250 Hz. Specific data concerning structural modes are not discussed in this study.

Kim & al. [7] conducted an experimental modal analysis on a Hyundai mid-size utility car frame to reduce the coupling of structural and acoustic modes. A single vertical shaker was attached on the front left lower arm suspension link. Accelerometers were placed on the frame to measure structural response while microphones measured acoustic response inside the car cabin. The analysis was done between 20 & 200 Hz. Table 2-4 shows the results of the analysis.

<table>
<thead>
<tr>
<th>Structural resonance (Hz)</th>
<th>Acoustic resonance (Hz)</th>
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<tbody>
<tr>
<td>#</td>
<td>Exp.</td>
</tr>
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</tr>
<tr>
<td>2</td>
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<tr>
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</tr>
<tr>
<td>4</td>
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<tr>
<td>13</td>
<td>166.5</td>
</tr>
<tr>
<td>14</td>
<td>184.1</td>
</tr>
</tbody>
</table>

Table 2-4: Structural & Acoustic resonances of a mid-size car cabin [7]

As seen in Fig. 2-20, modes around 140 and 150 Hz are clearly amplified by the interaction of both components. To reduce structural/air cavity resonance coupling, cabin panels quadratic speed were measured at resonance frequencies and damping material were added to the most important contributors: the addition of damping changes the phase of a car panel’s response. This way, coupling coefficients are reduced. An average noise reduction of 5 dB(A) was achieved within the 0-200 Hz frequency band.

Considering that Kim used a mid-size automobile for his modal analysis, it is interesting to observe that Douville FEM/BEM also detected the 2nd, 4th, 5th & 8th acoustic modes found by Kim at 84, 150 & 190 Hz. But further analysis would be necessary to confirm a match between the modes found by Douville and Kim because of the much higher number of modes found by Douville.
2.3) Noise and vibration reduction methods

Many articles discuss car sound quality, but when it comes to road-noise reduction technologies, three main approaches can be used: passive, semi-active and active control. The next sections will review recent acoustic control techniques while concentrating on semi-active active control technologies. Simultaneously, it will attempt to demonstrate the relevance of implementing ASAC on a suspension to reduce road-induced vibrations.

2.3.1) Passive noise and vibration control

Passive methods are of common use in cars generally because of their low cost. There are two types of passive reduction methods: alleviation of airborne noise or damping of structural vibrations.

2.3.1.1) Sound absorbing materials

Acoustic materials are included in car cabins to reduce noise: foams are included inside ceilings & roofs; carpets are placed on floors, etc. These materials absorb sound waves inside the cabin and are very efficient for high frequencies. However, acoustic materials need to be thick if low frequencies are to be absorbed. This is problematic since manufacturers want to maximize space inside cars while keeping weight and cost to a minimum.

2.3.1.2) Passive suspensions

Most modern cars are equipped with passive suspensions. A coil is combined with a damper to allow isolation of wheel vertical displacements while bushings placed at principal chassis links damp road irregularities. Passive suspensions will be described in details in Chapter 2, but the physical limits of a passive suspension lies in the compromise between road handling (rigid suspension) and ride comfort (flexible suspension). Passive suspension vibroacoustic optimization articles have already been mentioned in previous sections: Kido & al. [8] adjusted bushing rigidity, Ni. & al. [4] optimized wheel rim stiffness, etc. All these articles adapt stiffness and damping of structures to reduce road noise while trying to minimize effects on handling.
2.3.2) Semi-active noise and vibration control

The main distinction between semi-active suspensions and passive suspensions is their capability to change their stiffness or damping over time. The objective of a semi-active suspension is to dissipate kinetic energy while minimizing efforts applied to the car frame. These technology are widely used for ride comfort objectives, but are also used for low-frequency noise control.

2.3.2.1) Magnetorheological (MR) & Electrorheological (ER) materials

The stiffness of MR or ER materials vary proportionally to the current that passes through it. This principle was used to create a semi-active damper: conventional viscous dampers dissipate energy by friction and/or fluid displacement. By adjusting the MR/ER damper fluid viscosity as a function of time, it is possible to stiffen the suspension for better car handling while permitting suspension flexibility when a road irregularity is encountered. Fig. 2-21 shows Seung-Bok & al. [9] experimental semi-active damper. Their experiment successfully diminished suspension vibration up to 50 Hz.

Commonly known as « Sky-hook damping », the control principle of a semi-active suspension usually consists in simulating a virtual viscous damper with the four MR suspension damper. The virtual damper stabilizes the car frame vertical displacement in time while the wheels follow road irregularities (Fig. 2-22).

Damper reaction force is applied only when the relative displacement between the car frame and the wheel is diminishing (Eq. 2.5).
Where \( z \) is the relative displacement between the car frame and the car wheels: 
\[
    z = z_{frame} - z_{wheel}.
\]

A US patent by Watson for Ford Motor\(^{\circ} \) [23] controls the varying stiffness of an ER polymer bushing to reduce vibrations related to vehicle lurch. This occurs when the driven wheels move longitudinally relative to the chassis under forces applied by a large acceleration. These semi-active bushings solves this problem by applying skyhook damping to the suspension lower A-arm.

### 2.3.2.2) Electromechanical systems

There are two main types of semi-active electromechanical suspensions. The first type uses viscous friction to dissipate suspension energy: an electric motor is placed on a modified damper to regulate the diameter of the orifice by which fluid moves from one chamber to another. Teramura & al. [10] have implemented skyhook damping with an experimental damper shown at Fig. 2-23. His tests gave good results up to 75 Hz, but literature suggests that good system response can be expected up to 100 Hz. [12]

The second type uses dry friction: just like a standard car breaking system, hydraulic or piezoelectric actuators are used to control the contact pressure between two surfaces. Hydraulic damping control is not feasible over 75 Hz because of system reaction speed limitations, although piezoelectric devices can work at much higher frequencies: Chatterjee & al. [11] have demonstrated the feasibility of controlling by sky-hook damping low force, high frequency excitations up to 1 kHz. A general presentation of the piezoelectric damper is shown in Fig. 2-24.
2.3.3) Active noise and vibration control

Active noise control is different from passive or semi-active control by the fact that active control strategies inject energy out of phase of the disturbance signal to control it. There are two types of active noise control in cars, Active Noise Cancellation (ANC) and Active Structural Acoustic Control (ASAC).

2.3.3.1) Active Noise Cancellation (ANC)

Dehandschutter & al. have written many articles concerning car active noise reduction techniques [6, 14 & 15]. One of their articles discusses ANC: a typical car sound system is coupled with wheel reference sensors to actively cancel pressure waves created by road-induced vibrations inside a car cabin (Fig. 2-25).

Good noise reductions were obtained up to 250 Hz, but limitations were encountered at higher frequencies: more speakers are necessary to recreate exact sound pressure distribution in the air cavity. Consequently, this system can only create quiet zones around error microphone positions and can amplify noise in other areas of the cabin. This is a problem since passengers do not always have their head at the same position inside the car.

2.3.3.2) Active Structural Acoustic Control

Transmission path analysis has shown that car panels are part of the road-induced problem. Kim & al. [7] have attempted to control structural acoustic transmissions by adding piezoelectric actuator & sensors onto car panel surfaces (Fig. 2-26). This technique results in a good noise reduction for a large frequency bands, but the number of piezoelectric devices needed to fully control each panel is too prohibitive for a potential commercial application.
Another ASAC method is to add an actuator on the transmission path of the disturbance as near as possible to its source. This type of active control shows better performance than any other active control techniques because:

- If the vibrations are reduced before they can propagate, this will result in a global noise reduction inside the cabin;
- If actuator(s) position(s) is wisely selected, the number of actuators needed to apply control can be minimized since the number of transmission paths generally diminishes as one gets closer to the noise source;
- The controlled frequency band is function of actuator capabilities rather than the number of actuators and sensors used.

For all these reasons, the structural acoustic control of a car suspension appears to be the best approach to reduce road-induced interior noise. The following section will explain in detail existing active suspensions and comment on actual challenges and experimental results of ASAC.

### 2.4 Active suspensions

An active suspension can use one or many actuators, in parallel or in series with conventional suspension components, or use a modified suspension component (Fig. 2-27). However, to achieve good control performance, actuators must be combined with a robust algorithm using a realistic mathematical model of the suspension.

Many papers have been written on active suspension ASAC. Some are dedicated to suspension simulations; others are more concerned about control algorithms. This review concentrates on articles discussing experimental implementation of active suspension control: for each study, the main characteristics, the control algorithm, the experimental procedures and their experimental results will be presented.
2.4.1) Active suspension models

Suspension models can be accomplished analytically or by experimental measurements. However, simulation of an active suspension is relatively difficult because conventional passive suspensions are non-linear and the addition of an actuator only adds complexity to the system. Another important factor is that the vibroacoustic behaviour of the suspension can change with time: the number of passengers, tire inflation, road quality, etc. This means that one must be able to modify the model in real time to correctly respond to any intrinsic system variations.

Even if a suspension is non-linear, researchers usually prefer to use linear models because they are generally sufficient when systems are slightly non-linear, but also because higher order models need greater computational power. The type of model also needs to be chosen wisely: a state-space model is commonly used to observe a system's stability, but they require relatively important calculation power to be created compared to a simple time-domain input-output model. [24] Consequently, characterization of a state-space model must be done “off line”. In the case of a rapidly changing system, the controller will not be able to adapt. On the other hand, system FRFs techniques can be used to correct on-line, system variations in real time.

2.4.2) Active suspension controllers

An adaptive controller needs to characterize the noise source and the system itself in real time. Usually, sensors placed on the controlled system give this information. When looking at the transmission paths, this information can be measured upstream or downstream the actuator, which distinguishes the two main types of active controllers: feedback or feedforward controllers.

2.4.2.1) Feedforward controllers

A feedforward controller uses an input sensor located between the noise source $\nu$ and the actuator to anticipate the disturbance $d$ to control (Fig. 2-28). The transfer function between the noise source and the input sensor is called the sensor path $P_s$. To adapt the active controller an error sensor $e$ is used. The transfer function between the noise source and the error sensor is called the disturbance path $P_e$. The transfer function between the actuator and the error sensor is called the secondary path $G_e$. To control the disturbance, an electronic controller $H$ sends a control command $u$ to the actuator. The addition of the control signal to disturbance signal leaves gives the residual error signal $e$. In theory, this
signal should be equal to zero at all time. In practice however, this type of configuration has its constraints. When opposed to a random disturbance, causality must be respected: the time needed to evaluate the control command must be less important than the time needed for the disturbance to propagate to the actuator. In the case of a suspension, the input reference sensor should be placed as near as possible to the road excitation.

![Feedback system controller diagram](image)

**Fig. 2-28: Feedforward active control systems [24]**

The system can adapt its command in real-time with a LMS algorithm and can also use an uncorrelated noise source (typically a signal sent by the actuator itself) to identify transmission paths modifications. This allows characterization of the noise source and the secondary transmission path “on-line” in a relatively short period of time. However one must be cautious, if the adaptation coefficient is too important, the system can become unstable since the controller is a closed loop system.

### 2.4.2.2. Robust Feedback controllers

Robust feedback controllers are used when it is impossible to obtain time-advanced disturbance information (Fig. 2-29). Colleagues from McGill University are trying to implement LQR & $H_\infty$ controllers on a quarter-car suspension test bench within the AUTO21 project [25].

![Feedback system controller diagram](image)

**Fig. 2-29: Feedback system controller [24]**
Particular attention must be paid to the system poles position when executing feedback control over a system since it can become unstable because of the recursive nature of the controller. In a $H_\infty$ controller, errors from sensor’s sensitivity, systems non-linearity or neglected dynamic behavior are considered as uncertainties to prevent controller instability. In that case, a recursive filter is more efficient than a non-recursive filter because fewer coefficients are necessary.

2.4.3) Previous suspension ASAC experiments for road noise alleviation

Actuator(s) position(s) is a factor that cannot be neglected since the efficiency of the controller is also linked to the capacity of the actuator to control specific frequencies: in the transmission path analysis section, Douville and Kido did not detect all tire modes because of the position and direction of the perturbation. This is also true for the control force. The actuator type is also important because each type has advantages and limitations. This section will aim to look at methodologies to identify optimal actuator positioning and the results obtained as a function of actuator type.

2.4.3.1) Hydraulic active suspensions

Lauwerys & al. [13] have built a simplified quarter-car suspension with an active hydraulic damper (Fig. 2-31). A mass simulating the car weight is placed on the suspension and the tire is placed on a dynamic shaker in parallel with a support system (Fig. 2-30). The system is modeled by measuring FRFs and a $H_\infty$ controller is used to diminish car body vertical acceleration.
This study succeeded to reduce body vibrations generated between 0 to 50 Hz. A reduction of 6 dB was obtained at the first two wheel resonances (1.5 & 15 Hz). Lauwers explains the frequency limitation of his active suspension by an insufficient solenoid valve reaction time and because of the non-linear nature of the hydraulic pumping system.

2.4.3.2) Electromechanical active suspensions

Douville & al. [2] have tested active control using a dynamic shaker (Fig. 2-32). A first shaker is linked to the suspension’s spindle to create the primary disturbance up to 250 Hz. A second control shaker was attached to the suspension’s lower A-arm.

Force sensors placed at the end of each suspension frame link provides the resulting forces in the X, Y and Z-axis. A car frame FEM previously mentioned in section 1.3.3 provides the vibroacoustic transmission path. The optimization of the actuator location has been done through FRF analysis: a force sensor was placed in series with the actuator piano wire link. The suspension was excited at different locations uniquely with the actuator and the FRF were compared with primary FRFs: the selected location was chosen depending on the number of identical resonance excited by the two shakers (Fig. 2-33). The final position was at the centre of mass of the lower A-arm in the vertical direction.

Fig. 2-32: Douville & al. setup: quarter-car suspension setup [2]

Fig. 2-33: Disturbance and actuator FRF comparison [2]
Active control was implemented with an adaptive feedforward controller. A force sensor placed in the primary dynamic shaker link gave the reference signal. Even if this configuration is not possible on a real car, the experiment results showed a SPL reduction at the driver’s head of $20 \, dB(lin)$ at 25, 39, 70, 140, 212 & 243 Hz.

2.4.3.3) Inertial shakers

Inertial shakers are known to be very efficient when specific frequencies are to be controlled, but are less efficient for broadband noise. Dehandschutter & al. [6, 14 & 15] have used this type of actuator on a real car in combination with Kuo & Morgan multiple error Filtered-X LMS feedforward controller [26] (Fig. 2-34). The feedforward control scheme was selected after experimental attempts showed that a feedback strategy was not feasible due to instability of the control loop.

![Fig. 2-34: Dehandschutter & al. complete automobile ASAC setup [15]](image)

FRFs were used to model the car’s transmission path: a total of 120 actuator positions were observed for actuator positioning optimization (30 actuator positions for each wheel). A set of 12 reference sensors (4 tri-axis accelerometers placed on each wheel spindle) and 6 error microphones were used with a multiple reference function to implement active control. [15] Results show a $6.9 \, dB$ noise reduction inside the 75 to 105 Hz frequency band at the driver’s head and $5.5 \, dB$ between 225 and 250 Hz.

2.5) Chapter summary

This general overview of the road noise problem leads to the following conclusions:

- Suspension resonances below 80 Hz are mainly due to rigid body displacement of suspension components. Deformation of suspension components becomes more important above this frequency.

- Contribution of the suspension to road noise, below 250 Hz, is mainly due to global suspension, tire, strut tower and lower A-arm resonances.
• A suspension is a dynamic system which changes in time depending on various factors, such as tire inflation, car load, road conditions, etc.

• Noise inside a car cabin is, in part, a result of the coupling of the suspension, the car frame and acoustic cavity formed by the cabin.

As for noise reduction technologies, our investigation has lead to conclude that:

• Compared to existing passive, semi-active or active control techniques, ASAC control of a suspension offers best potential for noise reduction of suspension resonances between 100 and 250 Hz;

• Electromechanical actuators offer the best potential of noise reduction inside this frequency band. Hydraulic systems are limited to approximately 75 Hz and consequently are more efficient for ride comfort;

• The positioning of control actuators is generally done by comparing primary and secondary frequency response functions;

• Active control of a suspension can be achieved by a linear controller and must be adaptive to adapt the controller to variations of the disturbance or control path;

• A time-domain, transfer function type, controller is preferable to a state-space model for active control because of calculation costs;

• A feedback is preferable to feedback controllers if reference information can be retrieved upstream of the actuator and causality is respected.

This information will help guide our decisions for the following chapters. It is interesting to see that no method was suggested as to the exact optimal location for an actuator on a suspension. Therefore, the following chapters will attempt to create a suspension model, on which a virtual actuator can be added, that can be used to reduce the experimental burden of finding an optimized set of actuator position and orientation for a suspension ASAC.
CHAPTER 3

3) MODELING OF A THREE-DIMENSIONAL QUARTER-CAR SUSPENSION

Like many researchers, Douville has positioned his control actuator by comparing primary and secondary path frequency response functions (FRFs): if most resonances observed in the primary FRFs are found in the secondary FRFs, the controllability is adequate and the actuator position should offer good control efficiency. However, there is no way of proving that this specific position is the best unless every possible actuator position is tested. This is why an analytical suspension model is required. However, creating a model of a system without a prior understanding of its intrinsic characteristics could lead to an erroneous model. This chapter intends to describe the model that was used to simulate the dynamic behaviour of the quarter-car suspension and the used to optimize the actuator configuration.

3.1) Suspension to be modeled

Chapter 2 presented a quarter-car suspension test bench and a Honda Civic car body FEM/BEM conceived by Douville & al [28]. Both tools were implemented to reproduce the vibroacoustic behaviour of a car sub-system. The combination of these two sub-systems (Fig. 3-1) creates a complete image of road-induced interior noise from the road to the passenger ears.

![Fig. 3-1: FIN-03 project Transmission path tool, [28]](image)

The quarter-car test bench can reproduce forces created at the main suspension frame linkages: the strut tower and lower A-arm bushings. This tool reproduces a SIMO (single input, multiple outputs) time-domain system capable of simulating suspension linkage forces for any given primary excitation $F_p$, or control force $F_s$. The second part of the transmission path (car frame/panels/cabin air cavity) is modeled by finite elements (FEM) and boundary elements (BEM). Forces measured at the suspension linkages are
virtually applied on the FEM. The deformation of the car frame creates vibrations of the air cavity and sound pressure levels caused by road excitation or the control forces (force of the suspension links) can then be calculated throughout the cabin.

Douville used this tool in conjunction with a quadratic minimization method to perform an analytical study of active control. It allowed evaluating the potential of ASAC control onto car interior noise and also actuator requirements for an adequate control. However, he used only one control actuator configuration (a vertical force applied on the suspension’s lower A-arm). No optimization of the actuator location and orientation was done.

3.1.1) Description of a McPherson front drive suspension

There are many types of suspension design, but the test bench currently includes a front drive, McPherson strut front suspension. It can transmit engine torque to the wheel by the mean of a driving shaft and has only three direct links to the car frame: the strut tower, and the two lower arm bushings. For clarity, let us define a standard axis system (Fig. 3-2): the lateral direction (X) is parallel to wheel rotation axis, the longitudinal direction (Y) is the front/rear car axis and the vertical direction (Z) is the vertical axis.

1. Goodyear 185/70/R14 tire
2. Mounted steel rim
3. Driving Shaft
4. Wheel spindle
5. Break disk
6. Break calliper
7. Wheel Hub
8. Lower arm
9. Lower arm bushing
10. Trailing arm
11. Sway bar
12. Damper cylinder
13. Coil spring
14. Tower bushing
15. Steering rod

![Fig. 3-2: Front McPherson strut suspension](image)

The wheel (tire/rim sub-system) forms a flexible torus in which air is compressed to support the car’s weight and also damp vibration felt by passengers (tire air cavity). The wheel is bolted onto the spindle, squeezing the break disk in between. The driving shaft is clamped into the spindle. They constitute the rotating module of the suspension and
ensure car acceleration and displacement. The rotating section of the suspension is attached to the wheel hub by two conical bearings placed around the wheel spindle. The wheel hub is the heart of the suspension because it links the rotating components with the non-rotating components such as the strut tower and the lower arms.

The damper shaft, main bone of the strut tower, is combined with a coil spring (Fig. 3-3). The coil spring is compressed between the damper rod and cylinder by two base plates. It supports the mass of the car while permitting adequate damper movement to dissipate kinetic energy. The upper end of the damper is linked to the main frame by a thrust bearing to permit wheel rotation (car manoeuvring). The lower end is clamped to the wheel hub.

The lower arms take lateral and longitudinal loads exerted on the suspension while permitting vertical displacement of the wheel. Vibration damping is ensured by rubber bushings located at the intersection of the arms and their frame linkages. A ball joint links the lower arms to the wheel hub.

Fig. 3-3: Fixed parts of a McPherson suspension
Fig. 3-4 & 3-5 show a total of seven links to the car frame: the strut rod and lower arm can be combined into an “A” shaped lower arm. It links directly the suspension to the car frame by 4 bolts (the bolts are linkage 1 to 4 on the quarter-car test bench). The strut tower is squeezed under the top of the car frame (5\(^{th}\) direct link).

![Fig. 3-4: frame links of a front suspension](image1)

![Fig. 3-5: Sway bar, steering rod & Driving shaft link](image2)

The sway bar (Item 11) unites the two front strut towers and creates a non-direct link to the car frame (6\(^{th}\) link). Finally, the steering rod (Item 15) also makes a 7\(^{th}\) non-direct link: a pinion to the car frame fixes it.

<table>
<thead>
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<th>Min</th>
<th>Reference</th>
<th>Max</th>
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</thead>
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<tr>
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<td>+3.40°</td>
</tr>
<tr>
<td>Camber</td>
<td>-1.90°</td>
<td>-0.60°</td>
<td>+0.80°</td>
</tr>
</tbody>
</table>

Table 3-1: Suggested angles for a Ford Contour 1998 front suspension

![Fig. 3-6: Wheel hub kinetic angles](image3)

The suspension must also respect a specific geometry. Fig. 3-6 shows the caster, camber and toe-in angles. Their role will not be described here, but they are the wheel hub orientations in the Y-Z, X-Z and X-Y plane respectively (Table 3-1).
3.1.2) Laboratory test bench configuration

Since the suspension model will be based on test bench measurements, it is crucial to distinguish between road conditions and the laboratory setup. The quarter-car suspension is compressed between a three-legged table and a triangular frame shown in Fig. 3-7. Both components are mounted on a massive concrete block. This results in quasi-infinite impedance of the frame supporting the quarter-car suspension. In reality, a car frame has a flexible behaviour at the frame/suspension links.

A preload of 3.34 kN, approximating the car's mass distribution onto the strut tower, is applied onto the suspension by the mean of a vertically displaceable strut tower linkage. The standard suspension geometry (caster, camber & toe-in angles) can be reproduced with adjustable strut tower and lower A-arm linkages.

An MB-100 dynamic shaker is attached to the spindle by the means of a bearing. The wheel, being clamped to it, is unable to rotate. However, rotation of the wheel spindle is allowed in the Y-axis. Fig. 3-8 shows the reference force sensors inside the shaker's stinger. The fact that road excitation is not injected directly under the tire generates additional moments on the suspension because of the added lever between the shaker link and the tire's centre of mass.

Non-direct links are not considered on the test bench: the sway bar and steering rod are not present. Consequently, the rotation of the wheel in the vertical axis (Z) is mainly restrained by the shaker's link and direct link linkages. The driving shaft is screwed inside the triangular frame and is not instrumented.

Fig. 3-7: FIN-03 Quarter-car suspension test bench  
Fig. 3-8: Shaker link with reference sensor
Direct link linkages (lower A-arm and strut tower) are shown in Fig. 3-9 & 3-10. The strut tower link was designed with three unidirectional force sensors which, when combined, can measure forces in the X, Y & Z axes. Each lower A-arm link can be equipped with tri-axis force sensors to measure efforts transmitted by the lower A-arm to the car frame.

![Fig. 3-9: Strut Tower linkage & sensors](image)

![Fig. 3-10: Lower A-arm linkages & sensors](image)

Accelerometers can be placed on any suspension component to obtain force/acceleration FRFs. More details on the test bench can be found in Douville’s Master thesis. [28]

### 3.2) Experimental FRF modeling of the suspension test bench

An important assumption made by Douville was that only X, Y and Z-axis forces at frame/suspension links are significant in road-induced interior noise. To validate this hypothesis, the modeling tool used by Douville to model the vibroacoustic transmission path was modified to transmit both forces and moments to the car frame air cavity.

#### 3.2.1) Lower A-arm linkage FRF

The test bench lower A-arm linkages were slightly modified (Fig. 3-11): each linkage was implemented with a flat bar permitting the implementation of two tri-axis force sensors on each side (a total of eight force transducers is necessary to fully instrument the lower A-arm). The combination of force transducers 1A, 1B, 2A & 2B can measure efforts and moments induced to the bushing nearest to the car’s front while force sensors 3A, 3B, 4A & 4B are used to measured loads and moments on the bushing nearest to the back of the car.
This new sensor configuration prevents direct application of moments on the force sensors (which they are not capable to measure properly) while allowing Y-axis moments measurements (Fig. 3-12 & 3-13). Eq. 3.1 gives the transfer matrix for a single bushing between each of its four force transducers and the resultant efforts at the middle of the bushing centre rod.

\[
\begin{bmatrix}
1 & 0 & 0 & -D_z & +D_y \\
0 & 1 & 0 & +D_z & 0 \\
0 & 0 & 1 & -D_y & +D_x \\
1 & 0 & 0 & -D_z & +D_y \\
0 & 1 & 0 & +D_z & 0 \\
0 & 0 & 1 & -D_y & -D_x \\
1 & 0 & 0 & -D_z & -D_y \\
0 & 1 & 0 & +D_z & 0 \\
0 & 0 & 1 & +D_y & +D_x \\
1 & 0 & 0 & -D_z & -D_y \\
0 & 1 & 0 & +D_z & 0 \\
0 & 0 & 1 & +D_y & -D_x \\
0 & 1 & 0 & +D_z & 0 \\
0 & 0 & 1 & +D_y & -D_x
\end{bmatrix}
\begin{bmatrix}
F_{3AX} \\
F_{3AY} \\
F_{3AZ} \\
F_{3BX} \\
F_{3BY} \\
F_{3BZ} \\
F_{4AX} \\
F_{4AY} \\
F_{4AZ} \\
F_{4BX} \\
F_{4BY} \\
F_{4BZ}
\end{bmatrix}
= \begin{bmatrix}
F_{RX} \\
F_{RY} \\
F_{RZ} \\
M_{RX} \\
M_{RY} \\
M_{RZ}
\end{bmatrix}
\]  \hspace{1cm} (3.1)

Where \( D_x = 0.0254m \), \( D_y = 0.0571m \) & \( D_z = 0.762m \) are the orthogonal distances between each force transducer and the centre of their respective bushing.
To compare the relative importance of each FRF, moments were transformed into forces using Eq. 3.2.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1/D_y & 0 \\
0 & 0 & 0 & 0 & 1/D_z \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
F_{RX} \\
F_{RY} \\
F_{RZ} \\
M_{RX} \\
M_{RY} \\
M_{RZ}
\end{bmatrix}
= 
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
F_{MX} \\
F_{MY} \\
F_{MZ}
\end{bmatrix}
\]  
(3.2)
The transmissibility analysis does confirm the importance of moments concerning road-induced vibrations (Fig. 3-14). This means that important energies components were not taken into account in Douville’s transmissibility analysis.

3.2.2) Strut tower linkage FRF

The strut tower linkage did not require any modifications because it is capable of measuring X and Y-axis moments. (Fig. 3-15) Moments on the Z-axis are not measured, but the strut tower bearing prevents any efforts to be transferred through this DOF.

![Fig. 3-15: Strut Tower linkage force transducer configuration](image)

Eq. 3.3 presents the transfer matrix between each strut tower force sensor to obtain the global linkage forces and moments.

\[
\begin{bmatrix}
\cos(\theta_A)\cos(\phi_A) & \cos(\theta_B)\cos(\phi_B) & \cos(\theta_C)\cos(\phi_C) \\
\sin(\theta_A)\cos(\phi_A) & \sin(\theta_B)\cos(\phi_B) & 0 \\
\sin(\phi_A) & \sin(\phi_B) & \sin(\phi_C) \\
R_{AY}(\sin(\phi_A)) & R_{AY}(\sin(\phi_B)) & 0 \\
R_{AX}(\sin(\phi_A)) & R_{AX}(\sin(\phi_B)) & R_{AX}(\sin(\phi_C))
\end{bmatrix}
= \begin{bmatrix}
F_A \\
F_B \\
F_C \\
R_{AX} \\
R_{AY} \\
R_{AZ}
\end{bmatrix}
= \begin{bmatrix}
F_{RX} \\
F_{RY} \\
F_{RZ} \\
M_{RX} \\
M_{RY} \\
M_{RZ}
\end{bmatrix}
\]

(3.3)

Where \(\theta_A=120^\circ\), \(\theta_b=240^\circ\), \(\theta_c=0^\circ\) & \(\phi_A=\phi_B=\phi_C=45^\circ\). The orthogonal distances between each force sensor are: \(R_{AX}=R_{AX}=R^*\cos(\theta_A)\), \(R_{CX}=R\), \(R_{AY}=R_{BY}=R\sin(\theta_A)\). \(R=0.083m\) is the distance between strut tower force transducers and the centre of the strut tower linkage.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 1/R_{AY} & 0 \\
0 & 0 & 0 & 1/R
\end{bmatrix}
\begin{bmatrix}
F_{RX} \\
F_{RY} \\
F_{RZ} \\
M_{RX} \\
M_{RY}
\end{bmatrix}
= \begin{bmatrix}
F_X \\
F_Y \\
F_Z \\
F_{MX} \\
F_{MY}
\end{bmatrix}
\]

(3.4)
Like the bushing linkages, moments are converted into force with Eq. 3.4.

![Comparison of Strut tower transmissibility FRFs](image)

**Fig. 3-16: Strut tower linkage force transducer FRFs**

Results show that forces induced by moments on the strut tower force transducers are also an important contributor to road-induced vibrations (Fig. 3-16).

3.2.3) FEM/BEM vibroacoustic modeling of a car body

Moments transmitted by the suspension frame links can now be applied as input to Douville’s car body FEM/BEM to observe the resultant noise perceived by passengers. Every moment could be imposed on the model by using a combination of forces at specific nodes on the FEM model (Fig. 3-17).

![Desirable force & moment reproduction on car body FEM](image)

**Fig. 3-17: Desirable force & moment reproduction on car body FEM**
For each bushing, twelve FRFs (2 bushing extremities which are in contact with 4 frame surfaces that can transmit 3 orthogonal forces) are necessary to apply all forces measured in Eq. 3.1.

Unfortunately, the COMET® licence that is necessary to acquire the pressure distribution inside the car cabin has expired. This technical obstacle prevents the evaluation of new numerical FRFs. Consequently, only 12 FRFs are available (X, Y & Z-axis FRFs for each lower A-arm bolt linkage). Linkage 1/2 or 3/4 FRFs are combined to reproduce torques applied on the bushing centre rods (Fig. 3-18).

\[
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & -D_y & 0 & 0 & +D_y \\
0 & 0 & 0 & 0 & 0 & 0 \\
+D_y & 0 & 0 & -D_y & 0 & 0
\end{bmatrix}
\begin{bmatrix}
F_{1x} \\
F_{1y} \\
F_{1z} \\
F_{2x} \\
F_{2y} \\
F_{2z}
\end{bmatrix} =
\begin{bmatrix}
F_{RX} \\
F_{RY} \\
F_{RZ} \\
M_{RX} \\
M_{RY} \\
M_{RZ}
\end{bmatrix}
\] (3.5)

Measured forces at the bushing linkage in Eq. 3.1 are transposed on the car body FEM/BEM with Eq. 3.5. However, it is not possible to apply moments around the Y-axis with the actual FRF configuration since all four bushing linkages are aligned. Fig. 3-19 presents the bushing3d’s vibroacoustic transmissibility FRFs.
Fig. 3-19: Bushing SPL/Force FRFs comparison

\[
\begin{bmatrix}
P_{FX} \\
P_{FY} \\
P_{FZ} \\
P_{MX} \\
P_{MY}
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & \frac{1}{D_y} \\
0 & 0 & 0 & \frac{1}{D_y}
\end{bmatrix}
\begin{bmatrix}
F_{RX} \\
F_{RY} \\
F_{RZ} \\
M_{RX} \\
M_{RY}
\end{bmatrix}
= \begin{bmatrix}
H_{FX} \\
H_{FY} \\
H_{FZ} \\
H_{MX} \\
H_{MY}
\end{bmatrix}
\]  

(3.6)

To provide a common basis of comparison for the transmissibility FRFs, moments imposed on the bushing linkage have been transformed into forces using Eq. 3.6: each FRF of Fig. 3-19 is then in \( Pa/N \). We observe that the 2 moment FRFs have, in general, a less important impact on interior noise than pure efforts. But we cannot confirm that this tendency is applicable to the strut tower linkage. The proposed modifications of the car FEM/BEM should clarify this matter in further studies for a representative cabin sound pressure level.

As for the strut tower linkage, a configuration identical to the test bench sensors could be employed (Fig. 3-20): a total of 9 FRFs would be sufficient to reproduce all 5 DOF measured on the suspension. However, only three nodes FRFs (\( X, Y & Z \)) are available for a single strut tower node. Consequently, no torque can be transmitted on the FEM model by the strut tower linkage.
In brief, the quarter-car suspension test bench has been modified so that it can now measure all suspension transmissibility FRFs (Fig. 3-21). As for the car body, only a fraction of the transmissibility FRFs will be considered in the model: 3/5 for the strut tower ($X$, $Y$ & $Z$) and 5/6 for each bushing ($X$, $Y$, $Z$, $tX$ & $tZ$).

In addition to this, the FEM vibroacoustic FRFs are relative to a single node positioned at the driver’s head (no information is available on the rear passenger SPL).
3.3) Analytical modeling of the test bench suspension

As mentioned previously, the use of an analytical suspension model comes from the fact that actuator configurations can be tested onto it. An ideal suspension model should be able to reproduce the dynamics of each suspension component. To realize such a model, a FEM or experimental FRFs of each component would be necessary. Because finite element modeling is time consuming and needs much calculation power, FRF models will be preferred for their simple input/output transfer functions.

However, let us make an assumption to reduce the models complexity: stiff components - those who exhibit no or few modes within the 0-250 Hz frequency band - are assumed to be rigid. Discrete or continuous elements will be used to model these components. More flexible components will be modeled by experimental FRFs. This model, while requiring a minimum of calculation power, will then reproduce a large number of the suspension modes.

3.3.1) Discrete suspension model

At the beginning of this chapter, the description of the suspension has revealed that certain suspension parts can rotate or move relatively to others. Based on this analysis, it was decided to sub-structure the suspension model into four modules: the wheel, the strut tower, the lower A-arm and the driving shaft.

![Quarter-car suspension discrete element model](image)

**Fig. 3-22: Quarter-car suspension discrete element model.**

The tire and coil spring envelopes have been included in Fig. 3-22 for clarity. Table 3-2 gives a short description of the model’s colour and marker codes.
Mass distribution and discrete element configuration for each module is explained in details in Appendix A.

3.3.2) Dynamic stiffness of specific components

Douville’s OVSBA showed that the tire, the two lower A-arm bushings and the strut tower provide the principal modes between 0–250 Hz. To capture the modal behaviour of these components, an equivalent stiffness model can be used (Fig. 3.23). As an example, Fig. 3-23 and Eq. 3.8 present a typical single DOF system.

\[
(-\omega^2 M - j\omega C + K)x(\omega) = F(\omega)
\]  

(3.8)

![Fig. 3.23: Sketch of an equivalent rigidity model](image)

To create an equivalent stiffness model of this system, one must measure the displacement \(x\) imposed on the system and the reaction force \(F\) at the other end of the system. The transfer function between the force and the displacement gives the equivalent stiffness model (Eq. 3.9 & 3.10).

\[
(-\omega^2 M - j\omega C + K) = \tilde{K}(\omega)
\]

(3.9)

\[
\tilde{K}(\omega) = F(\omega)/x(\omega)
\]

(3.10)

These dynamic stiffness models will be implemented to the discrete element model to complete the quarter-car suspension model.
3.3.2.1) Lower A-arm bushing dynamic stiffness

To obtain the dynamic stiffness of bushings, accelerometers were placed on the outer cylinder of the bushings while resulting forces are taken at the suspension linkage. Appendix A describes in detail the methodology and position of sensors to measure forces and accelerations for each test bench linkage. Fig. 3-24 and 3-25 show typical experimental results.

![Coherence of FRF measurement of Link 3 - Fy/Dx](image)

Fig. 3-24: Coherence of the bushing linkage FRF

An average over 10 measurements was used to reduce the impact of coherence loss: at low frequencies (0-10 Hz), coherence losses can be explained by the inability of accelerometers to correctly measure low frequency vibrations. Fortunately, the passengers do not hear these frequencies. Other coherence loss can be explained by suspension or test bench resonance, non-linear behaviour of rubber parts or insufficient energy levels at certain frequencies.

![Equivalent rigidity of bushing linkage](image)

Fig. 3-25: Equivalent rigidity of bushing linkage

Another problem that could have lead to erroneous stiffness is the number of force transducers: only two tri-axis force transducers were available at the time of measurement (eight are necessary to fully instrument the lower A-arm). Consequently, only 6 FRFs could be measured simultaneously. The displacement of the force sensors from one linkage to another could have changed the intrinsic properties of the suspension. Therefore, the assumption that an linkage with no force has the same stiffness as an linkage with sensor is made.
3.3.2.2) Strut tower dynamic stiffness

The strut tower dynamic stiffness was measured with accelerometers under the lower coil base plate and force sensors on the strut tower linkage. Once again, the exact experimental procedure to obtain the model is fully detailed in Appendix A.

Coherence is better for this component than for the bushings (Fig. 3-26), which results in a better component model. Most important, coherence losses are detected at coil spring resonances (39 Hz and it’s harmonics). The fact that coherence levels are in average above 90% demonstrates that the viscous damper does not create a significant non-linear behaviour. Coherence loss below 10 Hz, seen in the lower A-arm curves, is also present for this component.

The strut tower equivalent rigidity was also averaged and the result is shown in Fig. 3-27. The fact that no sensor manipulation was required to make the model ensures that the intrinsic properties of the test bench are stable between measurements. For the strut tower, the energy level was appropriate through all the frequency band, resulting in a better equivalent rigidity model.

Fig. 3-26: Coherence of the strut tower linkage

Fig. 3-27: Equivalent rigidity of strut tower linkage
3.3.3) Tire dynamic stiffness identification with genetic algorithms

If, like the suspension linkages, the tire support had been designed with integrated force sensors, an experimental tire dynamic stiffness could have been retrieved. Since it is not the case, another technique must be used. Some researchers have used genetic algorithms to model systems even when some parameters are unknown. [27] One interesting aspect of the method is that it could avoid the need for additional instrumentation (test bench tire table modifications or, if studying a real car, test track censoring).

A genetic algorithm (GA) is a stochastic search algorithm based on the mechanics of natural selection. It performs its search balancing the need to retain population diversity (exploration), so that potentially important information is not lost, with the need to focus on fit portions of the population exploitation (optimization). Matlab® offers a “Genetic Algorithm and Direct Search” toolbox in which interface and standard genetic operators are already programmed. This can serve as a basis for GA optimization problems and will be used for both tire modeling and actuator optimization in future chapters. Many genetic operators can be used to create a functional evolution process. The mathematical concept of each genetic operator used for our optimization problems will be described in the following section.

3.3.3.1) Genetic operators

The selection algorithm allocates reproductive rights to individuals as a function of their strength: the probability of reproduction during a given generation is proportional to the fitness of the individual. The probability that an individual $i$, will be selected for mating is given simply by the individual's strength divided by the total strength of all the population (Eq. 3.11).

$$P_i = \frac{S_i}{\sum_{k=1}^{n} S_k}$$

(3.11)

Where $P_i$ is the probability of selection for individual $i$, $S_k$ is the strength of the individual and $n$ is the population of this generation. This gives every member of the population a finite probability of becoming a parent, with stronger classifiers having a better chance.

Other selection strategies are deterministic: elitism takes only a certain number of the strongest classifiers to create siblings. This also allows a certain individual to pass over to the next generation without any modifications. To ensure that the best solution is not lost through stochastic evolution, a combination of elitism and fitness proportionate reproduction is used for this work.
The **crossover algorithm** mates each individual, which was chosen by the selection algorithm. To illustrate the crossover mechanisms let’s take two individuals, one is made of letters, the other is made of numbers (Eq. 3.12 & 3.13).

\[
\text{Parent } #1 = [a \ b \ c \ d \ e \ f \ g \ h] \quad \text{(3.12)} \\
\text{Parent } #2 = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8] \quad \text{(3.13)}
\]

First, a random binary chromosome is created (Eq. 3.14). As shown in Eq. 3.15, each gene of this chromosome indicates to the algorithm which gene to take to create the child \((0 = 1^{st} \text{ parent}, 1 = 2^{nd} \text{ parent})\).

\[
\text{Random crossover vector} = [1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0] \quad \text{(3.14)} \\
\text{Child} = [a \ b \ 3 \ 4 \ e \ 6 \ 7 \ 8] \quad \text{(3.15)}
\]

Although the mechanics of selection and crossover operators are simple, the biased selection and structured, though stochastic, information exchange of crossover give genetic algorithms much of their power.

The random alteration of an individual’s chromosome performs a secondary role in the reproduction process. The **Mutation algorithm** is needed to guard against premature convergence, and to guarantee that any location in the search space may be reached. Again, many mutation functions are possible, but there are two specific functions that will be used here:

- The **Gaussian method** adds a random number to each vector entry of an individual. This random number is taken from a Gaussian distribution centred on zero. The variance of this distribution can be controlled in real time with certain parameters. The Gaussian approach will serve for the tire model identification in this chapter.

- The **Uniform method** is a two-step process. The algorithm selects a fraction of the vector entries of an individual for mutation, where each entry has a probability of being mutated. Then, the algorithm replaces each selected entry by a random number selected uniformly from the range for that entry. This mutation tool will be used to modify actuator-positioning chromosomes in Chapter 5.

The combination of any selection, crossover & mutation algorithm constitutes a fully functional genetic algorithm, which under the correct environment and subjective strength function will give an optimized (but not necessarily “the” best) solution to a given problem.
3.3.3.2) Environment & fitness rule description

The GA seeks tire dynamic stiffness that reproduces the experimental vibroacoustic behaviour of a pressurized, compressed, non-rotating tire as a function of frequency. In other words, the algorithm must find a tire model that will make the suspension model vibrate the same way that the test bench suspension linkages/dynamic shaker force sensors experimental FRFs. This represents a total of 6144 parameters to be identified (Table 3-3):

<table>
<thead>
<tr>
<th>Length of chromosome for one individual</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Six springs, one for each DOF</td>
<td>6</td>
</tr>
<tr>
<td>Each spring has an imaginary and real part: $k = k + j\eta$</td>
<td>x 2</td>
</tr>
<tr>
<td>A total of 512 frequencies between 0 &amp; 250 Hz</td>
<td>x 512</td>
</tr>
<tr>
<td>Number of Genes (Chromosome)</td>
<td>6144</td>
</tr>
</tbody>
</table>

Table 3-3: Evaluation of needed genes for dynamic tire identification

The quarter-car suspension model gives the environment in which populations will be evaluated: each individual will give a response to its environment by evaluating the corresponding vibroacoustic behaviour of each tire model. Fig. 3-28 shows the cycle of the evolution process:

![Fig. 3-28: Tire rigidities evolution process](image)

Each individual $I_n$ is placed inside the suspension model to obtain an analytical FRF. This frequency response is compared to the experimental results. The strength of an individual is based on its ability to mimic the 34-test bench force transmissibility FRFs. The errors between experimental and analytical responses, amplitude and phase, are penalized following Eq. 3.16.
\[
\lambda_e = \sum_{k=1}^{N_f} \left( \sum_{\omega_{im} \in \mathcal{W}} \left( \left| \log_{10}(H_{k,\text{exp}}(\omega)) - \log_{10}(H_k(\omega)) \right|^2 + \left( \angle(H_{k,\text{exp}}(\omega)) - \angle(H_k(\omega)) \right)^2 \right) \right)
\]

(3.16)

Where \( \angle(x) \) is the phase of \( x \). The best individual would theoretically achieve a score of 0. From these fitness values, parents are selected and the best solution is directly sent into the next generation.

The tire model is identified at each frequency separately. To justify this method, consider a 1x4 binary chromosome: we know that this vector can have 16 \((2^4 = 16)\) possible values. But if there is no connection between each scalar, there are then only two possibilities for each scalar \((2^*4 = 8)\). To do a similar process for our model, genes were optimized 12 genes at a time (tire rigidities for each DOF at a specific frequency). Although each frequency response is found separately, the stiffness found at a previous frequency is always taken as one of the first individual for the next frequency. This ensures good homogeneity in the rigidity response and accelerates the stochastic process.

Finally, the mutation factor is gradually decreased through time (starting at \( \mu = 1 \) and going down to \( \mu = 1 \times 10^{-5} \) for final convergence). This helps the algorithm to find the lowest value possible of the local or true minimum at each frequency.

### 3.3.3.3) Tire dynamic stiffness results analysis

In order to know if the stiffness found by the GA are physical, one can use a classic single DOF system: Cebo [21] stated that the tire vertical rigidity is around of 2 \( MN/m \). Knowing that the total mass of the suspension test bench suspension is approximately 50 \( kg \), it is possible to create an equivalent rigidity model of this system using Eq. 3.10. Fig. 3-29 presents the experimental equivalent rigidity model of the vertical tire gene response versus the equivalent rigidity model of a single DOF system.

![Fig. 3-29: Tire model Z-axis rigidity VS 1 DOF system](image)

The single DOF system has a mass of 50 \( kg \) and a spring constant of two \( MN/m \). We can observe good similarity between the tire model and the single DOF system: at the system’s resonance, equivalent rigidity tends to 0. Above the resonance, a steady 6 \( dB/Octave \) rigidity increase is observable.
As for modal frequencies, one can compare the anti-resonance of the tire on the X-axis with the resonances found from the OVSA conducted by Douville on the test bench tire surface. Similarly to the single DOF frequency response of Fig. 3-29, an anti-resonance of the dynamics stiffness model at Fig. 3-30 – measurement taken from the GA tire model - corresponds to a resonance of the suspension test bench.

![Tire X-axis dynamic stiffness model found by genetic algorithm](image)

**Fig. 3-30:** Identification of tire X-axis resonances predicted by Douville's OVSA

Tire resonance and global suspension resonance can be observed in the FRF obtained from the equivalent rigidity model. Unfortunately, measurement noise is important and blurs the model. Sensors used in the creation of FRF models are subjected to noise. It is also possible that proposed “rigid” components – test bench components or suspension modules – actually deform and create unexpected resonance. The genetic algorithm, trying to mimic the test bench FRFs, would adapt tire rigidities to reproduce experimental curves. This could explain some convergence problems or unexpected resonances found in the tire model. Still, these simple trends confirm that the tire dynamic stiffness proposed by the genetic algorithm is physically plausible.

3.3.4) Validation of the suspension model

Now that all model parameters are identified, we can now analyse the capability of the model to reproduce the experimental vibroacoustic behaviour of the test bench. Fig. 3-31 and 3-32 present the test bench VS suspension model transmissibility FRFs. In order to obtain these results, the genetic algorithm found the optimal tire dynamic rigidity to reproduce all 34 tests bench FRFs.
Since we have a total of 6 complex stiffnesses to identify per frequency with 34 FRFs, we are in the presence of an over determined problem. When asked to solve the problem two times in a row, the results are very similar. This leads to conclude that the GA does find an optimal solution, but approximations coming from the discrete suspension model or experimental dynamic stiffness force the GA to make a compromise between FRFs.

If the GA is asked to find a tire model only with a specific linkage FRFs, the problem becomes underdetermined (6 complex rigidities for 5 strut tower FRFs or 6 bushing FRFs). The GA results show a mean fitness of 25 points for the strut tower, but over 250 points for each bushing. This leads to two possible conclusions: the actual bushing modeling methodology should be revised, or the lower A-arm initially assumed to be rigid possesses an undeniable modal behaviour.

One simple (but expensive) way to clarify this matter would be to install eight tri-axis force transducers on bushing linkages. This way, all FRFs could be taken simultaneously and errors that were coming from sensor displacement could be removed.
3.4) Validation of the analytical model for secondary path FRFs

The main goal of this model being to optimize actuators locations, it is therefore important to evaluate if the model can correctly predict secondary (or control) path FRFs. As a validation test, a second dynamic shaker was installed on the test bench (Fig. 3-33). As shown on Fig. 3-34, the actuator (green) is linked to the lower A-arm (brown) of the suspension. This is the same configuration used by Douville for preliminary experimental active control study.

![Fig. 3-33: Actuator shaker setup](image)

![Fig. 3-34: Actuator Shaker installation sketch](image)

A 60 lbs preload (purple) is installed to ensure that the shaker wire is constantly under tension. A force transducer is placed vertically between the lower A-arm and the wire (red). A white noise signal is applied to the actuator to measure experimental Force/Force transmissibility FRFs.

![Fig. 3-35: Suspension model modifications for secondary path FRF reproduction](image)
To simulate the same configuration with the suspension model, the disturbance force was transferred to the lower A-arm by adding an additional discrete element (Fig. 3-35). This new element links the actuator force sensor with the centre of mass of the lower A-arm. A 1 N vertical force is induced at this point for each frequency. To mimic the rigidity of the test bench primary shaker link, a 0.1 MN vertical stiffness was included at the end of the shaker link. The stiffness of the electromechanical shakers was identified using a specific method discussed by Lang. [29]

Fig. 3-36 and 3-37 show the model FRFs compared to the ones measured experimentally. Some modes are well predicted by the model, others are not. One possible explanation for the FRF disparities is the approximated rigidity of the disturbance shaker.

![Graph showing FRF comparison](image)

**Fig. 3-36: Control actuator on lower A-arm, Bushing FRF of model VS test bench**

The Lang methodology uses an empirical technique to find the shaker table rigidity constant, but the rigidity is not dynamic and approximated. To eliminate the influence of the disturbance shaker on the actuator FRFs, experimental measurements should be executed without any disturbance shaker linked to the suspension. Or, an experimental dynamic stiffness of the shaker link should be conducted to accurately measure the forces generated by the shaker linked to the suspension.

Another important factor is the quality of the suspension model. Errors present in flexible component models, unexpected test bench resonances or GA adaptation errors will all have an impact on predicted actuator FRFs. If the tire equivalent rigidity model was measured directly on the test bench with force transducers on the tire table, a source of error could be removed. The impact of the actuator linking mechanisms is also unknown: the metallic wire has resonances that can modify the injected force.
Still, the analytical model is capable of reproducing most of the resonances measured within the principal noise contribution paths detected by Douville (the Z-axis for bushing #2 and the X-axis for the strut tower). The analytical suspension model will thus be used in Chapter 5 for the optimization of actuator locations for ASAC control optimization.

3.5) Chapter summary

In this chapter, a quarter-car McPherson strut front suspension was analytically modeled. This suspension model was validated against a modeling tool based on experimental measurements in a test bench for the suspension sub-system. It was explained how the transmission path tool was modified to reproduce all test bench FRFs and only 13/17 of the cabin’s FRFs.

The suspension was divided into four rigid modules with 12 DOF to reproduce global suspension’s resonance. Flexible components – the tire, the strut tower and the two lower A-arm bushings – were reproduced within the model with equivalent rigidity models. Parameters of instrumented linkages were measured and parameters of non-instrumented linkages were obtained with a genetic algorithm. Analysis shows that deduced FRF models are in reasonable agreement with measured models.

The overall conclusions are:

- The methodology used to create a quarter-suspension model mimics properly the vibroacoustic behaviour of the test bench. This model will consequently be used to test different actuators positions in the following chapters.

- The genetic algorithm tool and genetic operators have found physically plausible FRFs for the tire model. Similar operators could then be used for the optimization of actuator positions for control efficiency.

The experimental implementation of an actuator on the test bench has proven the ability of the model to evaluate correctly the secondary path FRFs. The suspension analytical model will serve as the reference in chapter 5 for actuation positioning optimization problems.
CHAPTER 4

4) EXPERIMENTAL STUDY OF ROAD EXCITATION

The literature review provided information for automobile noise sources. But what is of real concern here is specific data on road-induced interior noise (which was unfortunately very sporadic). Another concern that has not been discussed up to now is the test bench excitation signal: the tire supportable has a modal behaviour, which is not the case for a conventional road. The test bench excitation signal must consequently be adapted to minimize the effect of the tire support table. The instrumentation of a Chevrolet Epica LS and its road testing should help to gather valuable information related to these problems. This part of the work was made possible through a generous donation from General Motors Canada to this AUTO21 project and fruitful collaboration with Dr. Colin Novak (Mechanical Engineering, University of Windsor) and his research team.

4.1) Experimental setup: the Chevrolet Epica LS

A Chevrolet Epica LS is a mid-size car that has many similarities with the test bench suspension and FEM/BEM car frame model. Still, certain differences have to be pointed out because they can lead to different results. The geometric properties of the Epica wheels, suspensions and car frame are first presented in an attempt to predict possible differences with the test bench or FEM body model.

4.1.1) Wheel subsystem

We have discussed that the vibroacoustic behaviour of a wheel is a function of its geometry, inflation and load. Thompson proposed an analytic formula to approximate the first and second tire air cavity resonance based on tire operating conditions (Eq. 4.1). [30]

\[ f = \frac{c}{L_c \pm (1-m)l_{cp}} \]  

(4.1)

\( f = \) first \((f_1)\) & second \((f_2)\) natural frequencies of the deflected tire/air cavity;
\( c = \) velocity of sound \((331.5 \text{ m/s} + 0.6 \times \text{ Temperature in } ^{\circ}\text{C})\);
\( L_c = \) median circumferential length of the tire cavity;
\( m = \frac{S_{cp}}{S} = \) ratio of undeflected cavity cross sectional area to the cross sectional area in the contact patch.
\( l_{cp} = \) contact patch length.
Table 4-1 shows the specifications of the Epica & test bench tire.

<table>
<thead>
<tr>
<th>Type</th>
<th>Section (mm)</th>
<th>Aspect ratio (%)</th>
<th>Construction type</th>
<th>Rim dia. (in.)</th>
<th>Max. Load (Kg)</th>
<th>Max. Speed (km/h)</th>
<th>Pressure (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger</td>
<td>205</td>
<td>65</td>
<td>Radial</td>
<td>15</td>
<td>630</td>
<td>210</td>
<td>29</td>
</tr>
<tr>
<td>TEST BENCH</td>
<td>185</td>
<td>75</td>
<td>Radial</td>
<td>14</td>
<td>545</td>
<td>180</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 4-1: Tire geometry description and comparison

For a 1” load deflection and a 5” contact patch (measured test bench conditions), air cavity resonance are predicted 237 and 253 Hz at 25°C (Table 4-2). Using an OVSA (operational vibratory Shape analysis), Douville has identified that the test bench first air cavity resonance is at 238 Hz. The other air cavity resonance, if we follow Thompson’s prediction, should not be far from the test bench limit.

<table>
<thead>
<tr>
<th>Tire</th>
<th>Deflection</th>
<th>$L_c$</th>
<th>$m$</th>
<th>$l_{pv}$</th>
<th>$f_1$ (Hz)</th>
<th>$f_2$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted Test bench tire air cavity modes (temperature = 25°C)</td>
<td>1”</td>
<td>56.0”</td>
<td>0.8</td>
<td>5”</td>
<td>237</td>
<td>253</td>
</tr>
<tr>
<td>Predicted Epica LS air cavity resonances (temperature = 15°C)</td>
<td>1”</td>
<td>61.1”</td>
<td>0.8</td>
<td>5”</td>
<td>222</td>
<td>231</td>
</tr>
</tbody>
</table>

Table 4-2: Thompson test bench tire air cavity modes predictions

Since the Epica tire has a longer median circumferential length, resonance frequencies should be lower. For the same load and sectional ratio, Eq. 4.1 predicts air cavity resonances around 222 and 231 Hz.

4.1.2) Suspension sub-system

The Epica is equipped with two, front drive, McPherson strut front suspensions which also include lower A-arms. However, as compared to the test bench, one of their two bushings is inverted: the centre rod of one of the bushings is directly welded onto the lower A-arm (Fig. 4-1). This differs from our test bench lower A-arm which is linked to the car frame by two bushing outer metal cylinders (Fig. 4-2).

The distance between the lower A-arm linkages are also different: 276 mm for the Epica and 221 mm for the test bench. Assuming that bushing materials have the same stiffness, the lower A-arm of the Epica should have a stiffer response. Therefore, lower A-arm resonance frequencies should be higher.

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As for the strut tower linkages, both the test bench & Epica have similar mounting configurations. On the other hand, geometric properties of the coil spring (number of windings, diameter of the coil wire, diameter of the coil, etc.) should change strut tower vibroacoustic behaviour. Modal predictions are quite difficult for this component without an analytic model or experimental data.

4.1.3) Car body/air cavity sub-system

Fig. 4-3 & 4-4 show dimensions of the Epica LS and Honda Civic body that was used for the simulation. Despite that the Epica is slightly larger, both bodies are very similar. We can then anticipate that air cavity resonances of the model and the real car should be approximately in the same frequency range.

The Epica exterior panels were slightly altered because it was involved in an accident: the back bumper had to be removed and the front passenger door was slightly bumped. Although it is not presently possible to verify the influence of such damages on interior noise, these alterations will not be considered for road/interior noise studies.
The car interior is equipped with all conventional furniture: floor mattresses, headliners and seats cushions. These components are known to help the reduction of interior noise levels by reducing reverberation time.

4.2) Car instrumentation

To measure the interior noise levels and the road excitation, various sensors were installed on the Epica suspension and in the interior space. The following section describes the measurement instrumentation of the car.

4.2.1) Acoustic measurements

To measure Sound Pressure Levels (SPL) inside the cabin, two microphone systems were installed: two ICP microphone were placed on each side of the driver’s headrest. This configuration allows making measurements near the ears of the driver (Fig. 4-5).

As for the front passenger, a binaural head was installed onto the seat (Fig. 4-6). This dummy will allow comparing passenger and driver noise levels. At the same time, the binaural head is equipped with synthetic ears, head and torso that give the possibility of observing certain effects of the human body on incident pressure waves.
4.2.2) Vibration measurement

To evaluate road excitation characteristics, two tri-axis accelerometers were placed on the front left wheel hub with the help of aluminium plates. Fig. 4-7 and 4-8 show the exact position of each accelerometer.

![Fig. 4-7: Hub acc. bracket – Calliper side](image)

![Fig. 4-8: Hub acc. bracket – Direction rod side](image)

The combination of the two sensors provides information for all wheel hub rigid DOF. By comparing the wheel accelerations with microphones signals, measurement of the front left suspension noise contribution will be analysed.

4.2.3) Test track characteristics

Measurements were made on November 5th, 2005, (the temperature was 15°C and winds were below 10 km/h). The measurements were conducted at the Windsor Ford engine plant facility (Fig. 4-9).

![Fig. 4-9: Concrete test track, Ford Motor Company, Windsor engine plant, Ontario.](image)
The test track is made of 14’ wide concrete panels and is approximately 1 km long. To minimize wind effect while maximizing road noise, measurements were conducted at 50 km/h.

4.2.4) Data acquisition system

All data were recorded on an Orchestra 01dB-Stell system (Fig. 4-10). This system can record in real-time up to 32 channels simultaneously over a 20 kHz bandwidth.

![Fig. 4-10: Orchestra 01dB-Stell recording system](image)

Data measured at the wheel spindle and the driver’s head were limited to 12 kHz, which is the accelerometer frequency limit. The binaural head records were set to 20 kHz to have a more complete measurement of interior noise. It was decided to take 15 seconds segments because of the length of the test track.

For each test, the car passengers were: one back passenger (controlling the Orchestra system), one binaural head placed at the front passenger seat and one driver (engine on or off). More passengers inside the car provide more sound absorption. The experimental configuration gives a realistic idea of the sound absorption inside a car cabin. Tests were conducted in two sets: first, the vehicle engine propelled the car. Second, the Epica engine was turned off and towed with a truck.

4.3) **Excitation source characterization**

Road noise perceived by car passengers depends on the road quality and the transmission paths. To compare the SPL predicted by the transmission path tools to the SPL inside the Epica LS, test bench disturbance signal should be able to mimic the test track excitation. So far, road vibrations were simulated on the test bench by a white noise. The following section aims to adapt the test bench excitation to mimic a true road excitation.

4.3.1) Test bench tire table VS ISO road spectral density

In Chapter 2, standard ISO Spectral Density road displacement curves were presented for different road qualities [21]. To verify the spectral density of the tire table, an accelerometer was placed under the tire of the test bench. Standard road profiles at
$50\text{km/h} \ (13.9\text{m/s})$ are compared to the displacement measured under the tire in Fig. 4-11. At this speed, the wavelength of road defects exciting the suspension between 25 and 250 Hz are between 0.56 and 0.056 m. When the test bench shaker was set at 50\% of its maximal force, one can see that the energy is comparable in average to a “good” road profile. However, test bench/suspension resonances create large differences in certain frequency bands.

![Comparison of vertical road profiles displacement PSD](image1)

**Fig. 4-11: Suspension table displacement PSD VS Draft ISO road profiles**

In order to adapt the excitation of the test bench with ISO curves, an FRF is measured between the shaker voltage signal and the vertical displacement of the suspension table. To excite the test bench with a “Good road” profile, one can use this experimental FRF to create a corrected shaker command for road excitation reproduction (Fig. 4-11).

![“Good Road” filter VS Experimental FRF of shaker excitation](image2)

**Fig. 4-12: Comparison of ideal FRF and road filter**

To implement this new excitation on the test bench, a filter was created: a 2048 points inverse discrete Fourier transform is applied to the transfer function between the actual and desired voltage command for the test bench shaker. The resulting impulse signal is delayed by 1024 samples and multiplied by a Hanning window. The frequency response of the “Good road” FIR filter is shown in Fig. 4-12. This filter is convoluted in the time domain with the test bench white noise signal to create the desired power spectral density.
4.3.2) Acceleration of wheel hub – Test bench VS test track data

To compare the simulated road excitation on the test bench (white noise and “good road”) with the one applied on the Epica suspension, wheel hub acceleration PSD are analysed. Fig. 4-13 and 4-14 confirm that PSD measured on the test bench – filtered road excitation or not – are very different from experimental results measured on the vehicle.

Fig. 4-13: Test bench VS Epica wheel hub Z-Axis acceleration

Fig. 4-14: Test bench VS Epica wheel hub Y-Axis angular acceleration

Translations are, in average, less important on the test bench than in real conditions. On the other hand, rotations of the hub are greater on the test bench, especially on the Y-axis: the test bench shaker is linked to the spindle by a mechanical lever. This linking mechanism is equipped with a bearing allowing rotation on the Y-axis. Also, there is no direction rod or sway bar attached to the wheel hub on the test bench: rotations are then more easily allowed on the test bench than on the wheel hub of a complete vehicle.

To confirm that the filtering the excitation signal does bring the vibroacoustic behaviour of the test bench closer to the one measured on the Epica, each test bench wheel hub’s DOF PSD generated by a disturbance signal is subtracted form the Epica respective DOF. The disturbance shaker is set at 50% of its maximal force. (Eq. 4.2 and 4.3).

\[
\Delta PSD_{ROAD} = \sum_{f=\Delta f}^{250} [PSD_{ROAD}(f) - PSD_{EPICA}(f)] \tag{4.2}
\]

\[
\Delta PSD_{WHITE} = \sum_{f=\Delta f}^{250} [PSD_{WHITE}(f) - PSD_{EPICA}(f)] \tag{4.3}
\]
This procedure demonstrates that the filtering of the shaker excitation diminishes unwanted resonances coming from the test bench impedance and disturbance mechanism (especially above 175 Hz). The filtered road excitation produces wheel hub acceleration PSD that are in average closer to the Epica signal than a white noise excitation (Fig. 4-15 and 4-16).

Still, one must understand that, even with a filtered excitation, the shaker linking mechanism of the test bench creates important variations between the vibroacoustic behaviour of the test bench and the Epica suspension. To reproduce a correct road excitation, the excitation mechanisms should be placed under the tire of the test bench.
Finally, when primary shaker force PSD is observed (Fig. 4-18), one sees that the level of excitation is approximately $1 N^2/Hz$. We can then imagine that a small actuator could be sufficient to diminish road-induced vibrations. This goes in accordance with Dehandschutter experiments in which he used small inertial actuator (actuator had a maximum output force of 12 $N$ at resonance) to control narrow frequency bands (75-100 $Hz$ & 240-260 $Hz$).

4.4) Noise level inside a car cabin

The interior SPL below 10 $kHz$ clearly shows the importance of the 0-500 $Hz$ bandwidth (Fig. 4-19). Above 4.5 $kHz$ the noise level goes below the threshold of hearing. Data above this frequency is consequently uninteresting. A conventional car interior noise level is said to be between 40 and 75 $dB(A)$ [20], but no information is given on the car speed or the measurement procedure. Statistics show that people have a tendency to raise their voices (to be annoyed) when noise level is above 35 $dB(A)$.

Another observation is the difference of SPL measured by the binaural head and the driver microphones. As mentioned in section 2.1.1 the ear has a resonance around 3 $kHz$, which explains the higher noise level heard by the binaural head. Still, driver microphone signals will be used to compare the interior SPL predicted by the models to experimental
results. This is justified by the fact that FRFs obtained from the vibroacoustic simulation tool developed by Douville are obtained at the drivers’ head and also because, under 250 Hz, measurements are not significantly influenced by the location of the microphones.

When looking at passenger SPL within the 0-250 Hz bandwidth (Fig. 4-20), we can observe slight differences between passengers at certain frequencies. Cabin air cavity mode shapes can explain this observation: Douville’s Honda Civic FEM shows air cavity resonances at 75, 110, 150, 190, 210, 240 & 250 Hz. The 75, 110, 150, 190 & 210 Hz modes are observable approximately at the same positions. As for the 240 and 250 Hz modes, they are not present on experimental measurements. Considering that the air cavity resonances positions are very sensible to dimension variations and the presence of passengers and furniture within the automobile, these modes could be positioned above 250 Hz because of the intrinsic properties of the Epica.

![Diagram: Interior Sound Pressure Level in an Epica LS at 50 km/h]

Fig. 4-20: SPL in a towed car (50 km/h) – Driver’s head

On the other hand, SPL data clearly show both tire acoustic resonances at 220 and 230 Hz predicted by Thompson’s analytic formula. The level of both resonances confirms the importance of tire air cavity resonance as a contributor to cabin interior noise. Finally, Constant study clearly showed an influence of the second vertical resonance of a 15” tire to the SPL in the 80 and 125 Hz octave bands [3]. We observe this same tendency on the Epica pressure PSD.
4.4.1) Towed vehicle VS self-propelled vehicle

Fig. 4-21 compares the SPL inside a car moving at 50 km/h when it is self-propelled versus when it is towed by another vehicle. Since no specific information on engine revolution speed has been measured during experiments, it is difficult to identify noise sources. If a V6 engine runs at 2500 RPM, the rotational frequency is approximately 40 Hz and the firing frequency is around 120 Hz. [32]

![Graph showing SPL comparison between driving and towing](image)

**Fig. 4-21: Comparison of SPL – Driven VS Towed**

To have a better understanding of the contribution of engine to interior noise, a reference sensor (accelerometer and/or tachometer) should be placed on the power train components. Depending on engine revolution speed and torque, it could be possible to observe the importance of crankshaft or transmission gear interior noise contribution. This was not realized in this work.

4.4.2) Interior SPL of the Epica LS VS Civic FEM/BEM

The transmission simulations provide the pressure at the driver’s head for any type of test bench excitation. But the FEM/BEM model was never compared to a real interior noise measurement. Since the positioning of the actuators will also be tested using an interior SPL criterion, one must make this tool as realistic as possible to find the adequate actuator configuration.
Fig. 4-22 compares the Epica SPL to the prediction of the transmission simulation. We can see that all curves do not follow the same trend. On the other hand, one must keep in mind that not all car frame FRFs are included in the transmission tool at the moment. If all four wheels transmission paths would be considered, analytical energy level would be higher.

![Interior SPL at driver right ear level: vibroacoustic tool VS EPICA](image)

**Fig. 4-22: SPL comparison of the vibroacoustic model VS EPICA measurements**

The variance of the model is much more important than the experimental measurements. One explanation is insufficient damping in the virtual model: the presence of furniture and three persons inside the car contributes to increase damping and reduces reverberation time in the experimental results. Damping is a very important factor for noise level at low frequencies. In the FEM/BEM model, there are no stiffeners in car panels, no damping or absorption coefficients related to constituting materials and no bodies inside the air cavity.

We also observe that higher frequencies are more important with the model while experimental results tend to provide a flat response. The fact that experimental SPL are the result of excitation at four suspensions could explain the lower noise level predicted by the model at low frequencies. As for higher frequencies, the lack of damping can explain over-evaluation by the model.
4.5) Correlation of road-induced vibration with interior noise

FRFs are evaluated with the Epica measurements to study the potential of ASAC for reducing the interior noise from road-induced vibrations. The hub DOF entrance signal \( x \) was correlated with the cabin SPL measurement \( y \). Eq. 4.4 and 4.5 present the formula to obtain the FRF \( H_1 \) and \( H_2 \).

\[
H_1(f) = \frac{1}{T} \left[ \frac{1}{n_d} \sum_{i=1}^{n_d} X_i Y_i \right] \left/ \frac{1}{T} \left[ \frac{1}{n_d} \sum_{i=1}^{n_d} X_i X_i \right] \right. \tag{4.4}
\]

\( X_i \) corresponds to the Discrete Fourier Transform (DFT) coefficients of the input signal \( x(i) \) (wheel hub acceleration in this case) and \( Y_i \) to the DFT coefficients of the output signal \( y(i) \) (pressure at driver’s head). \( n_d \) is the total number of signal segments on which a DFT is made. \( T \) is the time period of each segment.

\[
H_2(f) = \frac{1}{T} \left[ \frac{1}{n_d} \sum_{i=1}^{n_d} Y_i Y_i \right] \left/ \frac{1}{T} \left[ \frac{1}{n_d} \sum_{i=1}^{n_d} Y_i X_i \right] \right. \tag{4.5}
\]

The FRF in Fig. 4.23 is found with a DFT of the time-domain impulse system response found with an LMS algorithm. The details of the LMS algorithm are explained in Douville Master’s thesis [28].

![FRF of Cabin Interior SPL at driver’s head / Vertical Wheel Hub Acceleration (Z-Axis)](image)

Fig. 4.23: FRF between Epica’s driver interior SPL & hub acceleration on the Z-Axis
Important variations exist between $H_1$ and $H_2$. Knowing that $H_2$ is influenced by output sensors noises (microphones) and that $H_1$ is very similar to $H$, this suggests that the microphone data contains an important amount of noise. Fig. 4-24 shows the coherence between the $X$, $Y$ and $Z$-Axis Epica hub acceleration and cabin interior noise (Eq. 4.6). The resulting coherence is very low (a coherence of 0.8 indicates that 64 % of the signal can be explained by this relation, a coherence of 0.4 means that only 16 % of the signal can be explained, etc.). With low level of coherence, active control has a very limited effect.

$$
\gamma^2(f) = \frac{H_1(f)}{H_2(f)}
$$

(4.6)

It’s interesting to observe coherence peaks at 75 and 150 Hz: they are also present within Park and al. coherence results. [22] Literature review has identified global suspension lateral resonances at these frequencies.

![Graph showing coherence between road excitation and cabin SPL](image)

**Fig. 4-24: Coherence between spindle accelerations & and error microphone**

To increase coherence, Dehandschutter and al. implemented ASAC with a total of 12 reference sensors and a multiple coherence function. [6] This function uses power spectral and cross-spectral matrices between the $k$ reference signals and the $l$ error microphones (Eq. 4.7 to 4.9). Details of the function can be found in Elliott’s book “Signal processing for active control”. [24]
\[ S_{xx} = E[X(f)X(f)^H] \]  
(4.7)  
\[ S_{yy} = E[Y(f)Y(f)^H] \]  
(4.8)  
\[ S_{xy} = E[Y(f)X(f)^H] \]  
(4.9)

Here, \( x \) is DFT of the vector of \( k \) reference signals (6 wheel hub acceleration signals), \( y \) is the DFT of the vector of \( l \) error signals (error microphones). \( S_{xx} \) is the \( k \times k \) matrix of power & cross-spectral densities for the reference signals at a specific frequency. \( S_{xy} \) is the \( l \times k \) matrix of cross-spectral densities between the reference and disturbance signals. \( S_{yy} \) is the \( l \times l \) matrix of power & cross-spectral densities of the disturbance signals. Eq. 4.10 present the multiple coherence function in the special case of a single disturbance signal (one microphone):

\[ \gamma_{xy}^2 = \frac{S_{xy}S_{xx}^{-1}S_{xy}^H}{S_{yy}} \]  
(4.10)

Where \( S_{xx}^{-1} \) is the pseudo-inverse of \( S_{xx} \).

![Multiple coherence function](image)

**Fig. 4-25: Multiple coherence function with the 6 DOF of the Epica Wheel Hub**

With this method, the coherence is slightly better (Fig. 4-25), it therefore appears that it might be possible to control minimally the 130 – 160 Hz frequency band. Still, if any global noise reduction is desired, coherence must be improved.
In order to minimize coherence loss due to output sensor (microphone) noise, the number of signal averages should be increased (take longer segments). Also, different input sensor (accelerometers) position should be tested: for example, sensors could be positioned at each suspension linkage: the bushings and strut towers are flexible components that lower the impedance of suspension links, but also increase the acceleration at these specific locations. This larger acceleration levels could give a sufficient energy level to sensor data and consequently keep data at higher levels than the sensor sensibility thresholds.

Another factor to consider is that there are four wheels on a complete car: the suspension test bench has a single disturbance. In a real car on the other hand, the road disturbs each wheel. The resultant noise is therefore a sum of each wheel road-induced vibrations. Dehandschutter measured the accelerations of each wheel hub to realize the ASAC of a complete car. However, in a straight line, the rear wheels are receiving the same disturbance as the front wheels with a time delay that is function of the car speed. Consequently, there are only two uncorrelated noise sources: the left and right side road-induced vibrations. The measurement of the two front wheel acceleration should help to increase the coherence levels between the wheel hubs accelerations and pressures levels at the driver’s head.

4.6) Chapter summary

The interior noise analysis of a Chevrolet Epica LS has revealed certain characteristics of road-induced noise:

- In a car moving at 50 km/h with three passengers, 0-500 Hz frequency band is the most important contributor to interior noise;
- In a car moving at 50 km/h on a slightly inclined road, engine noise is negligible compared to other sources of interior noise;
- The coherence of spindle acceleration with interior noise is more important on global suspension and wheel resonances.
- Even if the effects are not observable in the 0-250 Hz frequency band, we have also observed the effects of passenger’s ears on perceived noise levels.

The dB(A) correction curve corrects standard microphones measurements to include psychoacoustics effects in the perceived sound pressure level.
With respect to the vibroacoustic transmission path simulation tool, this study has analysed the following aspects:

- The vibroacoustic behaviour of road noise transfer paths is dependent on the geometry of each sub-system and their interaction. Therefore, the analysis of global car FRFs does not provide enough information to anticipate the behaviour of specific car sub-systems.

- The 50% test bench energy level is appropriate for a “Good road” profile;

- The measurements taken under the 3-legged table supporting the tire helped to adapt primary disturbance frequency content to reproduce a “Good road” profile;

- The filtered road disturbance has a PSD between 1 to 5 $N^2/Hz$. Depending on the chosen frequency band to be controlled, relatively small actuators could be employed to control road-induced vibrations;

- The damping of the FEM/BEM model is insufficient, the bodies of passengers, car furniture and material damping should be taken into account to correct the reverberation time of the air cavity.

The FRF analysis confirmed that the key in any experimental ASAC is the identification of sensor and actuator positions to obtain a sufficient coherence level. Additional coherence investigation could be executed with accelerometers placed on the suspension linkages to measure suspension FRFs and their multiple coherence function.
CHAPTER 5

5) OPTIMIZATION OF ACTIVE CONTROL CONFIGURATION

Chapter 2 provided some indications on the origins of road-induced interior noise and its propagation mechanisms. In Chapter 3, a simplified model of a quarter-car suspension was created to reproduce the vibroacoustic behaviour of its components. Chapter 4 exploited experimental measurements on a complete car to adapt the road excitation applied to the test bench. The next logical step is to use the information gathered in previous chapters to study the effects of an active suspension on road-induced interior noise.

The quarter-car suspension model is now used to test certain actuator configurations without any experimental burden. The optimization algorithm is implemented into the model so a variety of actuator positions and orientations, as well as control commands, could be tested in a short amount of time. This way, noise inside the car cabin can be reduced with a maximized efficiency for a specific type of actuator. This information will give clues on the right combination and position of actuators to implement on conventional McPherson suspensions to maximize noise control efficiency.

5.1) Possible actuator configurations

There are two ways to transform a conventional suspension into an active suspension: add an actuator onto a conventional suspension (parallel configuration) or modify one of its components so energy can be injected in series with a suspension component (fully integrated configuration).

An example of a parallel configuration could be inertial shakers attached on the suspension (Fig. 5-1 and 5-2). If the actuator has only one attachment point to the suspension, it has the advantage of not creating another transmission path for road-induced vibrations. This solution is also the easiest to implement on our suspension model: an actuator can be included in the suspension model by creating a link between the suspension and the actuator. However, if the actuator has two suspension linkages (like a power cylinder), it will add a supplementary transmission path that will have to be controlled. This inevitably adds complexity to the problem and the ASAC will be harder to achieve.
An example of a parallel actuator with two suspension linkages is the Bose\textsuperscript{©} suspension [34]. An electromagnetic actuator replaces the conventional strut tower viscous damper; it applies a control force on both ends of the strut tower in the Z-axis (Fig. 5-3 and 5-4). Unfortunately, the actual suspension model is unable to evaluate this exact configuration: because the model is based on a quasi-infinite impedance test bench, no displacement at the linkage level is possible. One would need to consider the impedance of a real car for each mainframe linkage using a 12 DOF equivalent rigidity model. [35]

An example of actuator in series would be a set of stacked piezoelectric devices placed between the strut tower linkage and it’s car frame linkages. This configuration could reduce vibrations levels by absorbing the energy coming form the linkage (like the case of a speaker inside a duct). To realise such a configuration on the suspension model, an actuator would need to be placed between suspension discrete elements, thus adding DOF to the model. Although feasible, such actuator configurations were not considered in this work. As for implementing an actuator between a suspension linkage and the car frame, 12 DOF mainframe linkages models would also be necessary.
5.1.1) Genetic algorithms for ASAC optimization

Genetic algorithms are already used in problems were the position of actuator(s) must be optimized for ASAC. [27] As for the actuator(s) control input, Douville has demonstrated it can be optimized when expressed in a simple quadratic form.

![Genetic Algorithm Diagram](image)

**Fig. 5-5: Actuator positioning optimization with genetic algorithms**

The combination of genetic algorithms and the optimal control theory has already been realised for active noise control optimization. [36]: the stochastic process of the genetic algorithm chooses a certain actuator configuration (individual). The optimal control for each configuration is evaluated (Fig. 5-5). The strength of each individual is evaluated on its potential for specific criterion reduction. The actuator configurations showing the best noise reducing potential are mated (crossover and mutation process) to create the next generation of actuator configuration. Generation by generation the optimal actuator configuration will be obtained.

5.1.2) Forces optimization for optimal control

Depending on the number of control actuators and error sensors in the active control system, the problem can be under-determined or over-determined. The methodology to use to find the optimal control inputs in each case is explained in Nelson & Elliot’s “Active Control of Sound” volume. [33] The minimization demonstration will not be done here, but the displacement, force and pressure criterion $J$ are detailed in the following sections. For the sake of efficiency, the frequency dependence in the following equations is omitted.
5.1.2.1) Displacement cost function

Appendix A presents the equations of motion of the suspension model. They can be used to create an active control criterion based on the displacement of each suspension DOF. The Eq. 5.1 presents the equations of motion of the suspension model. Where $\{\tilde{x}\}$ is the displacement vector of each suspension discrete element. In Eq. 5.3 the actuator vectors are added.

$$\{\tilde{x}\} = (-\omega^2 [M] + [K])^{-1} [\tilde{F}]$$  \hspace{1cm} (5.1)

$$[\tilde{B}] = (-\omega^2 [M] + [K])^{-1}$$  \hspace{1cm} (5.2)

$$\{\tilde{x}\} = [\tilde{B}] \{F_r\} + [F_s] \{\tilde{\alpha}\}$$  \hspace{1cm} (5.3)

The matrix $[F_s]$ is the generalized unitary control force (projected over the suspension DOF) and the vector $\{\tilde{\alpha}\}$ is the complex vector of each control force for a given frequency. The quadratic cost function minimizing the displacement of all suspension DOFs is:

$$J_{\text{DOF}} = \|\{\tilde{x}\}\|^2 = \{\tilde{x}\}^T [\tilde{x}]$$  \hspace{1cm} (5.4)

$$J_{\text{DOF}} = \left( [\tilde{B}] \left( \{ F_r \} + [F_s] \{\tilde{\alpha}\} \right) \right)^T \left( [\tilde{B}] \left( \{ F_r \} + [F_s] \{\tilde{\alpha}\} \right) \right)$$  \hspace{1cm} (5.5)

$$J_{\text{DOF}} = \{\tilde{\alpha}\}^T [F_s]^T [\tilde{B}]^T [\tilde{B}] [F_s] \{\tilde{\alpha}\} + \{\tilde{\alpha}\}^T [F_r]^T [\tilde{B}]^T [F_r] \{\tilde{\alpha}\} + \{N\}^T \{\tilde{\alpha}\} + \{E\}$$  \hspace{1cm} (5.6)

The optimal command input $\{\tilde{\alpha}_{\text{opt}}\}$ is obtained with:

$$\{\tilde{\alpha}_{\text{opt}}\} = - [J]^{-1} [N]$$  \hspace{1cm} (5.7)

Now, if the displacement of a specific link is to be minimized, the transfer matrices $[T_{\text{node}}]$ described in Appendix A give the exact displacement of each suspension model discrete element. As an example, the displacement of the strut tower linkage will be minimized $J_{\text{DBH}}$. The first step is to obtain the displacement vector of the strut tower link (Eq. 5.8).

$$\{\tilde{x}_{\text{BH}}\} = [T_{\text{BH}}] \{\tilde{x}\}$$  \hspace{1cm} (5.8)
where \( \{T_{BH}\} \) is the transfer matrix for the strut tower linkage (denoted \( BH \)) and \( \{\vec{X}_{BH}\} \) is the resulting 6 DOF vector describing the displacement of the strut tower node. Using Eq. 5.3 with Eq. 5.8, we can obtain the displacement of the strut tower linkage with the control matrix:

\[
\{\vec{X}_{BH}\} = [T_{BH}] \begin{bmatrix} \vec{B} \\ \{F_p\} + \{F_s\} \{\vec{\alpha}\} \end{bmatrix}
\]  

(5.9)

Eq. 5.11 gives the strut tower displacement cost function:

\[
J_{DBH} = \|\{\vec{X}_{BH}\}\|^2 = \{\vec{X}_{BH}\}^T \begin{bmatrix} \vec{X}_{BH} \end{bmatrix}
\]  

(5.10)

\[
J_{DBH} = \{T_{BH} \begin{bmatrix} \vec{B} \\ \{F_p\} + \{F_s\} \{\vec{\alpha}\} \end{bmatrix}\}^T \begin{bmatrix} \vec{B} \\ \{F_p\} + \{F_s\} \{\vec{\alpha}\} \end{bmatrix} \{T_{BH} \begin{bmatrix} \vec{B} \\ \{F_p\} + \{F_s\} \{\vec{\alpha}\} \end{bmatrix}\}
\]  

(5.11)

To obtain the displacement of any suspension discrete element, one has to use a form similar to Eq. 5.8 and replace the transfer matrix by the appropriate one.

5.1.2.2) Force cost function

If the displacement of a chassis link is known, the forces generated on the frame by the linkage can also be identified using its equivalent rigidity model. For example, if the force transmitted by the strut tower is to be minimized, one has simply to multiply the displacement vector of the link (Eq. 5.9) with its equivalent stiffness model \( \begin{bmatrix} \vec{K}_{BH} \end{bmatrix} \). This results in Eq. 5.12.

\[
\{\vec{F}_{BH}\} = \begin{bmatrix} \vec{K}_{BH} \end{bmatrix} \begin{bmatrix} \vec{X}_{BH} \end{bmatrix} = \begin{bmatrix} \vec{K}_{BH} \end{bmatrix} \{\vec{X}_{BH}\}
\]  

(5.12)

\( \begin{bmatrix} \vec{K}_{BH} \end{bmatrix} \) is a 6x6 diagonal matrix holding the experimentally measured strut tower equivalent dynamic stiffness and \( \{\vec{F}_{BH}\} \) is the 6x1 resultant forces vector of the strut tower linkage. Forces generated at specific chassis links can be minimized with:

\[
\{\vec{F}_{BH}\} = \begin{bmatrix} \vec{K}_{BH} \end{bmatrix} \begin{bmatrix} \vec{B} \end{bmatrix} \begin{bmatrix} \{F_p\} + \{F_s\} \{\vec{\alpha}\} \end{bmatrix}
\]  

(5.13)

\[
J_{FBH} = \|\{\vec{F}_{BH}\}\|^2 = \begin{bmatrix} \vec{K}_{BH} \end{bmatrix} \{\vec{B}\} \begin{bmatrix} \{F_p\} + \{F_s\} \{\vec{\alpha}\}\end{bmatrix}^T \begin{bmatrix} \vec{K}_{BH} \end{bmatrix} \begin{bmatrix} \vec{B}\end{bmatrix} \begin{bmatrix} \{F_p\} + \{F_s\} \{\vec{\alpha}\}\end{bmatrix}
\]  

(5.14)

The content of the linkage model can be adapted to control specific forces and/or moments. For example, if only Z-axis forces of the suspension are to be minimized, then only the third diagonal term of \( \begin{bmatrix} \vec{K}_{BH} \end{bmatrix} \) should be considered. All other terms within the matrix would be changed to zero:

\[
\vec{F}_{BH2} = \begin{bmatrix} \vec{K}_{BH2} \end{bmatrix} \begin{bmatrix} \vec{B} \end{bmatrix} \begin{bmatrix} \{F_p\} + \{F_s\} \{\vec{\alpha}\}\end{bmatrix}
\]  

(5.15)
Now, if all Z-axis transmitted forces are to be minimized, the sum of each force linkage cost function must be summed:

$$J_{FZ} = \|F_Z^2 + \|F_{BHZ}\|^2 + \|F_{BIZ}\|^2 + \|F_{BZ}\|^2$$  \hspace{1cm} (5.16)$$

Where $\|F_{BHZ}\|^2$, $\|F_{BIZ}\|^2$ and $\|F_{BZ}\|^2$ are the Z-axis efforts cost functions for the strut tower, left and right bushing respectively.

### 5.1.2.3) Pressure cost function

Doville’s vibroacoustic FEM provides numerical transfer functions from suspensions linkage forces up to pressure level at the driver’s head $\{H_{BH}\}$. It is possible to evaluate the noise perceived by the driver caused by the forces at each suspension link (Eq. 5.17):

$$p_{BH} = \{H_{BH}\}^T [K_{BH}] [T_{BH}] \{\ddot{X}\} = \{H_{BH}\}^T \{F_{BH}\}$$  \hspace{1cm} (5.17)$$

Since we only have the X, Y & Z-axis FRFs for the strut tower, the last 3 terms of $\{H_{BH}\}$ are empty. The same problem exists for the sixth term of $\{H_{B1}\}$ and $\{H_{B2}\}$. The total sound pressure at the driver’s head is a sum of the pressure caused by all the chassis links:

$$J_{pDriver} = \|p_{DRIVER}\|^2 = \|p_{BH} + p_{B1} + p_{B2}\|^2$$  \hspace{1cm} (5.18)$$

$$J_{pDriver} = \begin{bmatrix} G_{BH} \end{bmatrix} \begin{bmatrix} F_{BH} \end{bmatrix} = \begin{bmatrix} \ddot{X} \end{bmatrix} \begin{bmatrix} H_{BH} \end{bmatrix}^T [K_{BH}] [T_{BH}] \begin{bmatrix} B \end{bmatrix}$$  \hspace{1cm} (5.19)$$

$$J_{pDriver} = (((G_{BH} + [G_{B1}] + [G_{B2}] + \begin{bmatrix} F_p \end{bmatrix} + \begin{bmatrix} F_s \end{bmatrix} + \begin{bmatrix} \alpha \end{bmatrix} + \begin{bmatrix} \beta \end{bmatrix} + \begin{bmatrix} \gamma \end{bmatrix} + \begin{bmatrix} \delta \end{bmatrix}), ((G_{BH} + [G_{B1}] + [G_{B2}] + \begin{bmatrix} F_p \end{bmatrix} + \begin{bmatrix} F_s \end{bmatrix} + \begin{bmatrix} \alpha \end{bmatrix} + \begin{bmatrix} \beta \end{bmatrix} + \begin{bmatrix} \gamma \end{bmatrix} + \begin{bmatrix} \delta \end{bmatrix})))$$  \hspace{1cm} (5.20)$$

For this specific quadratic cost function, the $\{\alpha\}$ value will seek an optimal control input to reduce the pressure at the driver’s head depending on actuator location. If the noise level at all passengers were to be controlled, the cost function would be:

$$J_{pPassengers} = \|p_{DRIVER}\|^2 + \|p_{FRONTRIGHT}\|^2 + \|p_{BACKLEFT}\|^2 + \|p_{BACKRIGHT}\|^2$$  \hspace{1cm} (5.21)$$

To realise such a cost function, FRFs between forces at each suspension linkage and sound pressure at passengers’ locations would be necessary. This cost function would be more adapted to observe global noise reduction inside the car cabin.
5.2) Validation of the optimization tool

The combination of the genetic algorithm with optimal control provides an optimization tool to obtain configurations of actuators to ensure the best possible performance of the active suspension. However, the optimization tool must first be validated. To do so, certain tests are conducted to understand the behaviour of the optimization tool and the general trends suggested by this algorithm for an actuator positioning optimization.

5.2.1) Single actuator – unlimited force - broadband

This section observes the effect of a single control force on the suspension. An additional arm is linked to any point on the suspension. It can be of any length \( (L_b \text{ up to } 0.25 \text{ m}) \) and in any direction \( (\theta_b \text{ and } \phi_b \text{ assign the orientation lever in spherical coordinates}) \). The actuator force is applied at the extremity of this arm. It can also be in any orientation \( (\theta_f \text{ and } \phi_f \text{ assign the orientation of the force}) \). Table 5-1 presents the chromosomes of an individual. Binary genes were taken for this problem. This arbitrary choice was made by the user and has no incidence on the problem itself.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Possibilities</th>
<th># Of Genes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspension linkage</td>
<td>12 points in total</td>
<td>4</td>
</tr>
<tr>
<td>Bar angle ( (\theta_b) )</td>
<td>( 360^\circ / 1^\circ = 360 )</td>
<td>9</td>
</tr>
<tr>
<td>Bar angle ( (\phi_b) )</td>
<td>( 360^\circ / 1^\circ = 360 )</td>
<td>9</td>
</tr>
<tr>
<td>Bar length ( (L_b) )</td>
<td>( 0.25 \text{ m} / 0.4883 \text{ mm} = 512 )</td>
<td>9</td>
</tr>
<tr>
<td>Force angle ( (\theta_f) )</td>
<td>( 360^\circ / 1^\circ = 360 )</td>
<td>9</td>
</tr>
<tr>
<td>Force angle ( (\phi_f) )</td>
<td>( 360^\circ / 1^\circ = 360 )</td>
<td>9</td>
</tr>
<tr>
<td># Of Genes</td>
<td>Total (chromosome)</td>
<td>49</td>
</tr>
</tbody>
</table>

Table 5-1: Genes description for single force genetic optimization

As a first test, the optimization algorithm was asked to minimize the global displacements of the suspension \( J_{DOF} \) (Eq. 5.1). For this specific criterion, the genetic algorithm converges to the actuator force right on top of the primary shaker link (Fig. 5-6). As for the control force, it is the exact opposite of the road excitation. This seems natural because the selected position does reduce the interior noise level to the threshold of Matlab\textsuperscript{®} numerical resolution.
On the other hand, it points out the problems related to the test bench excitation mechanism: all the solutions which will be presented in this chapter will be optimized to control the forces or sound pressure created by the test bench disturbance, not a real suspension. If the suspension model had been based on a test bench where road excitation was exerted directly on the tire (displacement of the disturbance under the tire module), control positions would be more adapted to a real car.

Also, the genetic algorithm alone has no understanding as to where it is possible to position an actuator (the actual optimized actuator position imposes to modify some car components and is outside the car conventional volume). To prevent undesirable solutions, the spatial constraint of a car must be included as an element of the evolution environment: the tire, the coil spring, car panels and the road clearance volumes have been set around the suspension model to limit the genetic algorithm spatial evolution (Fig. 5-7).

Any actuator proposed outside the available space gets automatically an infinite penalty in its strength evaluation. This ensures that this individual is not selected as a parent for future generations. The same penalty is given to a solution proposing an actuator linked to any rotating component of the suspension (tire & driving shaft).
5.2.2) Single actuator – limited force - broadband

The next section will minimize $J_{dOFR}$ with the geometric constraints discussed in the previous section. Also, the optimization will be conducted for three specific cases: no force limitation, limited to 50% of the primary disturbance and limited to 10% of the primary disturbance. In each case, the primary disturbance is set to 1N for all frequencies.

If we compare the suggested actuator positions (Fig. 5-8 & Table 5-2), the actuators that can deliver an important amount of force are placed to oppose the disturbance: we know that around 56% of road-induced energy passes through the vertical direction, 34% in the X-axis and the rest in the Y-axis. The actuator placed at the $F_{\text{inf}}$ position is placed directly in the X-Z plane. Also, since there are no DOF between the tire and strut tower, the force directly counters the disturbance force.

![Fig. 5-8: Control force comparison](image)

![Fig. 5-9: Optimal control command as a function of actuator maximal force](image)

When looking at the control command (Fig. 5-9), we see that an important amount of energy is required to achieve optimal control. However, when the force is limited to 10%, the algorithm uses an additional lever to reduce the amount of force required to control the suspension. At the same time, our test bench study has proven that the rotating components of the suspension are very flexible compared to a real suspension. The algorithm uses this angular flexibility to amplify the displacement of the strut tower module and use it’s inertia to control other DOF displacement.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>$F_{\infty}$</th>
<th>$F_{50%}$</th>
<th>$F_{10%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspension linkage point</td>
<td>Wheel hub CM</td>
<td>Damper top</td>
<td>Damper top</td>
</tr>
<tr>
<td>Bar angle ($\theta_b$)</td>
<td>208°</td>
<td>17°</td>
<td>88°</td>
</tr>
<tr>
<td>Bar angle ($\phi_b$)</td>
<td>159°</td>
<td>48°</td>
<td>19°</td>
</tr>
<tr>
<td>Bar length ($L_b$)</td>
<td>0.1245$m$</td>
<td>0.1895$m$</td>
<td>0.2495$m$</td>
</tr>
<tr>
<td>Force angle ($\theta_f$)</td>
<td>192°</td>
<td>191°</td>
<td>191°</td>
</tr>
<tr>
<td>Force angle ($\phi_f$)</td>
<td>251°</td>
<td>63°</td>
<td>31°</td>
</tr>
</tbody>
</table>

Table 5-2: Optimal genes for actuator of $F_{\infty}$, $F_{50\%}$ and $F_{10\%}$ maximal force

Another interesting aspect is the actuator orientation: when the force is unlimited, the actuator is placed directly in the same plane as the disturbance. However, when the actuator force is constrained to be less than the disturbance, the algorithm disregards the $Z$-axis excitation and gives more importance to the $X$ & $Y$-axis disturbance (the 10% actuator is oriented in the $X$-$Y$ plane).

![Criterion reduction: 1 Actuator - $J_{\text{dop}}$ criterion](image)

Fig. 5-10: Criterion minimization depending on actuator maximal force

When looking at $J_{\text{dop}}$ (Fig. 5-10), one can see that the criterion reduction is proportional to the actuator force. However, Fig. 5-11 compares the criterion reduction in three cases: the performance of a 50% actuator placed at its optimal position, a 10% actuator placed in the same position and orientation as the 50% actuator (non optimal position) and a 10% actuator placed at its optimal position.
The results show that the optimization algorithm finds an actuator orientation and position that does increase an actuator efficiency depending on its maximal force. 50% of the disturbance force will be used for future cases since it is the maximal force of the experimental actuator used in Chapter 3.

5.2.3) Single actuator – limited force – frequency limited

This study will minimize $J_{DOF}$ with the help of a single actuator having a maximum force of 50% of the primary disturbance. However, the criterion will be minimized below 25 Hz in the first case and above 25 Hz in the second case. Table 5-3 and Fig. 5-12 show the suggested actuator positions for each case.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$J_{DOF}(0\text{--}25\text{Hz})$</th>
<th>$J_{DOF}(25\text{--}250\text{Hz})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linkage point</td>
<td>Damper bottom.</td>
<td>Damper top.</td>
</tr>
<tr>
<td>Bar angle ($\theta_B$)</td>
<td>266°</td>
<td>17°</td>
</tr>
<tr>
<td>Bar angle ($\phi_B$)</td>
<td>188°</td>
<td>48°</td>
</tr>
<tr>
<td>Bar length ($L_B$)</td>
<td>0.2495m</td>
<td>0.1895m</td>
</tr>
<tr>
<td>Force angle ($\theta_F$)</td>
<td>25°</td>
<td>191°</td>
</tr>
<tr>
<td>Force angle ($\phi_F$)</td>
<td>77°</td>
<td>63°</td>
</tr>
</tbody>
</table>

Table 5-3: Optimization of $J_{Def}$ between 0-25 Hz and 25-250 Hz
When looking at the criterion for each case (Fig. 5-13), one can observe that the reduction performance below 25 Hz is slightly improved in the first case (on the other hand, the compromise at other frequencies is also noticeable). The algorithm can consequently provide an optimized actuator position taking into account both the actuator characteristics and the modal shapes of the suspension.

The following studies will optimize uniquely the $25 - 250$ Hz band to observe possible noise reduction. The $0 - 25$ Hz frequency band must be taken out also because this study points out that the algorithm response could be modified by these frequencies: they possess an important amount of energy and could flaw the optimized actuator position.

### 5.3) Optimization of actuators configuration

Now that the performance of the optimization tool has been evaluated, it will be used with specific criterion to observe their effects on the internal cabin sound pressure at the driver’s head. The actuator positions will be optimized under the following conditions:

- A “Good road” profile disturbance having an average force of $1 \text{ N/Hz}$;
- Actuator maximal force of $0.5 \text{ N/Hz}$;
- Road, tire, coil and car panels volume constraints;
- Control of the $25 - 250$ Hz frequency band;

For each criterion three curves will be presented: the “before control” curve will display the criterion before control. The “optimal control” curve will show the possible criterion reduction of an actuator, which could deliver an infinite amount of force. The “constrained control” curve will present the actual criterion reduction potential of the actuator.

#### 5.3.1) Single actuator

For each actuator combination, three different cost functions will be optimized: $J_F^x$, $J_F^z$, & $J_{\text{Driver}}$. This is justified by the fact that Douville found the largest noise reduction potential with this criterion [28].

---

![Fig. 5-13: Criterion optimization at resonances](image-url)
5.3.1.1) Minimisation of X-axis transmitted forces

For this first case, a single actuator was placed on the suspension to reduce all X-axis forces transmitted by the suspension. Fig. 5-14 & Table 5-4 present the results from the optimization algorithm.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimized response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspension linkage</td>
<td>Damper top</td>
</tr>
<tr>
<td>Bar angle ($\theta_B$)</td>
<td>32°</td>
</tr>
<tr>
<td>Bar angle ($\phi_B$)</td>
<td>55°</td>
</tr>
<tr>
<td>Bar length ($L_B$)</td>
<td>0.2495m</td>
</tr>
<tr>
<td>Force angle ($\theta_F$)</td>
<td>189°</td>
</tr>
<tr>
<td>Force angle ($\phi_F$)</td>
<td>213°</td>
</tr>
</tbody>
</table>

Table 5-4: Optimization single actuator genes for $J_{FX}$

The actuator is placed on top of the tire (at the front of the coil spring) and slightly to the left section of the suspension. Since the centre of mass of the strut tower is on the right side of the suspension, the distance between the two will create moments around the X-axis. Also, the actuator is oriented mainly towards the excitation mechanism. This creates a rotation around the Y-Axis. These two factors can explain why a vertical excitation can generate lateral forces.

Fig. 5-15: SPL at driver’s head with $J_{FX}$

Fig. 5-16: Criterion $J_{FX}$ - single actuator
As shown in Fig. 5-16, it seems that the actuator is powerful enough to reduce $J_{FX}$ because it is brought to its optimal level in the entire frequency band. This indicates that additional force would be useless and that the actuator is correctly designed for the actual case. When looking at resonances more specifically, Table 5-5 presents interesting noise reductions.

<table>
<thead>
<tr>
<th>Resonance (Hz)</th>
<th>Reduction (dB(A))</th>
<th>Resonance (Hz)</th>
<th>Reduction (dB(A))</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>24.6</td>
<td>190</td>
<td>4.7</td>
</tr>
<tr>
<td>110</td>
<td>14.7</td>
<td>230</td>
<td>7.0</td>
</tr>
<tr>
<td>140</td>
<td>9.4</td>
<td>240</td>
<td>2.0</td>
</tr>
<tr>
<td>160</td>
<td>7.3</td>
<td>250</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table 5-5: Transmission path resonance noise reduction: single actuator - criterion $J_{FX}$

What can be deduced from these results is that, given a primary disturbance force of 1 N/Hz, a 0.5N/Hz dynamic actuator is capable of reducing the pressure level at the driver’s head of 5.12 dB(A) in the 25/250-frequency band (analytically). Alternatively, inertial actuators capable of generating 0.5 N at their resonance could reduce specific transmission path resonance (up to 24.6 dB(A) if trying to control the 75 Hz resonance).

5.3.1.2) Minimisation of Z-axis transmitted forces

The second study will aim to reduce the suspension Z-axis force transmissibility: $J_{FZ}$. Fig. 5-17 and Table 5-6 show the optimized actuator configuration.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimized response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspension linkage point</td>
<td>Wheel Hub CM</td>
</tr>
<tr>
<td>Bar angle ($\theta_b$)</td>
<td>127°</td>
</tr>
<tr>
<td>Bar angle ($\phi_b$)</td>
<td>135°</td>
</tr>
<tr>
<td>Bar length ($L_B$)</td>
<td>0.2495m</td>
</tr>
<tr>
<td>Force angle ($\theta_F$)</td>
<td>197°</td>
</tr>
<tr>
<td>Force angle ($\phi_F$)</td>
<td>55°</td>
</tr>
</tbody>
</table>

Table 5-6: Optimization single actuator genes for $J_{FZ}$

For this case, the actuator has been localized on top of the tire on the left side of the suspension. The orientation of the actuator is approximately at 45° in the X-Z plane. The actuator therefore produces a rotation around the Y-axis.
The criterion $J_{FZ}$ is more difficult to reduce for an actuator because the optimal command offers a lower criterion value in average than the maximal criterion achieved by the actuator (Fig. 5-19). Noise reductions confirm this trend (Fig. 5-18): the actuator can reduce noise globally by 3.7 $dB(A)$ this time at the driver’s head. Table 5-7 shows the possible noise reduction at the transmission path resonances.

<table>
<thead>
<tr>
<th>Resonance (Hz)</th>
<th>Reduction (dB(A))</th>
<th>Resonance (Hz)</th>
<th>Reduction (dB(A))</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>29.9</td>
<td>190</td>
<td>6.2</td>
</tr>
<tr>
<td>110</td>
<td>4.2</td>
<td>230</td>
<td>9.4</td>
</tr>
<tr>
<td>140</td>
<td>4.2</td>
<td>240</td>
<td>18.3</td>
</tr>
<tr>
<td>160</td>
<td>9.6</td>
<td>250</td>
<td>9.7</td>
</tr>
</tbody>
</table>

Table 5-7: Transmission path resonance noise reduction: single actuator - criterion $J_{FZ}$

It is interesting to observe that, even if the criterion suggests increasing the maximum force of the actuator, the values of the criterion at transmission path resonances are the same for the maximal and optimal value. A bigger actuator would therefore result in a global noise reduction, but no changes would be noted at the resonances.
5.3.1.3) Minimisation of the SPL at the driver’s head

The last case using a single actuator will attempt to reduce the sound pressure level at the driver’s head: \( J_{\text{driver}} \). Table 5-8 and Fig. 5-20 show the proposed actuator position.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimized response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suspension linkage point</td>
<td>Bushing #2</td>
</tr>
<tr>
<td>Bar angle (( \theta_B ))</td>
<td>91°</td>
</tr>
<tr>
<td>Bar angle (( \phi_B ))</td>
<td>43°</td>
</tr>
<tr>
<td>Bar length (( L_B ))</td>
<td>0.2427 m</td>
</tr>
<tr>
<td>Force angle (( \theta_F ))</td>
<td>0°</td>
</tr>
<tr>
<td>Force angle (( \phi_F ))</td>
<td>4°</td>
</tr>
</tbody>
</table>

Table 5-8: Optimization single actuator genes for \( J_{\text{driver}} \)

The actuator has been placed on the lower A-arm at the extreme right of the suspension. This is quite interesting since Douville had chosen, from the beginning, to implement his control actuator on this component. However, the actuator is oriented in the X-axis. This configuration uses the Z-axis angular DOF of the table to reduce interior noise.

![Fig. 5-20: Actuator pos. for \( J_{\text{driver}} \)](image)

![Fig. 5-21: SPL at driver's head with \( J_{\text{driver}} \)](image)

![Fig. 5-22: Actuator input for \( J_{\text{driver}} \)](image)

However, in this case, the actuator configuration is subjected to discussion: by looking at Fig. 5-22, one can see that the criterion has almost reduced between 50 and 250 Hz with an actuator force often below 0.15 N! This can be explained by the fact that the present criterion reduces noise at a specific point inside the cabin cavity. What we see in Fig. 5-21 is then a point of zero sound pressure. As explained in Chapter 2, noise is reduced at the driver’s head, but could be increased in other areas. Such a drastic reduction is also due to by the lack of damping inside the car FEM/BEM model.
5.3.2) Combination of two parallel actuators

For this actuator configuration, the length of the chromosome will be \( n \times 49 \) long, where \( n \) corresponds to the number of actuators (49 is the chromosome length in the previous section, see Table 5-1). The algorithm could optimize a configuration of four or eight actuators on a single quarter-car suspension, but one goal of ASAC control is to reduce the number of necessary actuators for noise control. This thesis will therefore limit itself to actuator configurations having less than three actuators.

5.3.2.1) Minimisation of X-axis transmitted forces

Here, two actuators will reduce suspension X-axis force transmissibility. Fig. 5-23 and Table 5-9 present the genetically optimized actuator positions.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1(^{st}) Actuator</th>
<th>2(^{nd}) Actuator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linkage point</td>
<td>Damper top</td>
<td>Wheel Hub</td>
</tr>
<tr>
<td>Bar angle ( (\theta_B) )</td>
<td>157°</td>
<td>328°</td>
</tr>
<tr>
<td>Bar angle ( (\phi_B) )</td>
<td>95°</td>
<td>36°</td>
</tr>
<tr>
<td>Bar length ( (L_B) )</td>
<td>0.1509m</td>
<td>0.0464m</td>
</tr>
<tr>
<td>Force angle ( (\theta_F) )</td>
<td>64°</td>
<td>191°</td>
</tr>
<tr>
<td>Force angle ( (\phi_F) )</td>
<td>255°</td>
<td>223°</td>
</tr>
</tbody>
</table>

Table 5-9: Optimization of two actuators genes for \( J_{FX} \)

Both actuators have been placed on the X-Z plane: one point in the vertical axis while the other points approximately in the X-axis. Depending on the modal shape to control, actuators are in phase to produce translation in that plane or out of phase to produce a rotation around a certain axis.
As shown in Fig. 5-25, two independent actuators can greatly reduce the $J_{FX}$ criterion. This demonstrates that setting two independent control actuators can greatly enhance ASAC noise reduction. However, the constrained actuator is not sufficient to reduce the criterion to its optimal command: the actuators would need to be more powerful to attain optimal control.

<table>
<thead>
<tr>
<th>Resonance (Hz)</th>
<th>Reduction (dB(A))</th>
<th>Resonance (Hz)</th>
<th>Reduction (dB(A))</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>24.9</td>
<td>190</td>
<td>9.31</td>
</tr>
<tr>
<td>110</td>
<td>24.9</td>
<td>230</td>
<td>35.9</td>
</tr>
<tr>
<td>140</td>
<td>7.56</td>
<td>240</td>
<td>25.2</td>
</tr>
<tr>
<td>160</td>
<td>14.0</td>
<td>250</td>
<td>17.5</td>
</tr>
</tbody>
</table>

Table 5-10: Transmission path resonance noise reduction: two actuator - criterion $J_{FX}$

Fig. 5-24, showing the cabin SPL at the driver’s head, confirms this tendency. There is a 13.2 dB(A) global noise reduction at the driver’s head and Table 5-10 shows resonance noise reductions above 20 dB(A) at 75, 110, 230, 240 & 250 Hz.

5.3.2.2) Minimisation of Z-axis transmitted forces

For this case, the optimization tool reduces the suspension Z-axis force transmissibility using two actuators. Fig. 5-26 and Table 5-11 display the optimized actuator configuration.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>1st Actuator</th>
<th>2nd Actuator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linkage point</td>
<td>Wheel hub CM</td>
<td>Wheel Hub CM</td>
</tr>
<tr>
<td>Bar angle (θₜ)</td>
<td>31°</td>
<td>192°</td>
</tr>
<tr>
<td>Bar angle (φₗ)</td>
<td>64°</td>
<td>235°</td>
</tr>
<tr>
<td>Bar length (Lₚ)</td>
<td>0.2495m</td>
<td>0.1567m</td>
</tr>
<tr>
<td>Force angle (θₗ)</td>
<td>23°</td>
<td>195°</td>
</tr>
<tr>
<td>Force angle (φₗ)</td>
<td>128°</td>
<td>111°</td>
</tr>
</tbody>
</table>

Table 5-11: Optimization two actuators genes for J₉Z

Actuators are both placed approximately in the X-Z plane and in the axis of the wheel and strut tower CM. However, they have been oriented 90° apart: this results in the creation of a rotation of the suspension around they-axis. Once again the actuators can work in phase or out of phase thus adding complexity to possibilities forces and moments injection into the suspension.

![Actuator pos. for J₉Z](image)

As seen in Fig. 5-28 this actuator configuration reduces very importantly the criterion (up to 80 dB at specific frequencies). On the other and, the reduction is not constant in all the frequency band and is suggesting that more powerful actuator are required to reduce the criterion to it’s optimal level.

<table>
<thead>
<tr>
<th>Resonance (Hz)</th>
<th>Reduction (dB(A))</th>
<th>Resonance (Hz)</th>
<th>Reduction (dB(A))</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>48.3</td>
<td>190</td>
<td>11.5</td>
</tr>
<tr>
<td>110</td>
<td>31.7</td>
<td>230</td>
<td>42.9</td>
</tr>
<tr>
<td>140</td>
<td>13.7</td>
<td>240</td>
<td>40.3</td>
</tr>
<tr>
<td>160</td>
<td>16.8</td>
<td>250</td>
<td>20.7</td>
</tr>
</tbody>
</table>

Table 5-12: Transmission path resonance noise reduction: two actuators - criterion J₉Z

Fig. 5-27: SPL at driver’s head – 2 actuators J₉Z

Fig. 5-28: J₉Z reduction potential – two actuators
The noise reduction (Fig. 5-27) is very interesting: actuators produce a global noise reduction of 21.1 dB(A) at the driver’s head. Resonance noise reductions are shown on Table 5-12 presents a better performance than the $J_{FX}$ criterion of the previous section. This configuration is, up to now, the best possible actuator configuration given by the algorithm.

5.3.2.3) **Minimisation of the SPL at the driver’s head**

For this case, the optimization tool reduces the SPL at the driver’s head with the help of two actuators. Fig. 5-29 and Table 5-13 display the optimized actuator configuration.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1st Actuator</th>
<th>2nd Actuator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linkage point</td>
<td>Lower Arm CM</td>
<td>Bushing #1</td>
</tr>
<tr>
<td>Bar angle ($\theta^\circ$)</td>
<td>87°</td>
<td>351°</td>
</tr>
<tr>
<td>Bar angle ($\phi^\circ$)</td>
<td>20°</td>
<td>47°</td>
</tr>
<tr>
<td>Bar length ($L_g$)</td>
<td>0.1709m</td>
<td>0.0894m</td>
</tr>
<tr>
<td>Force angle ($\theta_F^\circ$)</td>
<td>8°</td>
<td>89°</td>
</tr>
<tr>
<td>Force angle ($\phi_F^\circ$)</td>
<td>192°</td>
<td>147°</td>
</tr>
</tbody>
</table>

Table 5-13: Optimization two actuators genes for $J_{Pdriver}$

Both actuators are placed on the lower A-arm. Actuator 1 is oriented on the X-axis while actuator 2 is oriented on the Y-axis. The actuators are then using of lower A-arm rotation DOF with displacement suspension DOF to control suspension vibrations.

Fig. 5-30: SPL at driver’s head – 2 act. & $J_{Pdriver}$

Fig. 5-31: Ctrl. command – two actuators & $J_{Pdriver}$
The magnitude of the silent area (Fig. 5-30 and 5-31) displays no important differences compared to the single actuator configuration. In addition to this, total force required for the reduction of the criterion has increased: because of the orientation of the actuators, they can possibly work against each other at certain frequencies. A second actuator is consequently not interesting when if efficiency is a priority.

5.3.3) Link actuator (two opposite forces)

A combination of two dependent forces will be tested. This configuration corresponds to a control actuator that reacts on the suspension at two distinct locations, such as a power cylinder. Details of the corresponding chromosome are in Table 5-14.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Possibilities</th>
<th># of Genes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Act. #1 linkage point</td>
<td>12 points in total</td>
<td>4</td>
</tr>
<tr>
<td>Act. #1 bar angle (θ₁₁)</td>
<td>360° / 1° = 360</td>
<td>9</td>
</tr>
<tr>
<td>Act. #1 bar angle (ϕ₁₁)</td>
<td>360° / 1° = 360</td>
<td>9</td>
</tr>
<tr>
<td>Act. #1 bar length (L₁₁)</td>
<td>0.25 m / 0.488 mm = 512</td>
<td>9</td>
</tr>
<tr>
<td>Act. #2 linkage point</td>
<td>12 points in total</td>
<td>4</td>
</tr>
<tr>
<td>Act. #2 bar angle (θ₁₂)</td>
<td>360° / 1° = 360</td>
<td>9</td>
</tr>
<tr>
<td>Act. #2 bar angle (ϕ₁₂)</td>
<td>360° / 1° = 360</td>
<td>9</td>
</tr>
<tr>
<td>Act. #2 bar length (L₁₂)</td>
<td>0.25 m / 0.488 mm = 512</td>
<td>9</td>
</tr>
<tr>
<td># of genes (chromosome)</td>
<td>Total</td>
<td>62</td>
</tr>
</tbody>
</table>

Table 5-14: Gene’s description for a link actuator

The main difference in the chromosome for this optimization compared to the two previous configurations is that the orientation of the forces is dictated by the position of the suspension links.
5.3.3.1) Minimisation of X-axis transmitted forces

For this study, a link actuator will be simulated on the suspension model to reduce suspension X-axis force transmissibility. Fig. 5-32 and Table 5-15 give the results.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1st attachment</th>
<th>2nd attachment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linkage point</td>
<td>Damper bottom</td>
<td>Bushing #2</td>
</tr>
<tr>
<td>Bar angle ($\theta_b$)</td>
<td>336°</td>
<td>140°</td>
</tr>
<tr>
<td>Bar angle ($\phi_b$)</td>
<td>217°</td>
<td>95°</td>
</tr>
<tr>
<td>Bar length ($L_b$)</td>
<td>0.25m</td>
<td>0.0587m</td>
</tr>
<tr>
<td>Force angle ($\theta_F$)</td>
<td>73.9°</td>
<td>73.9°</td>
</tr>
<tr>
<td>Force angle ($\phi_F$)</td>
<td>-17.27°</td>
<td>162.72°</td>
</tr>
</tbody>
</table>

Table 5-15: Optimization link actuator genes for $J_{FX}$

One link is placed on the bottom of the damper and the other is attached to the left bushing. The algorithm is consequently using the DOFs between the strut tower module and the lower A-arm module to correct the vibroacoustic behaviour of the suspension. The fact that the actuator forces are always in opposite directions with each other removes the possibility of controlling the relative phase of the two actuators, as in the previous sections.

![Fig. 5-32: Actuator pos. for $J_{FX}$](image)

![Fig. 5-33: SPL at driver's head with $J_{FX}$](image)

![Fig. 5-34: $J_{FX}$ reduction potential: link actuator](image)

Results from Fig. 5-33 & 5-34 are not encouraging: first, the optimal criterion is not lower than as single actuator with the same force. Second, the noise level inside the cabin has increased by 1.1 $dB(A)$ at the driver’s head. Resonances are reduced at low frequencies but are amplified above 175 Hz.
<table>
<thead>
<tr>
<th>Resonance (Hz)</th>
<th>Reduction (dB(A))</th>
<th>Resonance (Hz)</th>
<th>Reduction (dB(A))</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>6.98</td>
<td>190</td>
<td>-0.5</td>
</tr>
<tr>
<td>110</td>
<td>4.8</td>
<td>230</td>
<td>-6.3</td>
</tr>
<tr>
<td>140</td>
<td>0.1</td>
<td>240</td>
<td>-1.1</td>
</tr>
<tr>
<td>160</td>
<td>6.7</td>
<td>250</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 5-16: Transmission path resonance noise reduction: link actuator - criterion $J_{FZ}$

This suggests that this actuator configuration should be disregarded for this criterion.

### 5.3.3.2 Minimisation of Z-axis transmitted forces

For this study, a link actuator will be simulated on the suspension model to reduce suspension Z-axis force transmissibility. Fig. 5-35 and Table 5-17 give the results.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1st Link</th>
<th>2nd Link</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linkage point</td>
<td>Damper bottom</td>
<td>Bushing #1</td>
</tr>
<tr>
<td>Bar angle ($\theta_B$)</td>
<td>319°</td>
<td>310°</td>
</tr>
<tr>
<td>Bar angle ($\phi_B$)</td>
<td>215°</td>
<td>168°</td>
</tr>
<tr>
<td>Bar length ($L_B$)</td>
<td>0.2495m</td>
<td>0m</td>
</tr>
<tr>
<td>Force angle ($\theta_F$)</td>
<td>-96.82°</td>
<td>-96.82°</td>
</tr>
<tr>
<td>Force angle ($\phi_F$)</td>
<td>12.56°</td>
<td>192.56°</td>
</tr>
</tbody>
</table>

Table 5-17: Optimization link actuator genes for $J_{FZ}$

The actuator is linked by the algorithm to the left bushing and to the bottom end of the damper cylinder. The fact that the strut tower must rotate to allow car handling will eliminate the experimental implementation of this configuration.

Fig. 5-36: SPL at driver's head with $J_{FZ}$ criterion

Fig. 5-37: Actuator command with $J_{FZ}$ criterion

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The $J_{iz}$ criterion is reduced to its optimal value over the entire frequency band (Fig. 5-37). This suggests a smaller actuator could provide a similar result. On the other hand, noise reduction is negligible and even negative at most resonances (Fig. 5-36). This can be explained by the fact that adding a power cylinder between the lower A-arm and the strut tower creates another noise transmission path which did not exist on the conventional suspension.

### 5.3.3.3) Minimisation of the SPL at the driver’s head

For this case, the optimization tool reduces the SPL at the driver’s head with the help of a double linked actuator. Fig. 5-38 and Table 5-18 display the optimized actuator configuration.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1st Link</th>
<th>2nd Link</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linkage point</td>
<td>7 (Damper)</td>
<td>12 (Left bush.)</td>
</tr>
<tr>
<td>Bar angle ($\theta_b$)</td>
<td>33°</td>
<td>159°</td>
</tr>
<tr>
<td>Bar angle ($\phi_b$)</td>
<td>175°</td>
<td>128°</td>
</tr>
<tr>
<td>Bar length ($L_b$)</td>
<td>0.2183m</td>
<td>0.1011m</td>
</tr>
<tr>
<td>Force angle ($\theta_p$)</td>
<td>253.4°</td>
<td>253.4°</td>
</tr>
<tr>
<td>Force angle ($\phi_p$)</td>
<td>-30°</td>
<td>149°</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fig. 5-38: Actuator pos. for $J_{pdriver}$</th>
</tr>
</thead>
</table>

Table 5-18: Optimization link actuator genes for $J_{pdriver}$

The actuator links the lower end of the damper with the left bushing (nearest to the driver). This should generate a rotation around their X-axis for both components.

<table>
<thead>
<tr>
<th>Fig. 5-39: SPL at driver’s head – Cylinder $J_{pdriver}$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Fig. 5-40: Control command – link actuator - $J_{pdriver}$</th>
</tr>
</thead>
</table>
The cost function (Fig. 5-39) displays no important differences compared to the criterion reduction with a single actuator. Once again, a point of zero sound pressure is ensured with a minimum of force (Fig. 5-40).

5.4) Chapter summary

On the optimization algorithm:

- The combination of a genetic algorithm and optimal control has permitted to create an optimization tool capable of finding the optimal actuator position, orientation and control input for an active suspension when given a space constraint (conventional suspension in our case).

- This optimization tool has been validated and it was shown that the response of the algorithm encompasses the capabilities of the actuator and the vibroacoustic behaviour of the suspension to optimize one or several actuator configurations.

The genetic algorithm results suggest that:

- For a single inertial actuator, the reduction of the suspension X-axis forces offered the largest noise reduction and the best performance in terms of criterion minimisation;

- With two actuators, the reduction of the suspension Z-axis forces offered the most noise reduction potential and the most criterion reduction efficiency.

- Link actuator used on the suspension does not offer an interesting noise reduction potential in comparison to one or two inertial actuators.

The minimization of the pressure at the driver’s head was, for each configuration, the case where the best noise reduction was obtained. However, the lack of damping inside the car body model and the unavailability of a global sound reduction criterion resulted in a point of zero sound pressure, which is not appropriate for this study. This problem could also cause an underestimation of the required force to reduce the SPL criterion to the level suggested by the model and lead to an erroneous actuator positioning.
6) CONCLUSIONS AND RECOMMENDATIONS

A literature review on modern car noise problems identified important sources of vibrations and interior noise between 250 and 500 Hz: engines, road/suspension interaction and Heat, Ventilation & Air Conditioning (HVAC) systems. When cars are moving at approximately 50 km/h (which is the standard speed in most North American cities), the road/suspension interaction was identified as the most important source of interior noise in modern cars.

To reduce road-induced vibration and noises, researchers have successfully developed various passive, semi-active & active techniques to alleviate noise levels in car cabins. The literature review has shown that the Active Structural Acoustic Control (ASAC) of a car suspension was the technique offering the most noise reduction potential if certain obstacles (suspension modeling, controller causality/stability and actuator/sensor positioning optimization) could be overcome.

With the aim of solving certain obstacles related to the experimental burden of experimental implementation of suspension ASAC, the overall objective of this thesis was to **elaborate an analytic active suspension optimization tool that reduces the time needed to locate actuator(s) and optimize their efficiency.** To do so the thesis was divided into three sections:

- Elaborate a suspension semi-analytic model capable of simulating the vibroacoustic behaviour of a quarter-car suspension below 250 Hz;

- Compare the tools elaborated by the **AUTO21** project with experimental measurements on a complete car. Also, study, characterize & adapt test bench excitation to represent a realistic road excitation;

- Elaborate & validate an algorithm capable of optimizing the position, orientation and control command of one or many actuators of an active suspension to increase ASAC efficiency.

The gathered information could then be used to test various actuator types and controllers to study their effects on modern cars interior noise.
6.1) Elaboration of an analytic quarter-car suspension model

The creation of a quarter-car suspension model on which analytic actuators have been implemented has been conducted in four principal steps:

- The analysis of a conventional McPherson strut front suspension & the improvement of the transmission path tool (quarter-car suspension test bench & car body FEM/BEM) previously elaborated by Douville;
- The creation of a 12 degree-of-freedom discrete element model the quarter-car suspension components;
- The development of equivalent rigidity models of flexible & instrumented suspension components (strut tower and lower A-arm bushings);
- The development of the equivalent rigidity model of the last flexible (which was not instrumented) suspension component: the tire.

The results of the suspension test bench modeling have revealed that genetic algorithms could deduce adequately the tire equivalent rigidity model. Also, the suspension modeling technique is capable of reproducing the vibroacoustic of a specific active suspension configuration.

When asked to reproduce the vibroacoustic behaviour of a suspension excited by an actuator placed on the lower A-arm of the test bench suspension, the general trends are reproduced, but some differences are noticeable between experimental and analytic FRFs. Some of the variations can be explained by the fact that the disturbance shaker link was modeled by equivalent rigidity constant in the frequency domain or measuring environment resonances (test bench resonances originating from other components that the suspension itself). However, some errors could also be coming from the rigidity hypothesis: the Vibratory Operational Shape Analysis (OVSA) realised by Douville has identified most of the suspension resonance origins. Still, it is not impossible that important resonances were left behind in the modeling procedure. A modal analysis of certain test bench component (wheel hub & lower A-arm) could potentially improve the understanding of the suspension modal behaviour and help to correct/adapt the discrete element model.
6.2) Experimental study of road excitation

The study of road excitation on a complete car has allowed investigating certain aspects of experimental ASAC:

- Study the main characteristics of a road excitation;
- Measure experimental road/cabin SPL FRFs and compare them to transmission path tool or the suspension model predictions;
- Evaluate the feasibility of experimental ASAC over a conventional suspension.

The experimental results have shown that the excitation mechanism of the test bench does not reproduce correctly a conventional road excitation. On the other hand, with the help of analytic road profiles, it was possible to create a road filter and adapt the test bench disturbance frequency content to a more realistic level.

With this new excitation, the Sound Pressure Level inside a real car was compared to the one predicted by the transmission path tool. As explained in Chapter 3, the FEM/BEM model predicts most of the structural/air cavity resonances found in the Epica LS. However, the model’s damping is insufficient and creates important resonance/anti-resonance peaks in the model’s predicted SPL.

As for the experimental implementation of an adaptive feedforward controller on the suspension, the coherence study has revealed an important challenge: without the multiple coherence function, no modes or resonance showed enough coherence with error microphones inside the car cabin to be controlled. Even with this multiple error function only one 10 Hz frequency band showed possible controllability. This points out one potential challenge for ASAC control in future experimental studies.

6.3) Active suspension optimization tool

Through the following steps, a suspension ASAC optimization tool was elaborated:

- A genetic algorithm environment was created with the suspension model created in Chapter 3. The tire, coil spring, lower arm and car panels’ were virtually implemented around the suspension model.

- Different optimal control criteria were created to optimize different actuator configuration for optimal control.
The results have been found for the test bench, which does not necessarily suggest actuator positions that are adequate for a conventional suspension (the shaker linking mechanism induces additional moments on the test bench suspension and changes its vibroacoustic behaviour).

The results of the optimization tool against a disturbance force of 1 N/Hz presented the following conclusions:

- For a single actuator with no force restriction, an actuator placed right over the tire in the X-Z plane gives the best results;

- When a single actuator is limited to 0.5 N/Hz, the minimization of the X-axis suspension force transmissibility offers the best performance;

- When two actuators are limited to 0.5 N/Hz, the minimization of the Z-axis suspension force transmissibility offers the best performance. However, the minimization of the X-axis suspension force transmissibility offers the best efficiency;

Link actuator does not present good noise reduction potential. The minimization of the SPL at the driver’s head has shown the impact of the lack of damping inside the car body FEM model: an underestimated actuator command creates an overestimated noise reduction. This could result in impaired actuator configurations because it has been proven that the actuator positions are optimized depending on the necessary actuator command magnitude.

### 6.4) Recommendations

As a result of the experience gained in this project, here are some suggestions for future studies of suspension ASAC within the F204 project.

6.4.1) Test bench, suspension model & FEM/BEM improvements

The suspension model will be as good as the experimental measurement protocol or the test bench capability of reproducing realistic road conditions. One interesting test bench modification could be to adapt the road excitation mechanism so it is placed directly under the test bench tire. Analytic road profiles could then be directly injected into the test bench without any excitation filtering.
FEM/BEM model should be corrected to add beams on doors, roof and floors to correct resonance frequency domain positions and resonance amplitudes. Damping should also be corrected. Multiple Frequency Response Functions (FRFs) should be measured at each direct suspension linkage (four for each bushing linkage, three for each strut tower) to complete the transmission path tool. Second, all (or a sparse 3D matrix) cabin air cavity nodes FRFs should be measured to create a global noise reduction criterion capable of measuring the mean noise reduction into the car cabin.

For the suspension model, a modal analysis should be performed on the test bench. It could help to identify some undesirable test bench resonance and the suspension model would be improved. Also, if two force actuators could be bought to increase the number of force transducers to four on the lower A-arm linkages. This would increase the stability of the bushing dynamic stiffness models since no sensor switch would be necessary. If experimental modeling of a real suspension could be done on a real car, it would be possible to test actuators in series: stack piezoelectric components between the damper and the car frame would probably have a good noise reduction potential. This improved model could also test the Bose\textsuperscript{\textregistered} suspension configuration (fully integrated actuators).

6.4.2) Experimental realisation of multiple error F-XLMS suspension control

The transmissibility FRFs show very good coherence level. Therefore, it could be interesting to evaluate such a configuration on a conventional suspension: if the system is causal, one sensor could be placed at the wheel spindle, error sensors could be placed at each direct suspension frame linkage and one or more actuator could be placed on the transmission paths between the reference sensor(s) an error sensor(s). This system could be placed on the suspension test bench and optimized. Inertial actuators placed on top of the wheel or piezoelectric devices placed between the strut tower linkages have a good noise reduction potential between 100 and 500 Hz. The Kuo & Morgan multiple errors filtered LMS algorithm could be tested [26].

6.4.3) Optimization of actuator components with genetic algorithms

It would also be of interest to create electromagnetic models of actuators that can be modified by a genetic algorithm (Coil size, actuator resonances, etc.). This way the optimization tool could serve to optimize actuator, position, orientation, electromechanical properties and actuator command all at once.
APPENDIX

A) SUSPENSION DISCRETE ELEMENTS AND FLEXIBLE COMPONENTS FRF MODEL

This appendix contains the details of each suspension modules and the methodology used to create equivalent rigidity models of flexible components held inside some modules. If the modules were free to move in space, a total of 24 DOF would be necessary to evaluate all motions of the 4 modules (3 translations & 3 rotations per module). Fortunately, links between modules reduces the number of DOF needed:

- The wheel module will be considered free to move in any translation or rotation. This result in a total 6 DOF for this module;
- One end of the driving shaft is linked to the spindle by a universal joint. Rotations of this module are then independent of the wheel module, except for the wheel-rotating axis (7th & 8th DOF);
- Two bearings are linking the wheel to the strut tower module. Consequently, only the wheel-rotating axis is independent from the wheel module (9th DOF);
- The lower A-arm, being linked to the wheel hub by a ball joint, is free to rotate on any axis (10th, 11th & 12th DOF).

Fig. A1 shows the resulting 12 DOF suspension discrete models. The combination of this model with FRF model of the tyre, bushings and strut tower will make a complete vibroacoustic model of a quarter-car suspension.

![Suspension Discrete Elements](image)

Fig. A-1: Complete quarter-car suspension wire model
A.1) Wheel Module

The Wheel module (Fig. A-2) contains: the tire – grey, the wheel rim – purple, the break disk – red, the spindle – cyan. All these components will be considered as a lumped mass. The mass of the tyre is included in the mass of the module. The excitation force being applied on the wheel spindle by the shaker link, we must include these components to the module as well (green, yellow, pink and gold). The tyre, being flexible, will be simulated by six dynamic stiffnesses. The tyre’s equivalent rigidities are found with the help of a genetic algorithm explained in details at Chapter 3.

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>$I_{xx}$ (kg m²)</th>
<th>$I_{yy}$ (kg m²)</th>
<th>$I_{zz}$ (kg m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>26.23</td>
<td>0.854783</td>
<td>0.525093</td>
<td>0.525093</td>
</tr>
</tbody>
</table>

Table A-1: Wheel Module inertia properties

![Fig. A-2: Wheel Module & its wire diagram](image)

Blue lines represent the discrete elements of the module, red dots are the nodes at the extremities of the discrete elements and the black & white dot is the Centre of Mass (CM) of the module. The wheel module has three discrete elements: one going from the disturbance shaker link to the centre of mass of the wheel. From the centre of the wheel, one element is linking the tire equivalent rigidity model to the wheel and the other is linking the wheel to the driving shaft.
The disturbance shaker link as 6 DOF: \( x_{Sk} \), \( y_{Sk} \), \( z_{Sk} \), \( \phi_{xSk} \), \( \phi_{ySk} \) & \( \phi_{zSk} \). The resulting discrete displacement matrix is presented at Eq. A.1:

\[
[I] \begin{bmatrix}
x_{Sk} \\
y_{Sk} \\
z_{Sk} \\
\phi_{xSk} \\
\phi_{ySk} \\
\phi_{zSk}
\end{bmatrix} = \begin{bmatrix}
x_{Sk} \\
y_{Sk} \\
z_{Sk} \\
\phi_{xSk} \\
\phi_{ySk} \\
\phi_{zSk}
\end{bmatrix}
\]  
(A.1)

An important hypothesis made inside geometric matrices is the linearization of rotations. Vibrations are usually small displacements and angles can then be correctly approximated by the simplified equations presented above.

The CM of the wheel is clamped to the shaker node by a discrete element. The angular displacements are therefore identical at each end. The displacements however are different because of the distance between the two nodes (Eq. A.2 and A.3).

\[
\begin{bmatrix}
1 & 0 & 0 & R_{ZWhl} & -R_{YWhl} \\
0 & 1 & 0 & -R_{ZWhl} & 0 \\
0 & 0 & 1 & R_{YWhl} & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_{Sk} \\
y_{Sk} \\
z_{Sk} \\
\phi_{xSk} \\
\phi_{ySk} \\
\phi_{zSk}
\end{bmatrix} = \begin{bmatrix}
x_{Whl} \\
y_{Whl} \\
z_{Whl} \\
\phi_{xWhl} \\
\phi_{yWhl} \\
\phi_{zWhl}
\end{bmatrix}
\]  
(A.2)

To measure the levers between tow nodes, one must know the distance between the 2 nodes \( R_{Whl} \) and the angles of this lever \( \theta_{Whl} \) and \( \phi_{Whl} \) (Eq. A.3)

\[
\begin{bmatrix}
\cos(\theta_{Whl}) & \cos(\phi_{Whl}) \\
\sin(\theta_{Whl}) & \cos(\phi_{Whl}) \\
\sin(\phi_{Whl}) & \end{bmatrix}
\begin{bmatrix}
x_{Whl} \\
y_{Whl} \\
z_{Whl} \\
\phi_{xWhl} \\
\phi_{yWhl} \\
\phi_{zWhl}
\end{bmatrix} = \begin{bmatrix}
R_{XWhl} \\
R_{YWhl} \\
R_{ZWhl}
\end{bmatrix}
\]  
(A.3)

Where: \( R_{Whl} = 0.14476m \), \( \theta_{Whl} = 180^\circ \) & \( \phi_{Whl} = 0^\circ \). Eq. A.3 will give the orthogonal coordinates of each discrete element in this section.

Like the passing from the shaker node to the wheel CM node, the passing from the wheel node to the tyre node is executed simply by adding the length of the discrete link the two nodes in the transfer matrix (Eq. A.4):
\[
\begin{bmatrix}
1 & 0 & 0 & (R_{ZWhl}+R_{ZTire}) & -(R_{YWhl}+R_{YTire}) \\
0 & 1 & 0 & (R_{ZWhl}+R_{ZTire}) & 0 \\
0 & 0 & 1 & (R_{YWhl}+R_{YTire}) & -(R_{XWhl}+R_{XTire}) \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_{Sr} \\
y_{Sr} \\
z_{Sr} \\
\phi_{XSr} \\
\phi_{ZSr}
\end{bmatrix} =
\begin{bmatrix}
x_{Tire} \\
y_{Tire} \\
z_{Tire} \\
\phi_{XTire} \\
\phi_{ZTire}
\end{bmatrix}
\]

(A.4)

Where: \( R_{Tire} = 0.1753m \), \( \theta_{Rtire} = 180^\circ \) & \( \phi_{Rtire} = 78^\circ \).

The final wheel module discrete element goes to the driving shaft (Eq. A.5):

\[
\begin{bmatrix}
1 & 0 & 0 & (R_{ZWhl}+R_{ZCO}) & -(R_{YWhl}+R_{YCO}) \\
0 & 1 & 0 & (R_{ZWhl}+R_{ZCO}) & 0 \\
0 & 0 & 1 & (R_{YWhl}+R_{YCO}) & -(R_{XWhl}+R_{XCO}) \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_{Sr} \\
y_{Sr} \\
z_{Sr} \\
\phi_{XSr} \\
\phi_{ZSr}
\end{bmatrix} =
\begin{bmatrix}
x_{CO} \\
y_{CO} \\
z_{CO} \\
\phi_{XCO} \\
\phi_{ZCO}
\end{bmatrix}
\]

(A.5)

Where: \( R_{CO} = 0.01662m \), \( \theta_{RCO} = 180^\circ \) & \( \phi_{RCO} = 0^\circ \).

### A.2) Driving shaft module

The driving shaft module links the wheel module to the triangular structure of the test bench by two universal joints (Fig. A-3). The driving shaft will be modeled as a light discrete element with six springs fixed at its transmission link.

The decision of using complex springs fixed in frequency is based on the suspension Operational Vibratory Shape Analysis (OVSA): only one mode was detected at 75 Hz for this component between up to 250 Hz. The fixed rigidities have been adjusted manually to obtain the correct resonance position and damping.

<table>
<thead>
<tr>
<th>( X )</th>
<th>( 70000+30000j )</th>
<th>( tX )</th>
<th>( 7000+300j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>( 70000+30000j )</td>
<td>( tY )</td>
<td>( 1+0j )</td>
</tr>
<tr>
<td>( Z )</td>
<td>( 70000+30000j )</td>
<td>( tZ )</td>
<td>( 1+0j )</td>
</tr>
</tbody>
</table>

Table A-2: Fixed rigidities of the driving shaft link
The driving shaft universal joints bring two new DOF: $\phi_{xsfr}$ & $\phi_{zsfr}$. Eq. A.6 foretells the equation of motion of this node:

$$
\begin{bmatrix}
1 & 0 & 0 & (R_{zwhl}+R_{xcl}) & -R_{ywhl} & -R_{xcld} & R_{zcl} & -R_{ycl}
0 & 1 & 0 & -R_{zwhl} & R_{xcl} & 0 & R_{ycl}
0 & 0 & 1 & (R_{ywhl}+R_{xcld}) & -R_{zwhl} & R_{xcl} & 0 & -R_{xcl}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x_{sfr}
y_{sfr}
z_{sfr}
\phi_{xsfr}
\phi_{ysfr}
\phi_{zsfr}
\phi_{xcl}
\phi_{ycl}
\phi_{zcl}
\end{bmatrix} = \begin{bmatrix}
x_{ycl}
y_{ycl}
z_{ycl}
\phi_{ycl}
\phi_{zcl}
\end{bmatrix} \quad \text{(A.6)}
$$

Where: $R_{ycl} = 0.29445m$, $\theta_{rcl} = 0^\circ$ & $\phi_{rcl} = 185^\circ$.

### A.3) Strut Tower module

The Strut Tower module (Fig. A-4) is made of the wheel hub – green, the break calliper – brown, the coil spring – yellow, the fixed damper part – red, the moving damper part – pink, the spring compression plate – cyan, a metallic cone designed to fix the strut tower into the fifth instrumented link – gold.
The moving parts of the strut tower (the coil spring, the damper rod, the top compression plate and the cone adaptor) will be replaced by equivalent springs.

A.3.1) Strut tower FRF model - Force measurements

The strut tower linkage is capable of measuring moments around the X and Y-axis. (Fig. A-5) Moments on the Z-axis cannot be measured, but the strut tower bearing prevents any efforts to be transferred in this DOF.
Eq. A.7 is presenting the transfer matrix between each strut tower force sensor to obtain the global linkage forces and moments.

\[
\begin{bmatrix}
\cos(\theta_x)\cos(\phi_x) & \cos(\theta_x)\cos(\phi_y) & \cos(\theta_x)\cos(\phi_z) \\
\sin(\theta_x)\cos(\phi_x) & \sin(\theta_x)\cos(\phi_y) & 0 \\
\sin(\phi_x) & \sin(\phi_y) & \sin(\phi_z) \\
R_{11}(\sin(\theta_x)) & R_{21}(\sin(\theta_x)) & 0 \\
R_{12}(\sin(\theta_x)) & R_{22}(\sin(\theta_x)) & R_{32}(\sin(\theta_z))
\end{bmatrix}
\begin{bmatrix}
F_x \\
F_y \\
F_z \\
M_{Rx} \\
M_{Ry} \\
M_{Rz}
\end{bmatrix}
\]

(A.7)

Where \( \theta_x=120^\circ, \theta_y=240^\circ, \theta_z=0^\circ \) and \( \phi_x=\phi_y=\phi_z=45^\circ \). The orthogonal distances between each force sensor are: \( R_{Ax}=R_{Ay}=R\cos(\theta_x), R_{Cx}=R, R_{Ay}=R_{By}=R\sin(\theta_x) \). \( R=0.083m \) is the distance between strut tower force transducers and the centre of the strut tower linkage.

A.3.2) Strut Tower FRF model - Acceleration measurements

Accelerometers were placed following Fig. A-6 & A-7 configuration to measure the acceleration of each strut tower DOF. The captors were not oriented in the same direction as the global axis system that we have established in Chapter 3, direction changes have to be made (Eq. A.8 to A.10).

\[ A_{x1}=-A_{x1} \]  
\[ A_{y1}=A_{y1} \]  
\[ A_{z1}=-A_{z1} \]  

(A.8)  
(A.9)  
(A.10)

Accelerations measurements can be used to identify resultant acceleration at the end of the damper cylinder (Eq. A.11 to A.15):
\[ A_x = \frac{D_2}{D_1} (A_{r2} - A_{r1}) + A_{r2} \]  
(A.11)

Where \( D_1 = 0.1757m \) and \( D_2 = 0.1121m \)

\[ A_y = \frac{D_2}{D_1} (A_{x2} - A_{x1}) + A_{x2} \]  
(A.12)

Fig. A-6: Strut Tower acceleration measurement configuration - Front view

Fig. A-7: Strut Tower acceleration measurement configuration – Side view

\[ A_z = A_{z2} = A_{z2} \]  
(A.13)

\[ A_{rx} = \frac{A_{r2} - A_{r1}}{D_1} \]  
(A.14)

\[ A_{ry} = \frac{A_{r2} - A_{r1}}{D_1} \]  
(A.15)

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A.3.3) Equivalent rigidity FRF model formulation

All signals are acquisitioned in the time domain. But, with a Discrete Fourier Transform (DFT), one can identify the strut tower’s force/acceleration FRFs (Eq. A.16 to A.18):

\[
a(t)\leftrightarrow DFT \Rightarrow A(\omega) \quad \text{(A.16)}
\]
\[
f(t)\leftrightarrow DFT \Rightarrow F(\omega) \quad \text{(A.17)}
\]
\[
\frac{F(\omega)}{A(\omega)} = H(\omega) \left( \frac{N}{m \cdot s^2} \right) \quad \text{(A.18)}
\]

Spring rigidities must be expressed in \(N/m\). We must integrate two times the acceleration results to have force/displacement FRFs (Eq. A.19 to A.23):

\[
x(t) = Xe^{-j\alpha} \quad \text{(A.19)}
\]
\[
ix(t) = v(t) = -j\omega Xe^{-j\alpha} \quad \text{(A.20)}
\]
\[
ix(t) = a(t) = -\omega^2 Xe^{-j\alpha} \quad \text{(A.21)}
\]
\[
a(t) = -\omega^2 x(t) \quad \text{(A.22)}
\]

The last demonstration is only valid when the frequency \(\omega\) is given.

\[
-\omega^2 \frac{F(\omega)}{A(\omega)} = \frac{F(\omega)}{X(\omega)} = H(\omega) \left( \frac{N}{m} \right) \quad \text{(A.23)}
\]

We now have an equivalent rigidity model of the strut tower for each frequency. Since there are 5 rigidities to measure, a total of 5 FRFs are found.

A.3.4) Discrete element description

Since two conical fixes the wheel in the suspension hub bearing only one DOF changes from the wheel node to the centre of mass of the strut tower: \(\phi_{X_{\text{Hub}}}\).

\[
\begin{bmatrix}
1 & 0 & 0 & \left( R_{Z_{\text{Hub}}} + R_{Z_{\text{Hub}}} \right) & -\left( R_{Z_{\text{Hub}}} + R_{Y_{\text{Hub}}} \right) & 0 \\
0 & 1 & 0 & -R_{Z_{\text{Hub}}} & 0 & \left( R_{X_{\text{Hub}}} + R_{Y_{\text{Hub}}} \right) & -R_{Z_{\text{Hub}}} \\
0 & 0 & 1 & \left( R_{X_{\text{Hub}}} + R_{X_{\text{Hub}}} \right) & 0 & R_{Y_{\text{Hub}}} \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_{\text{skr}} \\
y_{\text{skr}} \\
z_{\text{skr}} \\
\phi_{X_{\text{skr}}} \\
\phi_{Y_{\text{skr}}} \\
\phi_{Z_{\text{skr}}} \\
\phi_{X_{\text{Hub}}} \\
\phi_{Y_{\text{Hub}}} \\
\phi_{Z_{\text{Hub}}}
\end{bmatrix} =
\begin{bmatrix}
x_{\text{Hub}} \\
y_{\text{Hub}} \\
z_{\text{Hub}} \\
\phi_{X_{\text{Hub}}} \\
\phi_{Y_{\text{Hub}}} \\
\phi_{Z_{\text{Hub}}}
\end{bmatrix} \quad \text{(A.24)}
\]

Where: \(R_{\text{Hub}} = 0.29963m\), \(\theta_{\text{RHub}} = 195^\circ\) & \(\phi_{\text{RHub}} = 292.9^\circ\).

From the strut tower CM, one lever is clamped to the ball joint at the lower A-arm and one other is clamped to the lower and of the damper cylinder. For the first case:
\[
\begin{bmatrix}
1 & 0 & 0 & (R_{ZWhl}+R_{ZHub}+R_{ZBl}) & -(R_{YWhl}+R_{YHub}+R_{YBl}) & 0 \\
0 & 1 & 0 & -R_{ZWhl} & 0 & (R_{XWhl}+R_{XHub}+R_{XBl}) & -(R_{ZHub}+R_{ZBl}) \\
0 & 0 & 1 & R_{YWhl} & -(R_{XWhl}+R_{XHub}+R_{XBl}) & 0 & (R_{YHub}+R_{YBl}) \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x_{Skr} \\
y_{Skr} \\
z_{Skr} \\
\phi_{XSkr} \\
\phi_{YSkr} \\
\phi_{ZSkr} \\
\phi_{XHub} \\
\phi_{YHub} \\
\phi_{ZHub}
\end{bmatrix} = \begin{bmatrix}
x_{Bl} \\
y_{Bl} \\
z_{Bl} \\
\phi_{XBl} \\
\phi_{YBl} \\
\phi_{ZBl}
\end{bmatrix} \tag{A.25}
\]

Where: \( R_{Bl} = 0.344.321 m \), \( \theta_{RBl} = 15^\circ \) & \( \phi_{RBl} = 80.5^\circ \).

From the centre of mass of the strut tower to the lower end of the damper cylinder:

\[
R_{XSum} = R_{XWhl}+R_{XHub}+R_{XBl} \tag{A.26}
\]

\[
R_{YSum} = R_{YWhl}+R_{YHub}+R_{YBl} \tag{A.27}
\]

\[
R_{ZSum} = R_{ZWhl}+R_{ZHub}+R_{ZBl} \tag{A.28}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & (R_{ZSum}+R_{ZDl}) & -(R_{XSum}+R_{XDl}) & 0 \\
0 & 1 & 0 & -R_{ZWhl} & 0 & (R_{XSum}+R_{XDl}) & -(R_{ZHub}+R_{ZBl}+R_{ZDl}) \\
0 & 0 & 1 & R_{YWhl} & -(R_{XSum}+R_{XDl}) & 0 & (R_{YHub}+R_{YBl}+R_{YDl}) \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x_{Skr} \\
y_{Skr} \\
z_{Skr} \\
\phi_{XSkr} \\
\phi_{YSkr} \\
\phi_{ZSkr} \\
\phi_{XHub} \\
\phi_{YHub} \\
\phi_{ZHub}
\end{bmatrix} = \begin{bmatrix}
x_{Dl} \\
y_{Dl} \\
z_{Dl} \\
\phi_{XDl} \\
\phi_{YDl} \\
\phi_{ZDl}
\end{bmatrix} \tag{A.29}
\]

Where: \( R_{Dl} = 0.18825 m \), \( \theta_{RDl} = 150^\circ \) & \( \phi_{RDl} = 70.3^\circ \).

From the lower end of the damper cylinder to the lower coil base plate:

\[
R_{XSum} = R_{XWhl}+R_{XHub}+R_{XBl}+R_{XDl} \tag{A.30}
\]

\[
R_{YSum} = R_{YWhl}+R_{YHub}+R_{YBl}+R_{YDl} \tag{A.31}
\]

\[
R_{ZSum} = R_{ZWhl}+R_{ZHub}+R_{ZBl}+R_{ZDl} \tag{A.32}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & (R_{ZSum}+R_{ZDl}) & -(R_{XSum}+R_{XDl}) & 0 \\
0 & 1 & 0 & -R_{ZWhl} & 0 & (R_{XSum}+R_{XDl}) & -(R_{ZHub}+R_{ZBl}+R_{ZDl}+R_{ZDl}) \\
0 & 0 & 1 & R_{YWhl} & -(R_{XSum}+R_{XDl}) & 0 & (R_{YHub}+R_{YBl}+R_{YDl}+R_{YDl}) \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
x_{Skr} \\
y_{Skr} \\
z_{Skr} \\
\phi_{XSkr} \\
\phi_{YSkr} \\
\phi_{ZSkr} \\
\phi_{XHub} \\
\phi_{YHub} \\
\phi_{ZHub}
\end{bmatrix} = \begin{bmatrix}
x_{DL} \\
y_{DL} \\
z_{DL} \\
\phi_{XDL} \\
\phi_{YDL} \\
\phi_{ZDL}
\end{bmatrix} \tag{A.33}
\]

Where: \( R_{DL} = 0.30653 m \), \( \theta_{RDl} = 0^\circ \) & \( \phi_{RDl} = 270^\circ \).
A.4) Lower A-arm module

The Lower A–arm module (Fig. A-8) will contain: the lower A-arm & ball joint – green, the two rubber bushings – yellow. The tie rods (cyan) are not considered in the module because they are fixed to the frame.

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>$I_{xx}$ (kg m$^2$)</th>
<th>$I_{yy}$ (kg m$^2$)</th>
<th>$I_{zz}$ (kg m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>0.0199</td>
<td>0.0426</td>
<td>0.0617</td>
</tr>
</tbody>
</table>

Table A-4: Lower A-arm module inertial properties

The two bushings of the lower A-Arm module will be modeled by equivalent rigidity models because Douville’s OVSA has revealed rigid motion resonance of this component; this suggests that resonance are coming from global suspension resonance and bushing resonance.

Fig. A-8: Lower A-Arm module
A.4.1) Bushing FRF model - Force measurements

The combination of linkage 1A, 1B, 2A & 2B measures efforts and moment induced to the bushing nearest of the front bumper while linkage 3A, 3B, 4A & 4B are used to measured efforts on the bushing nearest to the driver (Fig. A-9 and A-10).

Eq. A.34 gives the transfer matrix between each lower A-arm force transducer and the resultant effort in the middle of the bushing centre rod.

\[
\begin{pmatrix}
1 & 0 & 0 & -D_z & +D_y \\
0 & 1 & 0 & +D_z & 0 & -D_x \\
0 & 0 & 1 & -D_y & +D_x & 0 \\
1 & 0 & 0 & -D_z & +D_y & 0 \\
0 & 0 & 0 & -D_z & -D_y & 0 \\
0 & 0 & 0 & -D_z & -D_y & 0 \\
0 & 0 & 0 & -D_z & +D_x & 0 \\
0 & 0 & 0 & -D_z & +D_y & 0 \\
0 & 0 & 1 & +D_y & -D_x & 0 \\
0 & 0 & 1 & +D_y & -D_x & 0
\end{pmatrix}
\begin{pmatrix}
F_{3AX} \\
F_{3AY} \\
F_{3AZ} \\
F_{4BX} \\
F_{4BY} \\
F_{4BZ} \\
F_{5AX} \\
F_{5AY} \\
F_{5AZ} \\
F_{6BX} \\
F_{6BY} \\
F_{6BZ}
\end{pmatrix}
= \begin{pmatrix}
F_{RX} \\
F_{RY} \\
F_{RZ} \\
F_{M_{RX}} \\
F_{M_{RY}} \\
F_{M_{RZ}}
\end{pmatrix}
(A.34)

Where \(D_x=1\"\), \(D_y=2.25\"\) & \(D_z=3\"\) are the Cartesian distances between each force transducer and the centre of their respective bushing. Unfortunately, only two force sensors are available. Therefore, force measurements had to done two by two. This supposes that fixed linkage points with no force captor have the same rigidity as an linkage point installed with force captors.
A.4.2) Acceleration measurement configuration

Accelerometers are placed on different emplacements on the lower A-arm to detect its rigid body motion. It will serve as the entrance signal for dynamic stiffness modeling of the bushings (Fig. A-11 and A-12).

First, direction corrections are made (Eq. A.35 to A.37):

\[ A_{x_1} = -A_{c_{x_1}} \quad (A.35) \]
\[ A_{y_1} = A_{c_{y_1}} \quad (A.36) \]
\[ A_{z_1} = A_{c_{z_1}} \quad (A.37) \]

Second, angular accelerations are retrieved from accelerations measured by accelerometers (Eq. A.38 to A.41):

\[ A_x = \frac{A_{y_2} - A_{z_1}}{2 R_1 \sin(\theta_{DIZ}) \cos(\theta_{DIX})} \quad (A.38) \]
\[ A_{AVG} = \frac{A_{y_2} - A_{z_1}}{2} \quad (A.39) \]
\[ A_y = \frac{A_{ZCM} - A_{AVG}}{R_1 \cos(\theta_{DIZ}) \cos(\theta_{DIY})} \quad (A.40) \]
\[ A_z = \frac{A_{y_2} - A_{x_1}}{2 R_1 \sin(\theta_{DIZ}) \cos(\theta_{DIX})} \quad (A.41) \]
A.4.3) Discrete model description

The lower A-arm module is attached to the strut tower module by the mean of a ball joint. This ball joint leaves all rotation of the lower A-arm independent of the strut tower module: $\phi_{XArm}$, $\phi_{YArm}$ and $\phi_{ZArm}$ are then the last three DOF of the suspension model. The centre of mass of the lower A-arm is clamped to the ball join node.

$$
R_{XAdd} = R_{XWhl} + R_{XHub} + R_{XB12} \tag{A.42}
$$
$$
R_{YAdd} = R_{YWhl} + R_{YHub} + R_{YB12} \tag{A.43}
$$
$$
R_{ZAdd} = R_{ZWhl} + R_{ZHub} + R_{ZB12} \tag{A.44}
$$

$$
\begin{bmatrix}
1 & 0 & 0 & R_{ZAdd} & -R_{YAdd} & 0 & 0 & R_{ZArm} & -R_{YArm} \\
0 & 1 & 0 & -R_{ZWhl} & 0 & R_{XAdd} & -(R_{ZHub} + R_{ZB12}) & -R_{ZArm} & 0 & R_{XArm} \\
0 & 0 & 1 & R_{YWhl} & -R_{XAdd} & 0 & (R_{YHub} + R_{YB12}) & R_{YArm} & -R_{XArm} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X_{Skr} \\
Y_{Skr} \\
Z_{Skr} \\
\phi_{XSkr} \\
\phi_{YSkr} \\
\phi_{ZSkr} \\
\phi_{XArm} \\
\phi_{YArm} \\
\phi_{ZArm}
\end{bmatrix}
= 
\begin{bmatrix}
X_{Arm} \\
Y_{Arm} \\
Z_{Arm} \\
\phi_{XArm} \\
\phi_{YArm} \\
\phi_{ZArm}
\end{bmatrix}
\tag{A.45}
$$

Where: $R_{Arm} = 0.1851m$, $\theta_{RArm} = 180^\circ$ & $\phi_{RArm} = 8^\circ$.

The two bushings are clamped to the CM of the lower A-arm, The dynamic equations of the 1/2 bushing are:

$$
R_{XBush} = R_{XArm} + R_{XB12} \tag{A.42}
$$
$$
R_{YBush} = R_{YArm} + R_{YB12} \tag{A.43}
$$
$$
R_{ZBush} = R_{ZArm} + R_{ZB12} \tag{A.44}
$$

$$
\begin{bmatrix}
1 & 0 & 0 & R_{ZAdd} & -R_{YAdd} & 0 & 0 & R_{ZBush} & -R_{YBush} \\
0 & 1 & 0 & -R_{ZWhl} & 0 & R_{XAdd} & -(R_{ZHub} + R_{ZB12}) & -R_{ZBush} & 0 & R_{XBush} \\
0 & 0 & 1 & R_{YWhl} & -R_{XAdd} & 0 & (R_{YHub} + R_{YB12}) & R_{YBush} & -R_{XBush} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X_{Skr} \\
Y_{Skr} \\
Z_{Skr} \\
\phi_{XSkr} \\
\phi_{YSkr} \\
\phi_{ZSkr} \\
\phi_{XArm} \\
\phi_{YArm} \\
\phi_{ZArm}
\end{bmatrix}
= 
\begin{bmatrix}
X_{B12} \\
Y_{B12} \\
Z_{B12} \\
\phi_{XB12} \\
\phi_{YB12} \\
\phi_{ZB12}
\end{bmatrix}
\tag{A.45}
$$

Where: $R_{B12} = 0.16467m$, $\theta_{RB12} = 223.5^\circ$ & $\phi_{RB12} = 0^\circ$.
For the bushing 3/4:

\[
\begin{align*}
R_{X\text{Bush}} &= R_{X\text{Arm}} + R_{X34} \\
R_{Y\text{Bush}} &= R_{Y\text{Arm}} + R_{Y34} \\
R_{Z\text{Bush}} &= R_{Z\text{Arm}} + R_{Z34}
\end{align*}
\] (A.46)

\[
\begin{bmatrix}
1 & 0 & 0 & R_{Z\text{Add}} & -R_{Y\text{Add}} & 0 & 0 & R_{Z\text{Bush}} & -R_{Y\text{Bush}} \\
0 & 1 & 0 & -R_{Z\text{Whl}} & 0 & R_{X\text{Add}} & -R_{Z\text{Hub}} & -R_{Z\text{Bi}} & R_{X\text{Whl}} & -R_{Z\text{Bi}} \\
0 & 0 & 1 & R_{Y\text{Whl}} & -R_{X\text{Add}} & 0 & (R_{Z\text{Hub}} + R_{Z\text{Bi}}) & R_{Y\text{Whl}} & -R_{X\text{Whl}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X_{\text{Skr}} \\
Y_{\text{Skr}} \\
Z_{\text{Skr}} \\
\phi_{\text{Skr}} \\
\psi_{\text{Skr}} \\
\chi_{\text{Bi12}} \\
\phi_{\text{Bi12}} \\
\theta_{\text{Bi12}} \\
\phi_{\text{XHub}} \\
\phi_{\text{ZHub}} \\
\phi_{\text{Arm}} \\
\phi_{\text{ZArm}}
\end{bmatrix}
\] (A.49)

Where: \( R_{B34} = 0.16467m \), \( \theta_{RB34} = 136.5^\circ \) & \( \phi_{RB34} = 0^\circ \).

**A.5) Suspension model dynamic equations derivation**

To derive the equations of motion for the discrete part of the suspension model, both the Newtonian and Lagrangian approach are applicable. However, in our case, we want to be able to add a control actuator in the model by the simplest possible way. Since the energy method permits the addition of two systems, the Lagrange approach was chosen.

All dynamic systems can be represented by a Hamiltonian function (Eq. A.50) formed by its potential, kinetic and work energies between arbitrary instants \( t_0 \) and \( t_f \):

\[
H_{\text{Sup}}(U_1...U_n) = \int_{t_0}^{t_f} \left( T_{\text{Sup}} + V_{\text{Sup}} + W_{\text{Sup}} \right) dt
\] (A.50)

In our case, each \( U_n \) corresponds to a system’s Degree Of Freedom (DOF). Kinetic energy is the sum of translations and rotations of lumped masses. In our case, we have three masses: the wheel module, the strut tower module and the lower A-arm module (Eq. A.51 to A.54).

\[
T_{\text{Whl}} = \frac{1}{2} \left( +m_{\text{Whl}} \left( \frac{\partial x_{\text{Whl}}}{\partial t} \right)^2 + \left( \frac{\partial y_{\text{Whl}}}{\partial t} \right)^2 + \left( \frac{\partial z_{\text{Whl}}}{\partial t} \right)^2 \right) \\
+ \left( I_{xx_{\text{Whl}}} \left( \frac{\partial \phi_{x_{\text{Whl}}}}{\partial t} \right)^2 + I_{yy_{\text{Whl}}} \left( \frac{\partial \phi_{y_{\text{Whl}}}}{\partial t} \right)^2 + I_{zz_{\text{Whl}}} \left( \frac{\partial \phi_{z_{\text{Whl}}}}{\partial t} \right)^2 \right)
\] (A.51)
\[ T_{Hub} = \frac{1}{2} \left( +m_{Hub}\left( \frac{\partial x_{Hub}}{\partial t} \right)^2 + \left( \frac{\partial y_{Hub}}{\partial t} \right)^2 + \left( \frac{\partial z_{Hub}}{\partial t} \right)^2 \right) \]

\[ T_{Arm} = \frac{1}{2} \left( +m_{Arm}\left( \frac{\partial x_{Arm}}{\partial t} \right)^2 + \left( \frac{\partial y_{Arm}}{\partial t} \right)^2 + \left( \frac{\partial z_{Arm}}{\partial t} \right)^2 \right) \]

\[ T_{Susp} = T_{Whl} + T_{Hub} + T_{Arm} \]

The potential energy is the sum of all deformations of the suspension model. These energies are coming from the different equivalent rigidity models elaborated from chapter 2 and the driving shaft constant rigidities (Eq. A.55 to A.60).

\[ V_{Tire} = -\frac{1}{2} \left( +\tilde{K}_{XTire} x_{Tire}^2 + \tilde{K}_{YTire} y_{Tire}^2 + \tilde{K}_{ZTire} z_{Tire}^2 \right) \]

\[ V_{B12} = -\frac{1}{2} \left( +\tilde{K}_{XB12} x_{B12}^2 + \tilde{K}_{YB12} y_{B12}^2 + \tilde{K}_{ZB12} z_{B12}^2 \right) \]

\[ V_{B34} = -\frac{1}{2} \left( +\tilde{K}_{XB34} x_{B34}^2 + \tilde{K}_{YB34} y_{B34}^2 + \tilde{K}_{ZB34} z_{B34}^2 \right) \]

\[ V_{DL} = -\frac{1}{2} \left( +\tilde{K}_{XDL} x_{DL}^2 + \tilde{K}_{YDL} y_{DL}^2 + \tilde{K}_{ZDL} z_{DL}^2 \right) \]

\[ V_{CL} = -\frac{1}{2} \left( +\tilde{K}_{XCL} x_{CL}^2 + \tilde{K}_{YCL} y_{CL}^2 + \tilde{K}_{ZCL} z_{CL}^2 \right) \]

\[ V_{Susp} = V_{Tire} + V_{B12} + V_{B34} + V_{DL} + V_{CL} \]

Work includes all exterior forces acting on the system (Eq. A.61). Actually, only the disturbance force is applied on the suspension model.

\[ W_{Susp} = F_{Sk} z_{Sk} \]

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The Lagrange equations are then used to derive the equations of motion. For a function $H$ having $n$ variables that depend only on time $t$, the Hamilton function takes the form of Eq. A.62.

$$H(U_1, \ldots, U_n) = \int_{t_0}^{t_1} F(U_1; U_1, \dot{U}_2; U_2, \dot{U}_3; \ldots; U_n, \dot{U}_n) \, dt$$
(A.62)

The minimisation of Eq. A.62 gives one equation for each DOF of the system (Eq. A.63).

$$\begin{bmatrix}
\frac{\partial F}{\partial U_1} \cdot \frac{d}{dt} \frac{\partial F}{\partial U_1} = 0 \\
\vdots \\
\frac{\partial F}{\partial U_n} \cdot \frac{d}{dt} \frac{\partial F}{\partial U_n} = 0
\end{bmatrix} \forall \ t \in [t_0, t_1]$$
(A.63)

For the Wheel module there are 6 DOF:

$$\sum E_{x_{skr}} = \frac{1}{2} \left( -\tilde{K}_{x_{tire}} x_{tire}^2 - \tilde{K}_{x_{bil}} x_{bil}^2 - \tilde{K}_{x_{bd}_1} x_{bd_1}^2 - \tilde{K}_{x_{bd}_2} x_{bd_2}^2 - \tilde{K}_{x_{dld}} x_{dld}^2 - \tilde{K}_{x_{dld}} x_{dld}^2 \\
+ m_{whl} \left( \frac{\partial x_{whl}}{\partial t} \right)^2 + m_{hub} \left( \frac{\partial x_{hub}}{\partial t} \right)^2 + m_{arm} \left( \frac{\partial x_{arm}}{\partial t} \right)^2 \right)$$
(A.64)

From Eq. A.2, the relation between $x_{tire}$ & $x_{skr}$ is:

$$x_{tire} = x_{skr} + \phi_{x_{skr}} (R_{zwhl} + R_{ztire}) + \phi_{z_{skr}} (R_{ywhl} + R_{ytire})$$
(A.65)

$$x_{skr} = x_{skr} + \phi_{x_{skr}} (R_{zwhl} + R_{ztire}) + x_{skr} \phi_{z_{skr}} (R_{zwhl} + R_{ztire})^2 + \phi_{x_{skr}} (R_{zwhl} + R_{ztire})^2 + \phi_{z_{skr}} (R_{zwhl} + R_{ztire})^2$$
(A.66)

$$\frac{\partial V_{tire}}{\partial x_{skr}} = -\tilde{K}_{x_{tire}} (x_{skr} + \phi_{x_{skr}} (R_{zwhl} + R_{ztire}) + \phi_{z_{skr}} (R_{zwhl} + R_{ztire}))$$
(A.67)

Eq. A.65 to A.66 is an example of the process that must be executed on each term of Eq. A.64 to derive $x_{skr}$ equations of motion. Eq. A.68 to A.78 presents the sum of the energy terms for each DOF to be derived.

$$\sum E_{y_{skr}} = \frac{1}{2} \left( -\tilde{K}_{y_{tire}} y_{tire}^2 - \tilde{K}_{y_{bil}} y_{bil}^2 - \tilde{K}_{y_{bd}_1} y_{bd_1}^2 - \tilde{K}_{y_{bd}_2} y_{bd_2}^2 - \tilde{K}_{y_{dld}} y_{dld}^2 - \tilde{K}_{y_{dld}} y_{dld}^2 \\
+ m_{whl} \left( \frac{\partial y_{whl}}{\partial t} \right)^2 + m_{hub} \left( \frac{\partial y_{hub}}{\partial t} \right)^2 + m_{arm} \left( \frac{\partial y_{arm}}{\partial t} \right)^2 \right)$$
(A.68)

$$\sum E_{z_{skr}} = \frac{1}{2} \left( -\tilde{K}_{z_{tire}} z_{tire}^2 - \tilde{K}_{z_{bil}} z_{bil}^2 - \tilde{K}_{z_{bd}_1} z_{bd_1}^2 - \tilde{K}_{z_{bd}_2} z_{bd_2}^2 - \tilde{K}_{z_{dld}} z_{dld}^2 - \tilde{K}_{z_{dld}} z_{dld}^2 + F_{skr} z_{skr} \\
+ m_{whl} \left( \frac{\partial z_{whl}}{\partial t} \right)^2 + m_{hub} \left( \frac{\partial z_{hub}}{\partial t} \right)^2 + m_{arm} \left( \frac{\partial z_{arm}}{\partial t} \right)^2 \right)$$
(A.69)
\[
\sum E_{\varphi S拉} = \frac{1}{2} \left( -\ddot{Q}_{\text{Tire}} \, \phi_{\text{Tire}}^3 - \ddot{Q}_{\text{XCL}} \, \phi_{\text{XCL}}^3 + I_{\text{xyWhl}} \left( \frac{\partial \phi_{\text{Whl}}}{\partial t} \right)^2 \right) ^2 \tag{A.70}
\]

\[
\sum E_{\varphi S拉} = \frac{1}{2} \left( -\ddot{Q}_{\text{Tire}} \, \phi_{\text{Tire}}^3 - \ddot{Q}_{\text{DL}} \, \phi_{\text{DL}}^3 + I_{\text{yyWhl}} \left( \frac{\partial \phi_{\text{Whl}}}{\partial t} \right)^2 + I_{\text{yyHub}} \left( \frac{\partial \phi_{\text{Hub}}}{\partial t} \right)^2 \right) \tag{A.71}
\]

\[
\sum E_{\varphi S拉} = \frac{1}{2} \left( -\ddot{Q}_{\text{Tire}} \, \phi_{\text{Tire}}^3 + I_{\text{zzWhl}} \left( \frac{\partial \phi_{\text{Whl}}}{\partial t} \right)^2 + I_{\text{zzHub}} \left( \frac{\partial \phi_{\text{Hub}}}{\partial t} \right)^2 \right) \tag{A.72}
\]

For the Strut tower module, there is only one DOF:

\[
\sum E_{\varphi \text{Hub}} = \frac{1}{2} \left( -\ddot{Q}_{\text{XCL}} \, \phi_{\text{XCL}}^3 + I_{\text{xyWhl}} \left( \frac{\partial \phi_{\text{Whl}}}{\partial t} \right)^2 \right) \tag{A.73}
\]

The driving shaft module has 2 DOF:

\[
\sum E_{\varphi CL} = \frac{1}{2} \left( -\ddot{Q}_{\text{XCL}} \, \phi_{\text{XCL}}^3 \right) \tag{A.74}
\]

\[
\sum E_{\varphi CL} = \frac{1}{2} \left( -\ddot{Q}_{\text{XCL}} \, \phi_{\text{XCL}}^3 \right) \tag{A.75}
\]

Finally, the Lower A-arm module has 3 DOF:

\[
\sum E_{\varphi \text{Arm}} = \frac{1}{2} \left( -\ddot{Q}_{\text{XY12}} \, \phi_{\text{XY12}}^3 - \ddot{Q}_{\text{YB34}} \, \phi_{\text{YB34}}^3 + I_{\text{xyArm}} \left( \frac{\partial \phi_{\text{Arm}}}{\partial t} \right)^2 \right) \tag{A.76}
\]

\[
\sum E_{\varphi \text{Arm}} = \frac{1}{2} \left( -\ddot{Q}_{\text{XY12}} \, \phi_{\text{XY12}}^3 - \ddot{Q}_{\text{YB34}} \, \phi_{\text{YB34}}^3 + I_{\text{yyArm}} \left( \frac{\partial \phi_{\text{Arm}}}{\partial t} \right)^2 \right) \tag{A.77}
\]

\[
\sum E_{\varphi \text{Arm}} = \frac{1}{2} \left( -\ddot{Q}_{\text{XY12}} \, \phi_{\text{XY12}}^3 - \ddot{Q}_{\text{YB34}} \, \phi_{\text{YB34}}^3 + I_{\text{zzArm}} \left( \frac{\partial \phi_{\text{Arm}}}{\partial t} \right)^2 \right) \tag{A.78}
\]

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Resulting equations of motion can be rearranged using matrices (Eq. A.79):

\[
\{\ddot{\mathbf{x}}\} = (-\omega^2 \mathbf{M} + \mathbf{K})^{-1} \{\mathbf{F}\}
\]  

(A.79)

\[\mathbf{M}\] is a 12x12 matrix containing all inertia terms (masses);
\[\mathbf{K}\] is a 12x12 matrix containing all deformation terms (springs).
\[\mathbf{F}\] is a 12x1 vector containing all work terms (forces);
\[\ddot{\mathbf{x}}\] is a 12x1 displacement vector, which is the unknown.

Eq. A.79 gives the resultant displacement vector \[\ddot{\mathbf{x}}\] of each degree of freedom for a given frequency \(\omega\).
REFERENCES


[34] http://www.bose.com/controller?event=VIEW_STATIC_PAGE_EVENT&url=/learning/project_sound/suspension_challenge.jsp&ck=0
