APPLICATION DE LA TOMOGRAPHIE SONIQUE AU DIAGNOSTIC DU BÉTON :
POSSIBILITÉS ET LIMITES

Mémoire de maîtrise ès sciences appliquées
Spécialité : génie civil

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To my parents Mehri and Manouchehr
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RÉSUMÉ

Depuis plusieurs années, une nouvelle méthode non destructive désignée "Tomographie" a été élaborée dans le but d'ausculter les ouvrages en béton. Cette technique est basée essentiellement sur la propagation d'ondes acoustiques afin de d'évaluer l'état interne de ces ouvrages. La variation relative de la vitesse de propagation de ces ondes sur la section d'étude, traduit les changements des caractéristiques mécaniques dans la structure qui reflètent donc l'état d'endommagement du matériau.

Cette étude concerne l'application de la tomographie sonique sur des modèles en béton au laboratoire et sur des structures en service. Les performances et les limites de détection de cette technique font l'objet de cette thèse et peuvent être résumées par les deux points suivants:

- Les endommagements causés superficiellement par divers aléas sont souvent marqués par la présence de fissures et de délaminages visibles à l'œil nu. Malheureusement, ces dégradations peuvent se produire à l'intérieur du béton ce qui rend nécessaire l'utilisation de la tomographie sonique comme un moyen adéquat pour remédier à cet inconvénient. En outre, cette technique est susceptible de suivre l'état de n'importe quel ouvrage en béton après réparation par injection de coulis de ciment.

- Les limites de détection se caractérisent par l'influence notable de certains paramètres (fréquence, taille de cellules, type de rais, pas de mesures, ...) clés sur la qualité de l'image finale. Comme objectif lointain, il est à prendre en considération l'influence de ces paramètres afin de mieux étayer cette étude.
ABSTRACT

Recognition of the many problems in concrete structures requires the reconstruction of their internal image. For this reason, some years ago, a new nondestructive method, sonic tomography, has been developed for scanning concrete structures. This method is most often based on transient stress wave propagation for verifying a concrete body, as well as measuring the wave velocity. The relative variation in wave propagation velocity in the material provides information about changes in the structure, and therefore the state of degradation.

Capacity and limitation of this method for reconstructing the internal image of the concrete structures are addressed in this thesis. This study discusses some experiments in which sonic tomography were performed on various concrete models in laboratory and the structures in service in-situ. In this case, two important aims are the main focus:

- Degraded area is usually marked by fracture and delaminations or poorly performed sections within the structure. Unfortunately, most of the degradation in concrete structures remain undetectable to the naked eye. Sonic tomography is a strong technique to show the damaged areas in body concrete. In addition, this technique is able to evaluate internal reparation (e.g. control of grout-injection zones).

- This is limited by the measurement and reconstruction of image conditions and their parameters (frequency, pixels size, rays type, measurement step, ...). For illustrating the real image it is necessary to know precisely these conditions and the quality of their influence on the tomographic image.
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<thead>
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<th>Description</th>
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<td>A</td>
<td>-amplitude</td>
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<tr>
<td>Ā</td>
<td>-scalar</td>
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<tr>
<td>a</td>
<td>-scalar</td>
</tr>
<tr>
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<td>-azimuth</td>
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<tr>
<td>C</td>
<td>-speed</td>
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<tr>
<td>E</td>
<td>-Yong's modulus of elasticity</td>
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<tr>
<td>F</td>
<td>-length</td>
</tr>
<tr>
<td>f</td>
<td>-frequency</td>
</tr>
<tr>
<td>f(x,y)</td>
<td>-function</td>
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<td>-compressive strength</td>
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<tr>
<td>G</td>
<td>-shear modulus of elasticity</td>
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<td>H_i</td>
<td>-vector of hyperplane</td>
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<td>-P-wave</td>
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<td>p</td>
<td>-vector</td>
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<td>-reflection coefficient</td>
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<td>-correction function vector</td>
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</tr>
<tr>
<td>υ</td>
<td>-Poisson's ratio</td>
</tr>
<tr>
<td>ξ</td>
<td>-constant Lagrange multiplier</td>
</tr>
<tr>
<td>Δt</td>
<td>-travel time</td>
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CHAPTER 1

INTRODUCTION

Concrete is a material which contains large numbers of flaws in the form of small voids and it is also a material which mechanical properties are not easily reproduced even under the best conditions. The main objective of nondestructive methods applied to concrete is to provide the engineering field with a reliable estimation of the structural quality of the concrete. By doing this, one does not have to rely solely on the results from test-specimens which are not necessarily representative of the structural concrete. The property taken to define the quality of the concrete differs for specific types of work; however, density, strength, or durability are usually implied. These properties are also of importance for performance of other structures and construction materials although in some cases, other specific properties may be required to be tested by nondestructive methods.

This work discusses theoretical and practical aspects of a relatively new method. This method can be used to visualize the internal condition of a concrete structure using nonintrusive data acquisition techniques and capable to show: (1) estimation of strength of concrete, (2) establishing homogeneity of concrete, (3) cracking position and (4) degraded zones.

Sonic tomography is an active acoustic technique which constructs an image of the interior of a body. It is performed by transmitting and receiving ultrasonic waves at a number of positions such that their respective raypaths cover a plane in the sample. This information can then be inverted to reconstruct an image of this interior plane. It has been widely used in medical applications (Brooks and De Chiro (1976) [41], Bats et al. (1983) [37], Herman (1980) [39], and Deans (1983) [42]) and in geophysical fields (Riker (1953) [38], Auld (1977) [44], Wong et al. (1983) [43], Fawcett (1983) [45], Anderson and Dziewonski (1984) [40]).
A back-projection method introduced by Bois et al (1972)[46], is a very simplified from of an algebraic reconstruction, but even this technique has rarely been applied. Data acquisition is expensive and in some ways inadequate and the acoustic waves used in tomography not only follow curved paths, but also experience diffraction, refraction, scattering and various forms of attenuation. Data acquisition methods are now at the point where collection of data adequate for reconstruction techniques is becoming feasible. The behavior of stress waves remains an obstacle in the applicability of these algorithms.

The wave may experience refraction and ray bending as well as attenuation and velocity changes, which complicate the interpretation. Two characteristics of the wave may be measured as data for tomographic reconstructions: the travel time between the source and receiver and the energy content contain additional information about the medium. However, it is difficult to extract the necessary data from the waveform. A wavelet generally contains energy due to different wavefronts arriving simultaneously along with effects of scattering, geometry, noise and so on. To eliminate the undesirable effects would take considerable computational effort as well as a good a priori knowledge of the medium. Travel times are easier to analyze than full waveforms, although they contain information relating primarily to the slowness or velocity of the medium. This work deal primarily with the stress travel time data.

The primary motivation of this study is to develop algorithms applicable to tomography used in concrete structures applications and the limitations and conditions of these algorithms will be verified. A thorough experimental study with real data from various velocity models is presented in this thesis.

The various types of nondestructive testing methods in concrete are discussed in chapter 2. Towards the end of the chapter, it is concluded that sonic tomography is one of the best methods for further development. In chapter 3, the principal of stress wave generation, propagation, reflection, refraction, diffraction and frequency are discussed. Towards the end, wave propagation in concrete is briefly described.
Basic principles of sonic tomography method are given in chapter 4. This method consists of two principal steps: 1) measurement and wave velocity calculation and 2) inversion and image drawing. In this chapter the inversion methods are discussed. Hence, it is concluded that the SIRT (Simultaneous Iterative Reconstruction Techniques) method is an excellent method for the reconstruction of images, which the basic principal and derivation of SIRT as applied to the tomography method. The program RAI-2D that is used in this thesis is explained at the end of this chapter.

In chapter 5, the experimental activity conducted in the laboratory is explained, and the details of the experimental models and experimental procedure are discussed. Afterwards, the results of the tests (tomographic images) are analyzed and discussed. In chapter 6, the in-situ activity is discussed. This activity includes the auscultation of a dam and the monitoring of a fissured slab grouting.

Finally, chapter 7 is the overall conclusion regarding the applicability of the sonic tomography method to the concrete structures evaluation.
CHAPTER 2

NONDESTRUCTIVE TESTING METHODS OF CONCRETE

2.1 Introduction

At the present time, standard methods for testing hardened concrete consist essentially of testing concrete specimens in tension, flexure, and compression. The main disadvantages of such methods are the delay in obtaining test results, the fact that the test specimens may not be truly representative of the concrete in the structure, the necessity of stressing the test specimens to failure, the lack of reproducibility in the test results, and the relatively high cost of testing. As a result, there have been a large number of attempts, over a period of about 30 years, to develop quick, inexpensive and nondestructive methods for testing concrete structures.

Because the determination of strength implies that concrete specimens must be loaded to failure, it becomes abundantly clear that the nondestructive methods of testing cannot be expected to yield absolute values of strength. These methods, therefore, attempt to measure some other physical property of structural concrete from which an estimate of the strength and elastic parameters of concrete is obtained. Some properties of concrete are its hardness, its resonant frequency, and its ability to absorb, scatter and transmit X-rays and gamma rays.

There are two types of nondestructive test methods for concrete. The first type consists of those methods which are used to estimate strength. The surface hardness, penetration, pullout, breakout, and maturity techniques belong to this category. Some of these methods are not truly nondestructive because they cause some surface damage, which is, however, minor compared with that produced by drilling a core. The second type includes these methods which measure other properties of concrete such as moisture content, density, thickness, and dynamic elastic modulus.
The latter two properties are used to determine changes in concrete when exposed to an aggressive environment such as cycles of freezing and thawing or aggressive chemicals. Also included in the second category are the stress wave, radar, and infrared thermography techniques which are used to locate delaminations, voids, and cracks in concrete.

The present chapter gives a general introduction to the current in-situ nondestructive methods and their applications in the concrete industry. Based on the literature review, the major in-situ nondestructive techniques can be divided according to their fundamental principles, into eight broad sections.

2-2 Sonic and ultrasonic methods

2.2.1 Pulse-echo methods

Pulse-echo methods are being used for thickness measurements, flaw detection and integrity testing of piles. In this method a stress pulse is introduced into an object at an accessible surface by a transmitter. The pulse propagates into the test object and is reflected by flaws or interfaces. The surface response caused by the arrival of reflected waves, or echoes, is monitored by either the transmitter acting as a receiver (true pulse-echo) or by a second transducer located near the pulse source (pitch-catch). Figure 2.1 illustrates the principle of these methods.

The receiver output is displayed on an oscilloscope, and the display is called a time-domain waveform. Using the time base of the display, the round-trip travel time of the pulse is determined. If the wave speed in the material is known, this travel time can be used to determine the depth of the reflecting interface using the following equation:

\[ L = \frac{1}{2} (\Delta t) \cdot V_p \]  

(2.1)
where $\Delta t$ is the round-trip travel time(s), $L$ is the depth(m), and $V_p$ is the P-wave speed(m/s). The factor of one-half is used because the actual depth is one-half the travel path of the wave. This equation is approximate for a pitch-catch system and is applicable only if the separation between the sending and receiving transducers is small [1].

2.2.2 Impact-echo methods

The principle of the impact-echo technique is illustrated in Figure 2.2. A transient stress pulse is introduced into a test object by mechanical impact on the surface. The stress pulse propagates into the object along spherical wavefronts as P- and S-waves. In addition, a surface wave (R-wave) travels along the surface away from the impact point. The P- and S- stress waves are reflected by internal interfaces or external boundaries. The arrival of these reflected waves, or echoes, at the

FIGURE 2.1. Schematic of pulse-echo and pitch-catch techniques [1].
surface where the impact was generated produces displacements which are measured by a receiving transducer and recorded on a digital oscilloscope. Because of the radiation patterns associated with P-waves and S-waves [2], if the receiver is placed close to the impact point, the waveform is dominated by the displacements caused by P-wave arrivals [1].

![Figure 2.2 Principle of the impact-echo method [1]](image)

2.2.3 Impulse-response methods

The principle of the impulse-response method is similar to the impact-echo method. A stress pulse is generated by mechanical impact on the surface of an object. In this method the force-time function of the impact is monitored by using an instrumented hammer or by using a hammer to strike a load cell. A transducer located near the impact monitors the velocity of the surface as it
vibrates in response to the arrival of reflected echoes. The waveforms of the force and velocity transducers are recorded and processed on a dynamic signal analyzer. The analyses reveal information about the condition of the structure. In this method, the time history of the structure’s response are recorded, and the impulse response is calculated. This can be accomplished by deconvolution or, in the frequency domain, by dividing the Fourier transform of the impact force-time function.

2.2.4 Pulse velocity methods

The basic idea upon which this method is established is that if the velocity of the longitudinal and traverse waves through a medium can be determined, then the dynamic modulus of elasticity and dynamic Poisson's ratio of the medium can be computed from equations:

$$v = \frac{V_L^2 - 2V_T^2}{2(V_L^2 - V_T^2)}$$  \hspace{1cm} (2.2)

$$E = \frac{\rho V_T (3V_L^2 - 4V_T^2)}{(V_L^2 - V_T^2)}$$  \hspace{1cm} (2.3)

Where: $V_L$ is the velocity of P-wave, $V_T$ is the velocity of S-wave and $\rho$ is the mass density of the material.

The transmitting transducer of the pulse velocity instrument transmits a pulse to one face of the concrete and the receiving transducer, at distance $L$, receives the pulse through the concrete. The pulse velocity instrument display indicates the time, $T$, it takes for the pulse to travel through the concrete, therefore the velocity is:
\[ V = \frac{L}{T} \]  

(2.4)

This method has been used for: (1) determination of dynamic modulus of elasticity and dynamic Poisson's ratio, (2) estimation of strength of concrete, (3) establishing homogeneity of concrete, (4) studies on the hydration of cement and (5) studies on durability of concrete [1].

2.2.5 Acoustic Emission methods

Acoustic emission is a general term used for any transient waveform released from a solid which is under stress. In concrete structures, the main source of wave emission would be crack development or the slip between concrete and reinforcement. On the concrete surface a number of transducers receive these stress waves. The variation of arrival time of waveforms registered by each transducer, is used to locate the source. Later interpretations can be performed using the intensity of wave emission. An increase in wave emission in a structure could be analyzed as an unsafe condition; however, this is not considered as a satisfactory deduction [1].

2.2.6 Spectral Analysis of Surface Waves (SASW) method

The SASW method is a technique for determining stiffness profiles and layer thickness in layered systems, such as pavement systems. The method developed out of past attempts to use R-waves to determine the properties of soil and pavement sites [33]. In the method, a transient stress pulse is generated by impact on the surface of the test site. Two receivers monitor the movement of the surface as the waves produced by the impact propagate past the receivers. Because the amplitude of particle motion in the R-wave is very large at the surface relative to the amplitude of motion in the P- and S-waves, surface movement caused by the R-wave dominates the measured response. The waveform measured by the two receivers contains information that is used to construct the stiffness profile of the underlying materials.
2.3 Others nondestructive methods

2.3.1 Surface hardness methods

Available equipment is capable of estimating the concrete hardness by impacting the surface and measuring the indentation or rebound value. Further correlation curves to the concrete's compressive strength have been constructed by various investigators. Conventional equipment is inexpensive and easy to use. However, the moisture content, surface conditions and aggregate size variations throughout the concrete have to be taken into consideration since these affect the results.

2.3.2 Magnetic/Electrical methods

Magnetic and electrical methods are being used in a number of ways to evaluate concrete structures. These methods are used (1) to locate reinforcement and measure member thickness by inductance; (2) measure the moisture content of concrete by means of its electrical properties and the nuclear magnetic resonance of hydrogen atoms; (3) measure the corrosion potential of reinforcement; (4) determine pavement thickness by electrical resistivity; and (5) locate defects and corrosion in reinforcement by measuring magnetic flux leakage [1].

Magnetic and electrical methods have received considerable attention in recent years. Their underlying principles range in complexity as do their practical applications in the field [1].

Magnetic nondestructive testing techniques used in conjunction with concrete involve the magnetic properties of the reinforcement and the response of the hydrogen nuclei to such fields. Because of the need to control the magnetic field, electromagnets are used in most instances. The changes in electrical properties of concrete have been investigated as a basis for electrical methods.
2.3.3 Nuclear and Radioactive methods

Radioactive and nuclear methods can be useful analytic or diagnostic tools, but, with an exception or two, are not widely used in concrete testing currently. The methods are based on directing radiation from sources such as radioisotopes and X-ray generators against or through fresh or hardened concrete samples. The radiation collected after interaction with the concrete provides information about physical characteristics such as construction, density, and structural integrity.

Gamma radiometry is the most widely used method, primarily for density determinations on roller-compacted and bridge deck concretes. Radiography is used occasionally in concrete laboratories for studying microstructure and in the field for confirming the integrity of structural concrete. Infrequently used, neutron-gamma techniques provide composition information on fresh or hardened concrete. The radioactive and nuclear methods are fast and accurate, but their use has been limited by the often complex technology involved, high initial costs, training and licensing requirements.

Although "radioactive" and "nuclear" have specific and distinct meanings, they are often used interchangeably in nondestructive testing contexts to refer to test methods which use the interaction of wave or particle radiation with matter to supply analytic or diagnostic information about the material. (In this chapter, the methods will be referred to, generically, as "nuclear methods.")

The nuclear methods used to test concrete can be separated into three categories: (1) radiometry; (2) radiography; and (3) neutron-gamma techniques. Radiometry describes techniques in which a radiation source and a detector are placed on the same or opposite sides of a concrete sample; a portion of the radiation from the source passes through the concrete and reaches the detector where it produces a series of electrical pulses. When these pulses are counted, the resulting count or count rate is a measure of the dimensions or physical characteristics, e.g., density or composition, of the concrete sample. Radiography describes techniques in which a radiation source
and a photographic film (the radiation detector) are placed on opposite sides of a concrete sample. After exposing the film, the result is a photographic image of the sample's interior, which is primarily used to locate defects in the concrete. Neutron-gamma techniques, rarely used in the concrete industry, are those in which a concrete sample is irradiated with neutrons, one type of radiation, and gamma rays, a second type, are emitted and detected. The result is a series of counts which are a measure of the composition of the concrete.

2.3.4 Infrared Thermographic methods

Infrared thermographic or infrared scanning is a technique which operates based on the capability of various materials to absorb heat. Solar radiation is the main source of heat for surface structures. As the solar ray flows through a concrete structure, air voids and fracture openings absorb a larger and more different percentage of heat than the surrounding concrete. This can be monitored and registered by an infrared camera. The same principle holds for steel reinforcement[1].

Using infrared thermography it is possible to locate voids, delaminations, fractures and steel reinforcement bars, but the exact position of such features is not possible to locate by using this method. Test results are highly affected by the surrounding conditions such as time of the day or seasonal changes. Moisture content of the concrete also affects the readings [1]. As for underground openings and shaft linings an artificial primary source of heat is needed.

2.3.5 Radar methods

Short-pulse radar is a powerful scientific tool with a wide range of applications in the testing of concrete. It is gaining acceptance as a useful and rapid technique for nondestructive detection of delaminations and other types of defects in bar or overlaid reinforced concrete decks. It also shows potential for other applications such as monitoring of cement hydration or strength development in
concrete, study of the effect of various admixtures on curing of concrete, determination of water
content in fresh concrete, and measurement of the thickness of concrete members.

To facilitate the understanding of these applications, the physical principles behind short-
pulse radar are required. Short pulse radar is the electromagnetic analog of sonic and ultrasonic
pulse-echo methods. It is governed by the processes involved in the propagation of electromagnetic
energy through materials of different dielectric constants.

2.4 Conclusion

Presently, nondestructive methods are valuable technologies for assessing damage to the
aging concrete structures. A wide range of methods are available to characterize materials and detect
defects in a variety of components. Applications of the appropriate methods will help assess the
condition of structures and allow cast effective solutions to repair and/or replace programs. Non of
the nondestructive testing techniques and equipment are capable of qualifying the quality and
determining the location of features in concrete.
CHAPTER 3

PRINCIPLES OF WAVE PROPAGATION IN SOLIDS

3.1 Introduction

When stress waves pass through a medium, the particles of that medium are set into vibration. Let one of these particles be displaced from its equilibrium position as a result of an external force associated with the wave. This motion is resisted by the elastic forces of the medium, thus causing the particle to come to rest after a finite displacement and then to return to its equilibrium position. Because of its inertia, the particle continues to move through this position with a given velocity and comes to rest after being displaced in the other direction, i.e. when its kinetic energy is converted into elastic energy. Hence the vibrations of the medium depend on its interior and its elasticity.

3.2 Elastic properties of solid and fluid media

Stress waves can be propagated in all kinds of elastic media-solid, gasses and liquids. Because of the differences between the elastic properties of solids and those of fluids, one would expect corresponding differences between acoustic properties. Changes in volume require the application of external forces in order to overcome the internal elastic forces and the medium is said to possess volume elasticity [3]. In order to obtain a change of shape of a solid body, external forces which exert a shear must be applied. The solid is then said to possess shear elasticity. This does not apply to fluids for which changes of shape can be effected without the application of any external forces.
3.3 Wave types

When a stress is applied suddenly to a solid, the disturbance which is generated travels through the solid as stress waves. There are three primary modes of stress wave propagation through isotropic, elastic media: dilatational, distortional, and Rayleigh waves (figure 3.1).

![Diagram of stress waves](image)

**FIGURE 3.1** Different modes of stress wave

Dilatational and distortional waves, commonly referred to as compression and shear waves, or P- and S-waves, are characterized by the direction of particle motion with respect to the direction the wavefront is propagating. P- and S-waves are known as body wave, which may propagate within or along the surface of solid. In a P-wave, motion is parallel to the direction of propagation; in the S-wave, motion is perpendicular to the direction of propagation. P-waves can propagate in all types of media; S-waves can propagate only in media with shear stiffness, i.e., in solids. Rayleigh waves, or R-waves, are waves which propagate along the surface of a solid. The particular motion in an R-wave near the surface is retrograde elliptical.
The wavefront defines the leading edge of a stress wave as it propagates through a medium. The shapes of the P-, S-, and R-wavefronts depend upon the characteristics of the source used to generate the waves. There are three idealized types of wavefronts: planar, spherical, and cylindrical. For example, when the stress waves are generated by impact at a point on the surface of a solid, the resulting P- and S-wavefronts are spherical and the R-wavefront is cylindrical [1].

3.4 Wave speed

In most applications of stress wave propagation, the input is a pulse of finite duration and the resulting disturbance propagates through the solid as transient waves. The propagation of transient stress waves through a heterogeneous bounded solid, such as a structural concrete member, is a complex phenomenon. However, a basic understanding of the relationship between the physical properties of a material and the wave speed can be acquired from the theory of wave propagation in isotropic elastic media[23].

In infinite elastic solids, the P-wave speed, \( V_p \), is a function of Young’s modulus of elasticity, \( E \), the mass density, \( \rho \), and Poisson’s ratio, \( v \):

\[
V_p = \sqrt{\frac{E(1-v)}{\rho(1+v)(1-2v)}}
\] (3.1)

In bounded solids, such as thin plates or long rods, P-wave speed can vary depending on the dimensions of the solid relative to the component wavelength(s) of the propagating wave[24]. For rod-like structures, such as piles, P-wave speed is independent of Poisson’s ratio if the rod diameter is much less than the wavelength(s) of the propagating wave. In this case, \( V_p \) is given by the following equation:
For a Poisson's ratio of 0.2, a typical value in concrete, the P-wave speed is five percent higher in an infinite solid than in a long thin rod.

The S-wave speed, \( V_s \), in an infinite solid is given by the following equation:

\[
V_s = \frac{G}{\sqrt{\rho}}
\]  

(3.3)

where \( G = \frac{E}{2(1-\nu)} \), (shear modulus of elasticity).

A useful parameter is the ratio, \( \alpha \), of the S- to P-wave speeds:

\[
\alpha = \frac{V_s}{V_p} = \frac{(1-2\nu)}{2(1-\nu)}
\]  

(3.4)

For a Poisson's ratio of 0.2, \( \alpha \) is 0.61.

R-waves propagate at a speed, \( V_R \), which can be determined from the following approximate formula[24]:

\[
V_R = \frac{0.87 + 1.12\nu}{1+\nu} V_s
\]  

(3.5)

For Poisson's ratio of 0.2, the R-wave speed is 92% of the S-wave speed, or 56% of the P-wave speed.
3.5 Wavelength and frequency

The simplest example of wave motion is that of the sinusoidal wave resulting from the vibration of the particles of a medium which execute simple harmonic motion, for which the particle displacement, $u$, from the position of equilibrium varies sinusoidally with time. At the same time the particle velocity, $v$, and the resulting stress, $\tau$, also vary in that manner. One should not confuse the particle velocity with the wave velocity.

Consider a sinusoidal wave propagated in the direction of the $x$-axis. A plot of particle displacement with distance along the $x$-axis at various time, $t$, is given in figure 3.2. The wavelength, $\lambda$, is given by the distance between two successive crests or troughs, and the wave amplitude, $A$, is the maximum or minimum value of the displacement measured from the position of equilibrium. The time taken for the particle to complete one vibration is called the time period, $T$, and the reciprocal of this is called the frequency, $f$.

![Diagram of a sinusoidal wave](image)

**FIGURE 3.2 Displacement of wave propagated in direction of $x$-axis**

The ability (sensitivity) of stress wave propagation methods to detect flaws or discontinuities depends on the component frequencies (or wavelengths) in the propagating wave and on the size of the flaw or discontinuity. A general rule is that the size of the flaw must be approximately equal to
or larger than the wavelengths in the propagating wave. Wave velocity, $C$, frequency, $f$, and wavelength, $\lambda$, are related by the following equation:

$$C = f \times \lambda$$

(3.6)

For example, to detect a flaw with a diameter of about 0.1 cm, it is necessary to introduce into the concrete (P-wave velocity of about 4000 m/s) a stress pulse that contains frequencies greater than approximately 40 kHz (wavelengths less than approximately 0.1 m).

3.6 Reflection and refraction

3.6.1 Interface problem

When a P- or S-wavefront is incident upon an interface between dissimilar media, "specular" reflection occurs. (The term specular reflection is used because the reflection of stress waves is similar to the reflection of light by a mirror.) As shown in Figure 3.3, stress waves can be visualized as propagating along ray paths.

The geometry of ray reflection is analogous to that of light rays, that is, the angle of reflection of any ray is equal to the angle of incidence, $\theta$, for that ray.

At a boundary between two different media only a portion of a stress wave is reflected. The remainder penetrates into the underlying medium (wave refraction). The angle of refraction, $\beta$, is a function of the angle of incidence, $\theta$, and the ratio of wave speeds, $V_2/V_1$, in the different media, and is given by Snell's law:

$$\sin \beta = \frac{V_1}{V_2} \sin \theta$$

(3.7)
Unlike light waves, stress waves can change their mode of propagation when striking the surface of a solid at an oblique angle. Depending on the angle of incidence, P-waves can be partially reflected as both P- and S-waves and can be refracted as both P- and S-waves. Since S-waves propagate at a lower velocity than P-waves, they will reflect and refract at angles (determined using Snell's law), $\theta_s$ and $\beta_s$, that are less than the angles of reflection and refraction for P-waves, as shown in figure 3.3 [1].

3.6.2 Impedance

The relative amplitudes of reflected waves depend upon the mismatch in specific acoustic
impedances at an interface, the angle of incidence, the distance of an interface from the pulse source, and the attenuation along the wave path. The influence of each of these factors is considered in the following discussion.

The portion of an incident ray of a P-wave that is reflected at an interface between two media depends on the specific acoustic impedances of each medium. The specific acoustic impedance, Z, of a medium is:

\[ Z = \rho \times V_p \]  \hspace{1cm} (3.7)

Specific acoustic impedance values for P-waves in selected materials are given in Table 3.1. Equation 3.7 is also valid for S-waves if the S-wave velocity is used to calculate specific acoustic impedance.

The amplitude in a reflected ray is maximum when the angle of incidence of the ray is normal to the interface. The amplitude of the reflected ray relative to the amplitude of the incident ray can be determined using the following equation:

\[ R_n = \frac{Z_2 - Z_1}{Z_2 + Z_1} \]  \hspace{1cm} (3.8)

where \( R_n \) is the reflection coefficient for normal incidence and \( Z_1 \), and \( Z_2 \) are the specific acoustic impedances of media 1 and 2, respectively. If \( Z_1 \) is greater than \( Z_2 \), then \( R_n \) is negative, indicating that the reflected wave will have the opposite sign, that is, a phase change occurs. For example, if the stress in an incident P-wave is compressive, then the stress in the reflected P-wave is tensile. If \( Z_2 \) is greater than \( Z_1 \), no phase change occurs.

For incident angles other than normal to an interface, the reflection coefficients are dependent on the angle of incidence, and they can be determined using the formulas in Krautkramer and
Krautkramer[25], which are applicable for plane waves incident upon plane boundaries. These formulas were used to calculate the reflection coefficients for a concrete/air interface.

Figure 3.4 shows reflection coefficients for an incident P-wave. It is assumed that the incident wave has an amplitude equal to unity. A similar figure can be constructed for an incident S-wave. The figure is composed of three graphs. The graph in the upper left gives the reflection coefficients, $R_P$, for the reflected P-wave. The graph in the lower right gives the reflection coefficients, $R_S$, for the mode-converted S-wave. The graph in the upper right gives the angular relationship between the incident wave and the mode-covered wave, which is determined by Snell's law. The drawing in the lower left gives an illustrative example. Note that for P-waves with a low angle of incidence, $R_P$ is approximately equal to one and $R_S$ is small.

FIGURE 3.4. Reflection coefficients at a concrete/air interface for an incident P-wave as a function of the incidence angle (Poisson's ratio = 0.2)[1]
### TABLE 3.1. SPECIFIC ACOUSTIC IMPEDANCES[1]

<table>
<thead>
<tr>
<th>Material</th>
<th>Mass Density (kg/m³)</th>
<th>P-wave velocity (m/s)</th>
<th>Specific acoustic impedance (kg/(m²-s))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.205</td>
<td>343</td>
<td>0.413</td>
</tr>
<tr>
<td>Concrete *</td>
<td>2300</td>
<td>300-4500</td>
<td>6.9-10.4 × 10⁶</td>
</tr>
<tr>
<td>Granite</td>
<td>2750</td>
<td>5500-6100</td>
<td>15.1—16.8 × 10⁶</td>
</tr>
<tr>
<td>Limestone</td>
<td>2690</td>
<td>2800-7000</td>
<td>7.5—18.8 × 10⁶</td>
</tr>
<tr>
<td>Marble</td>
<td>2650</td>
<td>3700-6900</td>
<td>9.18.3 × 10⁶</td>
</tr>
<tr>
<td>Quartzite</td>
<td>2620</td>
<td>5600-6100</td>
<td>14.7—16.0 × 10⁶</td>
</tr>
<tr>
<td>Soils</td>
<td>1400-2150</td>
<td>200-2000</td>
<td>0.286.3 × 10⁶</td>
</tr>
<tr>
<td>Steel</td>
<td>7850</td>
<td>5940</td>
<td>46.6 × 10⁶</td>
</tr>
<tr>
<td>water</td>
<td>1000</td>
<td>1480</td>
<td>1.48 × 10⁶</td>
</tr>
</tbody>
</table>

* The mass density of concrete depends on the mixture proportions and the specific gravities of the mix ingredients. The given density is for an average, normal weight concrete.

#### 3.6.3 Fissuring problem

In the wave propagation problem, when a wave comes into contact with a discontinuity (fissure), the reflection coefficient, \( R \), is given by Jones (1962):

\[
R = \frac{\rho_1 V_1 / \rho_2 V_2 - \rho_2 V_2 / \rho_1 V_1}{4 \cot^2 (2\pi c / \lambda_2) + \rho_1 V_1 / \rho_2 V_2 + \rho_2 V_2 / \rho_1 V_1}
\]  

(3.9)

Where: \( \lambda_2 \) is the wavelength in the fissure, and \( c \) is fissure thickness.

\( \rho_1 \) and \( \rho_2 \) are the mass density of the concrete and fissure (air, water, ...).

\( V_1 \) and \( V_2 \) are the longitude wave velocity in the concrete and fissure.
Therefore, the reflective quantity of the energy depends on the thickness of the discontinuity. For example, if a wave with a frequency of 50 kHz propagate in concrete medium, the reflection coefficient is 0.25 for a fissure of 2 mm full of water, and this coefficient is 0.81 for a thickness of 5 mm.

3.7 Wave propagation in concrete media

As a wave propagates through a solid its amplitude decreases with path length due to attenuation (scattering and absorption) and divergence (beam spreading). Divergence causes the amplitude of spherical waves to decrease in proportion to (the inverse of) the distance from the source. In evaluation of concrete, low frequency (long wavelength) waves must be used to reduce the attenuation of wave energy due to scattering (reflection and refraction from mortar-aggregate interfaces). If the wavelength of the propagating wave is less than the size of the aggregate, the mismatch in acoustic impedances between the mortar and the aggregate causes scattering of incident waves at each mortar-aggregate interface. For example, if the maximum size of the aggregate is 25 mm in a concrete with a P-wave speed of 4000 m/s, frequencies lower than 4000/0.025 = 160 kHz should be used to reduce scattering. The concrete will appear homogeneous to these lower frequency waves. However, use of low frequency waves reduces the sensitivity of the propagating waves to small flaws. Thus there is an inherent limitation in the flaw size that can be detected within concrete using stress wave propagation methods[1].
CHAPTER 4
SONIC TOMOGRAPHY METHOD

4.1 Introduction

Tomographic imaging is a technique to determine cross-section values of a spatially varying parameter from transmission data through an object. Figure (4.1) shows principal of the sonic tomography method.

In general, the measured transmission data must present a line integral of the parameter of interest along a known path through the sample. In the imaging process, a large number of these measurements are required between a series of source and receiver locations placed in appropriate positions about the object. These locations are chosen such that rays pass through as large a fraction of the object volume as possible, and must also conform to any requirements for regular positioning in the tomographic reconstruction procedure.

In the sonic and ultrasonic case of tomography, wave are propagated through the medium ausculted, travelling on a raypath from source to receiver. This raypath is usually assumed to be linear in the first approximation. Perhaps the most convenient parameter for use in imaging to the measured experimentally is the time of flight, the total time taken for the wave to propagate from source to receiver. The travel time, \( t \), represent the line integral of the path length divided by the velocity, \( V \),

\[
t_i = \int_{L_i} \frac{ds}{V(x, y)}
\]  

(4.1)

where \( L_i \) defines the \( i \)th raypath. The result, following the construction of a tomographic image, is then a two-dimensional distribution of the \``slowness\'' of the material on the object cross-section (slowness is defined as the inverse of velocity).
Figure 4.1 Sonic tomography principle
The remainder of this chapter describes the background theory for the tomographic reconstruction algorithms use in the study (RAI-2D).

4.2 Inversion methods

This section is principally concerned with tomography inversion in which a line integral relationship exists between the observed data and the material parameters. Tomography is the inversion of these line integral relationships (figure 4.2) to obtain estimates of the velocity field, \( V(x, y) \), within some region of space through which the rays have passed. Various approaches to this tomographical imaging problem will be considered, all of which have been successfully applied in a variety of medical and engineering applications [6]. Most of these approaches can be grouped together into three main categories:

\[
t_k = \int_{L_i} \frac{ds}{V(x, y)}
\]

Figure 4.2 Schematic of inversion relationship

4.2.1 Matrix inversion methods

a) Principle of the methods

The various methods of generalized matrix inversion, damped least squares or linear programming are all readily applied to this problem (Aki and Richards 1980, Jakson 1972)[15,27]. The region of interest is divided into a grid of pixels, and \( V(x, y) \) are assumed to be constant over
the area covered by any one pixel. Thaking the wave travel time equation as an example, the line integral may be approximated as:

\[ t_k = \sum_j \frac{\Delta s_j}{V_j} \tag{4.2} \]

where: \( \Delta s_j \) is the distance travelled by the ray through pixel \( j \), and \( V_j \) is the wave velocity within pixel \( j \), and summation is taken over the pixels interacted by the \( k \)th raypath.

In practice, the equations are normally set up so that one is solving for the perturbation of the velocity values from some starting model thus:

\[ \Delta t_k = t_{(observation)}_k - t_{(calculation)}_k = \sum_j \Delta s_j \Delta p_j \tag{4.3} \]

where:

\[ \Delta p_j = \left( \frac{1}{V_j} - \frac{1}{(V_{model})_j} \right) \tag{4.4} \]

is the perturbation of the slowness (reciprocal of velocity) within the \( j \)th pixel. This is because the number of distinct data are usually not sufficient to define uniquely all the parameter values within the grid of pixel. In other words, there are more unknowns than there are linearly independent equations. Under these circumstances, a solution to the set of equations is obtained by applying additional constrains and normally this involves scorching for a solution that not only satisfies the data but also is close, in some adjustable sense, to some initial model that must be specified [26].

In term of a matrix equation of the form (In general, this form is used for a great number of the inversion methods):

\[ t = AS \tag{4.5} \]
A is a \((k \times j)\) matrix of \(\Delta s\) values where \(k\) is the total number of rays crossing the region of interest and \(j\) is the total number of pixels. \(A\) is a relatively sparse matrix since any ray normally only intersects a small fraction of the pixel within the region under investigation. The equation 4.5 is solved using some matrix inversion procedures. Perturbations to the velocity contained within \(S\), are kept small by suitable damping within the inversion and the deviations to the raypaths that would result are also assumed small and ignored.

b) Conclusion of the Matrix inversion methods

The advantages of matrix inversion are that any source-receiver array can be handled easily and one is not restricted to straight rays. The disadvantage is that the method is very slow, sometimes quite unmanageably slow [26].

4.2.2 Analytic inversion methods

a) Fourier transform method

Fourier transform methods are based upon the projection slice theorem which state that: the one-dimension at an angle \(\theta\) is a slice at the same angle of the two-dimensional Fourier transform of the original object (Mersereau and Oppenheim 1974) [26]. In principle, this permits a very rapid method of backprojection. One simply Fourier transforms all the projections at the different angles in order to construct a two-dimensional Fourier transform field. Then one simply carries out a two-dimensional Fourier transform to obtain the original velocity field. However problems can arise when the data in wave-number space are interpolated from polar coordinates to the constant simple interval in Cartesian coordinate required for the final two-dimensional inverse Fourier transform. The number of data point per unit area decreases with distance from the origin. Therefore interpolation error will inevitably increase for larger values of wavenumber and the final image can
be seriously degraded as a result. In addition interpolation is computationally very expensive. For this raison an equivalent conventional approach turns out to be just as efficient.

b) *Filtered back-projection (convolution) method*

This method is identical to the summation method, except that the projection profiles are filtered optimally before being back projected. Several variations on the filter function have been reported (e.g. Robinson 1982) but in general this involves a convolution of the original data with another function, usually of the *sinus* type, hence the alternative name. Ideally modification performed on the profiles exactly counterbalances the blurring artifact encountered in simple back-projection.

c) *Conclusion of the analytic inversion methods*

Both analytic methods take a similar time to execute as one cycle of an iterative process, and produce exact results [18]. In general, these algorithms have the advantages both of being quite fast when compared to other reconstruction techniques, and also of being capable of handling a greater amount of data. A major disadvantage, however, is the stringent requirement placed on sampling geometry. This does not pose a problem in many applications, such as the immersion testing of materials, where it is possible to fix either a pair of single or array transducers at a constant separation and depend on the water to couple the ultrasonic energy into the sample. In many cases, though, immersion testing is feasible, and the transducers must be fixed to the surface of the object under test. In these instances, it may not be possible to ensure the preferred geometry [19].

4.2.3 *Iterative inversion methods*

This class of commonly used methods takes the opposite approach, and discretizes the system at beginning of the problem. The algorithms of this category generally involve the use of iterative procedures to arrive at an image, and are referred to as series expansion methods [21].
a) **Iterative Least-Squares Technique (ILST)**

In this form, developed to its current standing by Goitein (1972), as solution to equation 4.5 is [21, 28]:

\[ S = (A^T A + \sigma^2 I)^{-1} A^T t \]  \hspace{1cm} (4.6)

where \( t \) is the wave travel time and the superscript \( T \) signifies a matrix transpose and \( \sigma \) is fixed confidence bounds on individual datum [22]. In this method, all projections are calculated at the start of each iteration, then suitably damped (e.g. by a least-squares fit) to avoid excessive oscillation about the correct solution, before all corrections are made.

b) **Conjugate Direction Methods (CD)**

Conjugate direction (CD) methods have been considered by Herman and Lent (1976). The application to image reconstructions were considered, but the subject of placing constraints on the image vector was not addressed. In the following, conjugate direction methods are developed, and these methods are extended to handle constraints. These methods are a group of linear equation solvers that provide a means of creating a set of linearly independent vectors such that these vectors span the space to which the solution vector belongs [29]. For example, if the solution vector has \( n \) rows and 1 column, then \( n \) linearly independent vectors will be generated by a CD algorithm and the solution can be written as

\[ \bar{x} = a_1 p_1 + a_2 p_2 + \cdots + a_n p_n \]  \hspace{1cm} (4.7)

where \( a_i \)'s are scalars to be found, and the \( p_i \)'s are linearly independent vectors known as search direction vectors. In these methods, the solution of equation 4.5 is:

\[ \Phi(S) = \frac{1}{2} S^T A S - S^T t \]  \hspace{1cm} (4.8)
c) **Algebraic Reconstruction Techniques (ART)**

A very large number of variations on this general theme have been proposed and the reader is referred to the reference list of details (Dines and Lytle 1979 [7]; Gordon 1974 [6]; Herman 1980 [39]; Scudder 1978 [47]; Lytle and Dines 1980 [48]).

The starting point for ART is the same matrix equation 4.5 above. However, an iterative solution is sought rather than an exact inverse. The original ART algorithm was defined as follows: given \( m \) data in vector \( t \), \( t = AS \) where \( A \) is an \( m \times n^2 \) matrix where \((n\times n)\) are the dimension of the region to be imaged. Define calculated data:

\[
t_i^q = \sum_j A_{ij} S_j^q
\]  
(4.9)

for each ray \( i \) and each iteration \( q \) then:

\[
S_j^{q+1} = S_j^q + A_{ij} \frac{(t_i - t_i^q)}{\sum_j A_{ij}^2}
\]  
(4.10)

In other words, rays are traced through the region to be imaged and travel times calculated. The difference between the observed and calculated times is redistributed back along each raypath and new values for \( S \) are calculated according to equation 4.10. Rays are traced again through the revised velocity field and the process is repeated until an acceptable solution is obtained. Note that if pixel \( j \) is not intersected by any rays, then the value of \( S_j \) is left unchanged.

d) **Simultaneous Iterative Reconstruction Techniques (SIRT)**

First published by Gilbert (1972)[14], this is the opposite to ART, in that instead of corrections being made ray-by-ray, all rays passing through a certain pixel are determined [12]. The corrections that the pixel would have been given from each of these rays in the ART process then...
are summed and the pixel value adjusted only once. Successive pixels are covered, again using newly changed values in the ray calculation, and one iteration is complete when all pixels have been considered. This method will be explained in appendix II.

e) Conclusion of the Iterative methods

These algorithms are all iterative techniques, relying on row-action methods to arrive a solution. Because of this, they are less efficient than transform methods, in that they are slower and take up more computer memory. Their advantage is that they are more readily able to consider irregular geometries, and, as will be discussed later, raybending and anisotropy.

An iterative type of algorithm can consider both lateral and vertical inhomogeneities, can encompass easily known constraints, does not require geometrical symmetry and can produce solution within an acceptable amount of computer time whatever the nature of the system of equations, even if the number of pixels is large [18]

In this study, the SIRT algorithm is selected to use for reconstruction of the images because it is easier intuitively to understand and easier to implement.

4.3 Program RAI-2D

The program that was used for the inversion of data is RAI-2D [9]. Certain original characteristics were introduced in the function of algorithm by reporting classic geometric reconstruction (Côte P., 1988-89) [9, 11]. As was the case, this program calculated velocities using a variation of the SIRT.

4.3.1 Representation of the medium

As explained previously, SIRT is based on a back-projection method. The field of interest is divided into many pixels of constant physical properties. Source-receiver raypaths project
across this field as shown in figure 4.3 [10]. As the wave passes through each pixel, the properties of the pixel affect its velocity. The resulting travel time is thus dependent on the pixel properties and it is assumed that the contribution of each pixel can be deducted by the backprojectioning of the rays. For example, if the data were a single ray the best velocity model would be the average velocity of the ray as determined by its mean slowness (travel time divided by ray length). For two intersecting rays of different slowness, the pixel common to both ray, would have velocities dependent upon some weighted average of both rays. It follows that a data set consisting of many rays crossing at all angles may be backprojected to determine an estimate of the velocities in each pixel needed to produce the travel times. The alleviation of each pixel may be determined in a similar manner. Therefore the method of processing is based on the relation between propagation velocity and the total travel time.

4.3.2 The influence zone

This notion replaces each node by a circular influence zone. For the b given node the disc of influence center d at this point delimits the whole ray that will be selected for resolving. Figure 4.3 shows this model, the characteristics of medium are described by nodes of the grid. For solving the node with coordinates (i, j), the program selects rays that intercept influence zones corresponding to paths (S1, R1), (S1, R2) and (S2, R1).

The reconstruction of this manner averages the partial information taken by the rays which cross this zone. In backprojection, the slowness in point b is a weighting average of the slowness of Nb rays which cross this zone.

\[
S_b = \frac{\sum_{r=1}^{Nb} X_{rb} \frac{T_r}{L_r}}{\sum_{r=1}^{Nb} X_{rb}}
\]

(4.11)

Tr: time of ray propagation  
Lr: total length of ray
The disc's choice of radius will fix the lateral influence of each ray on each node. If this radius is chosen by the order of the wave length, the physical phenomenon will be better understood. In general, the propagation of the mechanical wave is presented by an alliance with a band larger than a thread. The proceeding manner allows discretisation and gives the rays one form similar to the physical behavior of waves. Inversion iterative will be applied in the same manner. This time, it is the residue \( R_{(es)r} \) that is carried by each ray inside the disc. The calculation of this residue corresponds to a ray \( r \); the difference between the measured time for this data and calculated time issues of the drawing of a ray:

\[
R_{(es)r} = T_{(observation)} r - T_{(calculation)} r
\]  

(4.12)

Correction of slowness for bringing in point \( b \) could be obtained by a simple average of variations of the slowness \( Nb \) in the influence disc.
\[ C_b = \frac{1}{Nb} \sum_{r=1}^{Nb} \frac{R_{(x)}^r}{L_r} \]  

(4.13)

4.3.3 The weightings

Some weightings were added to this correction to control the reconstruction. They intervene at two levels, the weighting of rays \( a_{rb} \) and the weighting of nodes \( A_{rb} \).

a) Weighting the rays

The weighting in the rays allows the importance that is needed to accord a particular ray in reconstitution to be quantified. It discriminates the manner in which the rays are selected by the influence zone. The global weighting \( a_{rb} \) has approximately four factors [10]:

- the length of ray in the zone of influence, \( X_{rb} \),
- the total length of ray, \( L_r \),
- azimuth of ray, \( AZ_{rb} \),
- the precision on the data, \( Confr \).

The ray intervenes with a different contribution which is proportional to the length of the ray in the disc and inversely proportional to its total length. The length of the ray in the influence zone depends on its distance from the center of the zone.

It is clear that in this zone, the rays with longer lengths have more influence. So the ray that passes at the border of this zone has less weight than the one which passes at center of the zone. Moreover, corresponding contribution has more chance, of being correctly relocated spatially.

It was also equally chosen to apply an azimuth weighting that allows the azimuth participation of the rays to be homogenized. In this manner, certain direction of rays were more
weakly represented than others. However, they are not at a disadvantage and come to the same point in reconstruction. The rays are classified according to their azimuth direction. Subsequently, each class is formed in relation to all of the azimuth directions presented, in order to obtain a coefficient of distribution for each class. The weighting assigned to each ray is:

\[ AZ_{rb} = 1 - \text{distr}_{rb} \]  

(4.14)

Finally, the last factor (Conf) intervenes by taking into consideration the imprecision of the data.

Each set of data is assumed to a number that it can be from 0, for the defective data, to 1, for the best data. It is therefore definitely possible to eliminate some data of reconstruction.

\[ 0 \leq \text{Conf}_r \leq 1 \]  

(4.15)

The global weighting of the rays \( a_{rb} \) is the combination of all its coefficients.

\[ a_{rb} = \frac{X_{rb}}{L_{rb}} \times AZ_{rb} \times \text{Conf}_r \]  

(4.16)

If this weighting is added to the expression of correction rays preceding (6), each term is weighted by the same coefficient \( a_{rb} \):

\[ C_b = \frac{\sum_{r=1}^{Nb} a_{rb} \frac{\text{Re}s_r}{L_r}}{\sum_{r=1}^{Nb} a_{rb}} \]  

(4.17)
b) Weighting the nodes

At this point in time, the correction does not make any distinction between the nodes. The additional weighting in the nodes ($A_{rb}$) was introduced to allow us to consider the variability of this model. This entails two steps of information:

- The global quality of rays for reconstructing a point $\Sigma a_{rb}$
- Validity of initial model $Conf_b$

Correction of the $C_b$ which is applied to each node depends on global quality of the information which it determines.

For each point, this coefficient corresponds to:

$$Min(1, \Sigma a_{rb})$$  \hspace{1cm} (4.18)

When the initial model is defined from a priori on knowledge of the medium each node is assigned to a liberty coefficient $Conf_b$ which depends on the quality of the information that we give it. We can supply knowledge about the middle of a program to determine the quality of this information. The total weighting on the nodes $A_{rb}$ is:

$$A_{rb} = Min(1, \Sigma a_{rb}) \times (1 - Conf_b)$$  \hspace{1cm} (4.19)

Real expression of the correction $C_b$ is given by:

$$C_b = A_{rb} \times \left( \frac{\Sigma_{r=1}^{Nb} \frac{Res_r}{L_r}}{\Sigma_{r=1}^{Nb} a_{rb}} \right)$$  \hspace{1cm} (4.20)
4.3.4 *Control of the convergence*

This correction, one time calculated for each node, can be applied and can modify respectively each velocity of the model. Certain restraints were nevertheless foreseen to assure the convergence of the solution. On the complex patterns, the iterative process can not converge. It is possible then to reduce calculation in modulating the model variation of each iteration. For this technique, one coefficient modulator which allows the convergence of the solution correctly is applied. Moreover, in order to avoid a velocity value to be excessively dispersed, the correction is limited to 10% of the previous value of one iteration to other. The new correction slowness can be written by:

\[
(S_b)^n = (S_b)^{n+1} + \text{modular} \times C_b \quad (4.21)
\]

In order to control the convergence of the solution, we calculate two statistical variables: the average value of residue and the gap-type of the residue. These statistics are calculated for every iteration and eventually either globally for all rays or for all rays issued by the same emitter or receiver. For a normal convergence of a solution, the two coefficients present a regular decrease and attain a stability threshold which constitutes a point to halt the iterative processes.

4.3.5 *Ray tracing*

From a representation of velocity model, we select analytically the minimum path of the arc circle time which passes through the source-receiver couple [30]. To trace a ray, two conditions have to be met:

- the ray must pass through a fixed source point and receiver point;
- the time of the path must be minimized.

To find the pseudo-ray, we connect a source point to a receiver point, so as to explore systematically all of the circles that pass through this point couple in order to converge the arc of
the optimum time. In RAI-2D, tracing algorithm is very simple. The ray path has to remain in the model plan, containing equally the source and receiver together with the circles which pass by S and R, can be covered to vary only one parameter (the arrow in figure 4.4). Figure 4.4 [10] shows analytic determination of the most rapid circular ray passing by R and S. The equations of circles are traced corresponding to the arcs of the circle to link the source and receiver.

![Source and Receiver](image)

**FIGURE 4.4 Schematic of passing circular ray**

To optimize this arrow (figure 4.4) we applied two methods successively. This model is systematically swept after a constant number of samples by varying regularly the length of the arrow. In this manner, we explore the sites of positive and negative arrows, on either side to the straight-ray of worthless arrow up to the maximum length arrow corresponding to source-receiver half-distance. This sweep allows a first optimum ray to be obtained. We proceed by a dichotomy which allows to converge toward the definitive trace. It starts from a pre-selected arrow and converges with a progress step 20 times smaller than the sweep. The duration of propagation of each circle sweeping the distribution of velocities in course is calculated step by step along the arc. The progress step is chosen as a fraction of the coating of stitch of discretisation such as:
progress step = \frac{\text{discretisation step}}{\text{progress coefficient}} \tag{4.22}

The time of path of each little arc element is calculated by the average velocity encountered at the two extremities. The velocities are described as the node of the discretisation grid. To obtain the value of these points, one must perform an linear interpolation from the velocities of four-corner nodes. Figure 4.5 shows the calculation of path time for little arc element, \( T_i \), from the velocities, \( V' \) and \( V'' \), of its end points.

\( x_1, x_2, y_1 \) and \( y_2 \) the coordinates surround \( M(x,y) \). If we want to interpolate the velocity of \( x \) belonging to the interval \((x_1, x_2)\), with \( V_1 = V(x_1) \) and, \( V_2 = V(x_2) \), the case being linear, it is sufficient to resolve an equation in the form of: \( V(x) = ax + b \). The expression of this equation is:

\[
V(x) = \frac{V_2 - V_1}{x_2 - x_1} \cdot x + \frac{V_1 x_2 - V_2 x_1}{x_2 - x_1} \tag{4.23}
\]

or:
\[ V(x) = \sum_{i=1}^{2} V_i - \left[ 1 - \left( \frac{x - x_i}{x_2 - x_1} \right) \right] \] \hspace{2cm} (4.24)

By interpolating in the same manner following \( y' \), the coordinates being independent, we obtain the velocity for point \( M(x,y) \) in the following way:

\[ V(M) = \sum_{i=1}^{2} \sum_{j=1}^{2} V(x_i, y_j) \cdot \left[ \left( 1 - \frac{x - x_i}{x_2 - x_1} \right) \cdot \left( 1 - \frac{y - y_j}{y_2 - y_1} \right) \right] \] \hspace{2cm} (4.25)

The path time of the complete arc, between \( S \) and \( R \), is obtained by adding the times of all of the little arc elements which compose it.
CHAPTER 5

EXPERIMENTAL ACTIVITIES IN LABORATORY

5.1 Introduction

The main objective of the experimental section of this research project is the investigation of the capabilities and the limitation of the sonic tomography technique in scanning a section of concrete structures.

To reach the objective of the experimental activity in laboratory, the methodology that is chosen is to perform tomography measurements and processing on concrete models that simulate the concrete degradation typically observed on concrete structures. Therefore the concrete models with the zones which present the problems in real concrete structures are tested.

The steps of the experimental activity algorithm that will be followed in this chapter are:

1- To verify the important problem of concrete structures as part of tomography testing (fissuring, degradation, ...),
2- To produce suitable experimental models,
3- To choose the suitable section that is capable to present the problem of each model,
4- To categorize the most important problem of the tomography method,
5- To do the tests and to give the data, on the basis of the step 4,
6- To calculate and inverse the data,
7- To illustrate the tomographic images,
8- To compare the images and to conclude.
5.2 Experimental models

5.2.1 Geometry

The first step of the laboratory work consisted of working on experimental concrete models. These models had to simulate the deteriorated zones of which the form and the mechanical state represented the reduced scale of the typical anomalies observed on the structure-in-service (i.e. fissuring, deterioration on surface).

The models that were made for these tests are concrete cubes. The dimension of each cube is 40×40×40 cm³ (0.064 m³).

5.2.2 Materials

The concrete which was used for making these experimental models are of two types. The first type (Type A) is a concrete characterized by a water /cement ratio of 0.5. The second type (Type B) possesses a 0.84 water /cement relationship pertaining to the mass proportion of the poorer aggregate. The concrete of type A represents a good quality concrete, whereas the concrete of type B simulates a poor concrete. In both cases, the average diameter of the aggregate equalled 10 mm. Table 5.1 shows the composition and the properties of these two concretes.

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Constituent (for 100 kg of mix)</th>
<th>$E_{28}$ (GPa)</th>
<th>$f'_{c28}$ (MPa)</th>
<th>$V_L$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water (kg)</td>
<td>7.3</td>
<td>8</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Cement (kg) (St-Laurent)</td>
<td>14.7</td>
<td>9.5</td>
<td>4400</td>
</tr>
<tr>
<td></td>
<td>Sand (kg)</td>
<td>34.7</td>
<td>48.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Aggregate (kg)</td>
<td>43.7</td>
<td>43.6</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 5.1 COMPOSITION OF THE CONCRETES
5.2.3 Form

Generally, in concrete structures, there are many global types of problems. In this work three main problems are fixed with sonic tomography: a degraded zone, a poor zone in middle of the structure, and fissuring. In this case, to present each problem in concrete structure, three different models are made.

The first model (model I) simulates a superficial deterioration of concrete. 62.5% of the volume of this bloc is held by type A concrete (25×40×40 cm³), and 37.5% of this volume is held by a type B concrete (15×40×40 cm³) (figure 5.1).

![Figure 5.1 Bloc model I](image)

Model II simulates a volumetric discontinuity. This pattern represents a cube of type A concrete in the center of which there is a material of low density (expanded polystyrene) with the dimension 30×30×4 cm³ (figure 5.2).
Model III represents the concrete characterized by one fissure clearing on the surface. The concrete of type A was used at first for the making of a cube. After one month of ripening, one vertical fissure is found in the median plan of this cube with a depth of 10 cm, and a thickness of 0.5 cm (figure 5.3).
5.2.4 Effect of coupling modes

The coupling between the structure and the sensor is a delicate problem in the domain of the auscultation by the acoustic techniques. Its quality governs the proportion of the vibration energy transmitted to the measurement instrument (the sensor) and consequently the clarity of the detected signals and the facility of their ulterior treatment. In this case, where the agent of coupling is inappropriate or wrong applied, the incidental wave acoustics suffer a bigger reflection to interfacing structure-coupling-sensor. The results is an increase of the signal length and the change of its amplitude.

In metallurgy, for example, the solution of coupling problem is to place the piece in the basin of water or oil. The emitters and the receivers are not in direct contact with the piece because the vibrations can pass through to the liquid medium. In civil engineering, this solution is inapplicable and the coupling is developed with the help of silicone grease or resin.

As to identify the most adequate coupling mode, testing and comparing were carried out between 3 modes of coupling that are shown in the figure 5.4:

- modes 1: structure- araldite- metallic washer- araldite-sensor. (This modes of coupling are generally used when the surface of the structure is degraded.)

- modes 2: structure- araldite- sensor.

- modes 3: structure- grease of silicone- sensor. (This mode of coupling is used when the measurement is carried out by soniscope.)
Three identical sensors were fixed on the center of the surface of the bloc model I (in strong zone) and for generating of the acoustic wave, a metallic hammer was used. Figure 5.4 (b) shows the result of this test. The signal number 1 corresponds to the answer of the sensor placed in the level of the impact point, the signals 2, 3 and 4 corresponds to the modes of coupling 1, 2 and 3 respectively.

The figure 5.4(b) reveals than the araldite (solid material) is a better agent of coupling than the grease of silicone. Because the amplitude of signals number 2 and 3 are bigger than the amplitude of signal number 4, it means that there is more energy that is received by the sensors with araldite than the energy that is received by the sensors with grease. Elsewhere, the modes of the coupling 1 (araldite and metallic washer) prove more efficient than the modes 2 (only araldite). The metallic washer improves the impedance acoustics of coupling. It attenuates the contrast of acoustic impedance between the structure and the sensor and insures a better energy transfer.

However for measurement of travel time there is no important difference between modes 1, 2 and 3.
Figure 5.4(b) Signals of the coupling test
5.3. **The experimental procedure**

As explained in chapter 2, the objective of this test is the verification of the influence of some parameters on the image tomography. We may classify these parameters in two categories:

- Measure parameters
- Reconstruction parameters

Table 5.2 shows the most important parameters of the two categories.

**TABLE 5.2 TOMOGRAPHY PARAMETERS**

| measurement parameters | - frequency  
|                        | - measurement step  
|                        | - measurement configuration  
| reconstruction parameters | - influence radius circle  
|                          | (influence zone)  
|                          | - pixel size  
|                          | - rays type |

5.3.1 **Measure parameters**

a) **Frequency**

Two measurement instruments were used for this test: low frequency (the accelerometer (Gagescope) system) and high frequency (soniscope system) (picture 5.1). The various components of these two systems are described in table 5.3.
-Soniscope (high frequency system):

Soniscope is an instrument that is designed to ultrasonically determine the condition and integrity of concrete. It can be used to detect and locate fissures or inclusions and can measure the thickness of the concrete structure. It is a signal acquisition apparatus that is supplied with two transducers piezoelectric (transmit and receive transducers) with a resonance frequency of 50 kHz which the strength of emission of the emitter is variable (0.5 kV, 1 kV, 2 kV and 4 kV). In this system the receiver gain can be made in one decibel steps from 5 dB to 120 dB. Also, the signal averaging function can help stabilize a received signal as well as eliminate non-periodic noise from it.

-Accelerometer (low frequency system)

Accelerometer is a measurement instrument that can be used in many domains. In sonic tomography this system may be used for acquisition and stoking data. When doing these tests, the accelerometer is controlled by Gagescope software package which allows multiple boards to operate together to perform simultaneous data acquisition on up to 32 different channels at rates as high as 50 MSPS (million samples per second). For this experiment, a 6 channel data-acquisition system with a high sampling frequency which can reach 10 MHz for each channel is used. In condition of the present application, the signals were acquired on all 6 channels simultaneously at a sampling frequency from 1 Hz to 10 MHz and it is capable to amplify up to 100 times.

During the tests with the Gagescope, the sensors were low frequency (1 kHz - 15 kHz). In general, we use four types of emission sources (see table 5.3); however, for this experiment, the first type of source is used (impact with the steel marble of 2 mm in diameter by a compressed air gun), with the frequency of 10 kHz (pictures 5.1 and 5.2).
TABLE 5.3. SPECIFICATION OF THE MEASUREMENT SYSTEMS

<table>
<thead>
<tr>
<th>System:</th>
<th>LOW FREQUENCY</th>
<th>HIGH FREQUENCY</th>
</tr>
</thead>
</table>
| **Source:** | - The impact of a marble of 2 mm in diameter is shot by a compressed air gun (10 kHz).  
- The impact of a marble of 8 mm in diameter is shot by a compressed spring (6 kHz).  
- Hammer (2 kHz).  
- Detonator (20 kHz) | Piezoelectric transducer (frequency of 50 kHz) |
| **Sensor:** | Accelerometer B&K (28 kHz) | Piezoelectric sensor with resonant of 50 kHz |
| **Filtering:** | Pass-low, 1 Hz - 15 kHz | Filtering by averaging of the signals |
| **Number of channels:** | 6 | 1 |
| **Sampling Frequency:** | 10 MHz/channel | 1 MHz |

b) Measurement step (m.s.)

The second important parameter in tomography measurement is the density of the measurement (or number of the measurement). For this test two measurement steps have been chosen. For the first step, the distance between of each sensor is 5 cm (m.s. is 5 cm) that is to say, there are 7 sensors on each sides (figure 5.5). In the second step, the density of the measurement was reduced, it means that the measurement step (m.s.) is 10 cm (figure 5.6).
Picture 5.1. The measurement instruments

Picture 5.2. The type of sources
Picture 5.3. Accelerometer system

Picture 5.4. Installation of the sensors on the bloc
c) **measurement configuration (m.c.)**

The third parameter that has been verified for these models is the measurement configuration (m.c.). In this case, the data collected in a sonic tomography problem are inherently incomplete, since the sides of the object ausculted do not completely surround the media under study [5]. As explained, there are 7 sensor positions on each side (14 source positions and 14 receiver positions). The first stage consists of all the measured rays (complete configuration), it means 196 seismic rays were measured. Figures 5.5 and 5.6 show the measurement configuration for the models tested. Afterwards, for all of the tests in volume, the sources on the one side have been eliminated and only three sides of the blocks have been used for getting the signals (partial configuration), it means 98 seismic rays were measured. This type of measurement configuration is shown in figure 5.7. As shown in the figures for each step, there are two types of configurations. The first type is the measurement configuration for model I and II and the second type is that the concentration of the measure is on the fissuring zone for model III.
Figure 5.5 Complete measurement configuration of the models (for m.s. = 5 cm)

a) model I and II, b) model III
Figure 5.6 Complete measurement configuration of the models (for m.s. = 10 cm)

a) model I and II, b) model III
Figure 5.7 Partial measurement configuration of the models

a) model I and II, b) model III
5.3.2 Reconstruction parameters

a) *Pixel size (p.s.)*

The size of the pixels used in the inversion in fact determines the limiting size of the velocity feature that can be recovered (chapter 4). A trade-off results of the fact that the solution becomes unstable with decreasing pixel size. Therefore, the optimal pixel size is small enough to cover the desired velocity features with the best possible resolution. Of course, the design of the experiment plays a large role in the trade-off, and the pixel size may be reduced with a greater number of rays. In these tests, to verify the results, the chosen pixel sizes are 1x1 cm², 2.5x2.5 cm², 5x5 cm², 10x10 cm² and 15x15 cm².

b) *Influence radius circle (i.r..)*

As explained in chapter 4, this parameter is the original reconstruction method followed by Côte [11]. This disc selects whole rays intercepting themselves. For calculating the velocity of the point of the node (center of the disc). The radius of this disc enables the possibility to increase or decrease the number of rays which intervene in the calculation of the velocity of the node. Choosing this radius depends on the pixel size measurement configuration, measurement step and ausculted zone. However, if we choose a radius too large, may be, the false information, for the node, will participate in the calculating the velocity. If the chosen radius is too small, may be, we will miss some information. The influence radiuses that were chosen are 5 cm and 10 cm.

c) *Ray type*

Another parameter which intervenes in inversion progress is the type of ray which expresses the mode of wave propagation. When the media is heterogeneous, the propagation between source and receiver develop indirectly and the rays are curved (curved ray). However in the homogeneous media, the wave propagation is direct (straight ray). Therefore, the type of ray must adopt a real
wave propagation in media. From the limited results given, the authors have tentatively concluded that velocity contrasts of 16 percent or less can be successfully interpreted using a multitude of tomography transmission paths and the straight-line data-interpretation techniques [7]. For verifying this parameter, the test was performed with both of the rays (curved ray and straight ray). The synthesis of the results analysis is shown in Table 5.4.

**Table 5.4. Synthesis of the experimental procedure**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Measurement or Calculation Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>50 kHz 10 kHz</td>
</tr>
<tr>
<td>measurement step (m.s.)</td>
<td>5 cm 10 cm</td>
</tr>
<tr>
<td>measurement configuration (m.c.)</td>
<td>complete partial</td>
</tr>
<tr>
<td>influence radius (i.r.)</td>
<td>5 cm 10 cm</td>
</tr>
<tr>
<td>pixel size (p.s.)</td>
<td>1x1 cm² 2.5x2.5 cm² 5x5 cm² 10x10 cm² 15x15 cm²</td>
</tr>
<tr>
<td>ray type</td>
<td>straight curved</td>
</tr>
</tbody>
</table>

5.4 **Results analysis**

The tomographic images of the models tested are presented in Appendix I.

5.4.1 **Pixel size (p.s.)**

a) **Model I**

Generally, in this model there are two completely different zones (figure 5.1), the weak zone (V ≤ 2500 m/s) and the strong zone (V ≥ 4000 m/s). With this method, we are not able to present a
weak zone exactly near a strong zone; thus, we need a transfer zone, and we know, in reality, this zone does not exist. Therefore, the best image has two particularities: firstly, it must show the weak zone and strong zone clearly, and secondly, it must have the smallest transfer zone.

The images of model I are illustrated in figures A1.1 to A1.18. In the images with small pixel sizes (p.s.= 1 cm and 2.5 cm), the limits of each zone are in the form of broken lines which are not realistic, but for p.s.= 5 cm (figures A1.14, A1.13 and A1.10) the images are nearer to reality and the images with p.s.= 10 cm present the best images among all of the images that we illustrated for this model (figures A1.4 and A1.5). These images approximately satisfy the two conditions which are mentioned above.

Figures A1.3, A1.4, A1.5, and A1.6 show us the image tomography of the model I with different pixel sizes. There is not a great difference between figures A1.3, A1.4 and A1.5 however, figure A1.6 is completely different (the form of each zone), specially in weak and transfer zones, and we can conclude that there is a limit in choosing the size of pixels.

b) model II

In several figures of this model, the weak zone which shows the polystyrene part in the middle of the image is seen. However in p.s. = 1 and 2.5 cm (figures A1.19, A1.24, A1.34 and A1.36) this zone is realistic and we can see the zone with velocity of 3000-3500 m/s, but other zones that show the concrete part are not homogeneous enough and have small spots. The reason for these spots is due to their very small pixel sizes: when the size of pixels is reduced, the number of the rays which pass in the pixels is reduced, resulting in a lower resolution for each pixel. For p.s.= 5 and 10 and 15 cm (figures A1.20, A1.21 and A1.23) in the concrete zone there is not the same problem, but the central zone (polystyrene part) is not clear and sometimes there is only one zone with velocity of 3500-4000 m/s and the zone with velocity of 3000-3500 m/s does not exist.
c) model III

However, in model I and II, we saw the changes of the tomography image owing to change of the pixel sizes, but model III is a very good example to show the influence of the pixel size on the tomography image. The thickness of the fissure, in this model, is small (1 cm) and for presentation of that, we need the pixel sizes to be small enough. For this reason, in this model, when the pixel size is very small (1x1 cm$^2$) the weakest zone ($v \leq 2000$ m/s) is seen (figures A1.42 and A1.54), but when the pixel size increases, this zone is missed gradually (figure A1.44 and A1.46), so that in figures A1.46 and A1.56 (p.s. = 10 cm) the weak zone is completely changed.

Secondly, in figures A1.42 and A1.50 (pixel sizes are small) there is a strong zone ($v \geq 5000$ m/s) just near the weak zone, but in figure A1.44 and A1.56 that are treated by pixel sizes 5x5 cm$^2$ and 10x10 cm$^2$ this strong zone is not seen. In fact, this strong zone does not exist however existence of this zone can be observed during treatment of an error.

5.4.2 influence radius (i.r.)

a) model I

The images of this model show that, generally, the results of the influence radius of 10 cm are better than the results of the influence radius equals 5 cm. Because this model is formed of two homogeneous parts in homogeneous medium, the velocity of the rays is nearly the same and the influence radius sweeps the rays with the same information. Therefore when the influence radius become larger, the information of each point is more.

b) model II

For this model, unlike model I, the results with i.r. = 5 cm (figures A1.24, A1.31 and A1.34) are better than with i.r. = 10 cm (figures A1.22 and A1.26), because the thickness of the
polystyrene is not large (4 cm) and if we use i.r. = 10 cm, undoubtedly, it sweeps many rays that only pass through the concrete part, so there is much information that is not correct for the polystyrene part and we can’t have a factual image.

c) model III

In this model, we have a small case (discontinuity because of a fissuring) presented in the image tomography; therefore, it is theoretically better to limit the data that is swept by influence radius. When figure A1.42 (treated with i.r. = 5 cm) is compared with A1.45 (treated with i.r. = 10 cm), or A1.54 with A1.55, it is clear that figures with i.r. = 10 cm do not present the zone with v ≤ 2500 m/s. Also, the weak zone and transfer zone are too big. On the contrary, in the figures with i.r. = 5 cm, the zone with v ≤ 2000 m/s is seen and the weak and transfer zones are limited. However, in figures with i.r. = 5 cm, the length of the weakness (zone v ≤ 3000 m/s) is only 5 cm, which is unrealistic because the length of the fissure is 10 cm. We can explain this problem in the following manner: when the influence radius is too big for the pixels which embrace the fissure the influence radius circles may sweep the rays that pass through the fissured part and non fissured part. This problem is more important for the pixels that are at the bottom part of the fissure; therefore, the calculation is accomplished with the correct information (rays that pass through fissured part) and incorrect information (rays that pass through non fissured part).

5.4.3 ray type

a) model I

In section 5.3, it was shown that straight rays are used for homogenous media and curved rays are used for heterogeneous media. In model I there are two homogeneous media in each side, therefore straight rays should be better for it and the images of this model confirm that. It is clear that for this model the images treated by straight rays are better than the images treated by curved rays. For example, figures A1.10 and A1.13 were treated with the same conditions except ray tracing. Figure A1.13 was treated by straight rays and figure A1.10 by curved rays. It is clear that
figure A1.13 is realistic, both in the form of each zone (each zone is more homogeneous) and in the form of the transfer zone (the transfer zone is smaller).

b) model II

For the same reason that is explained above, the images of curved rays are more realistic than the images of straight rays for this model (i.e. figures A1.24, A1.25 and A1.36) because this model is the same as an heterogeneous medium and if the images of straight rays are compared with the images of curved rays, we can see that in the former, the central zone (weak zone) is not in the correct direction and that it is turned on an angle of almost 90°, due to the influence rayon of 5 cm.

c) model III

With the manner of wave propagation in the fissuring medium (that was explained in chapter 3), we may infer that the curve ray has more harmony with wave propagation in medium of this model. In the image treated by curve rays (figures A1.42 and A1.54) in position of the fissure, the weak zone (v ≤ 2000 m/s) is seen clearly, whereas in the image treated by straight rays, (figures A1.47 and A1.57) this zone is lost.

Secondly, in the images treated by curve rays (figures A1.42 and A1.44) the strong zone (v ≥ 5000 m/s) that is located exactly after the weak zone is smaller than the same zone in the image treated by straight rays (figure A1.47 and A1.48).

5.4.4 frequency

a) model I

As it was explained in section 5.3, all of the tests were done with a low frequency system (10 kHz) and a high frequency system (50 kHz). If we compare the images of these two systems, in general, the transfer zone in images of the 50 kHz is smaller than the same zone with the 10 kHz
(for example figures A1.11 and A1.2). For m.s. = 10 cm the images of 50 kHz are clearer than the images of 10 kHz (for example figures A1.18 and A1.9). Sometimes in the images of 50 kHz, there is no important difference between images with m.s. = 10 cm and m.s. = 5 cm (figure A1.18 and A1.14)

b) model II

For this model the global form of the images of 10 kHz and 50 kHz are the same, especially for the weak zone. For the strong zone (concrete part), the images of 50 kHz show a more homogeneous medium.

c) model III

For this model, in comparison of low frequency system (10 kHz) images with high frequency system (50 kHz), there are three important points: first, in images of 10 kHz, the medium (in non fissuring part) is more homogeneous than the medium presented by 50 kHz images (figures A1.42 and A1.54). Second, in images of 10 kHz, the most weak zone (v ≤ 2000 m/s) is thicker than the same zone in images of 50 kHz, but the thickness of the transfer zone (exactly after the weakest zone) in 10 kHz is bigger than the same zone in images of 50 kHz (figures A1.42, A1.47, A1.54 and A1.57). In model I, we had the same transfer zone result. Thirdly, the images that were simultaneously treated by the large pixel size and the influence radius (p.s. = 10 cm and i.r. = 10 cm) the images of both systems (low and high frequency systems) are the same and there is not an important difference between the two (figures A1.46 and A1.56).

5.4.5 measurement configuration (m.c.)

a) model I

Figures A1.15, A1.16 and A1.17 are examples of images that were illustrated with the three-side condition. The most important point in all of this model using images with three-side measured is the existence of the strong zone (V ≥ 4500 m/s). We can see this zone where the density of the
rays is weaker (near the side C-D). In addition, the influence of short rays is seen on the weak zone (near the side B-C) which makes is clear that this zone is stronger. In this model, we can be assured that the existence of these strong zones is attributable to the inversion algorithm. This means that if a part of a continuum medium has a poor measurement density, the inversion method can not be used to calculate accurately and that this part is illustrated with a higher than realistic velocity zone. This fact is to be considered since this problem is more obvious in the image that are measured by the low frequency system (e.g. figures A1.7, A1.8).

b) **model II**

This model is a good example for the influence of measurement configuration parameter on tomography image. Figures A1.27 to A1.30 are the results of this model measured by the low frequency system (10 kHz) with the three sides condition. Figures A1.37 and A1.38 have the same conditions, however they are measured by high frequency system (50 kHz). In the first series (Figures A1.27 to A1.30) the images are altered completely: there is no weak zone in the center and the strong zones near the B-C and C-D sides are still seen. However in the second series (figures A1.37 and A1.38) the changes are less visible, as there is a small weak zone and the velocity of the strong zone in B-C and C-D sides is between 4500 m/s and 5000 m/s, this image is more realistic.

c) **model III**

If figure A1.43 (complete configuration) is compared with figure A1.49 (partial configuration) and figure A1.45 with A1.50, the following important point is seen: there is no important difference in the mid-side up of the image between each series of images that are compared and on the contrary, in the mid-side down, a great contrast is observed. The cause of this problem is the changing of the measurement density. As explained in section 5.3, in model III, the positions of the emitters and receivers are different than those in models I and II (figures 5.5, 5.6, 5.7). Furthermore, the rays distribution in mid-side up of model III in two positions of measurement (complete and partial) are almost the same; therefore, the form of the images does not
change. However in mid-side down, the rays density of the partial configuration is much weaker than the rays density of the complete configuration. Also, the zone with \( V \geq 5000 \text{ m/s} \) is bigger where the measurement density is poorer.

5.4.6 measurement step (m.s.)

a) model I

As it is already explained in the frequency section for the condition of m.s. = 10 cm, the images of the sonoscope system (50 kHz) are better than the images of the accelerometer system (10 kHz). The most important point about this parameter is to protect the global form of the zones with m.s. = 10 cm and m.s. = 5 cm, are the same; however, in the images of m.s. = 10 cm some details that exist in m.s. = 5 cm are missing. For example, we can see this matter by comparing the figures A1.14 and A1.18, in addition to A1.4 and A1.9.

b) model II

The images of this model which are measured by m.s. = 10 cm were illustrated in figures A1.31, A1.32, A1.33, A1.35, A1.39 and A1.41. These images can confirm the results of model I. In other words, if the images measured by m.s. = 10 cm are compared with the same condition except those measured by m.s. = 5 cm (A1.31 with A1.19, A1.32 with A1.20, A1.33 with A1.25) we can easily conclude that on the images measured by m.s. = 10 cm some details are missed and that these results are logical; because in m.s. = 10 cm the half of the data (rays) is omitted, resulting in the decreased resolution of the images. Otherwise, because the density of the rays is decreased homogeneously, the global form of the image is protected.

c) model III

The comparison of the images of m.s. = 5 cm with the images of m.s. = 10 cm (figures A1.43 with A1.51 and A1.54 with A1.58), demonstrates two points: firstly, in the image measured by m.s. = 5 cm, the density of the rays is greater; therefore, the thickness of the weaker zone
(v≤2000m/s) is limited. The thickness of m.s.= 5 cm is smaller than the same thickness found in images of m.s.= 10 cm because the density of the rays in images of m.s.= 10 cm is poorer. Consequently, the information is not sufficient to illustrate an exact image. Secondly, whenever the pixel size increases, changes in the images with small pixel sizes occur less (figures A1.46, A1.52, A1.57, A1.53, and A1.48).

5.4 Conclusion

In this chapter many parameters and applied conditions of the sonic tomography were verified. The results of this study are very interesting to use of this method for scanning the concrete structures. Hence, sometimes these parameters could completely change the tomographic image, therefore, it is always necessary to choose the best parameters. This method is limited to two kinds of conditions: the limitation of the algorithm method and the measurement conditions. The algorithm limitations are the limitations of the inversion theory used for the reconstruction of the images. For example, the existence of the unreal transfer zone (the images of model I). The measurement conditions are the limitations and conditions of the tomographic parameters. Sometimes, it is not possible to eliminate the limited problems, however, it is possible to improve the tomographic images by the choice of the suitable measurement parameters. The suitable parameters are not easy to be chosen. This choice is depending on the several conditions such as the geometry of the structure and the objective of the survey. However, some points have to be taken into consideration for choosing these conditions. In this case, the important points are explained as follows: i) for the homogeneous media it is better to consider the pixel size and influence radius to be sufficiently big and the type of ray which is used for these media could be straight-ray, ii) in contrast, for heterogeneous media the pixel size has to be small enough for presenting the discontinuities, on condition that the resolution is acceptable. The curved-ray type is suitable for these media and it is better to decrease influence radius as much as it is possible.
Generally, the results of the tomographic images of high frequency are better than those images of the low frequency. However, the use of sonoscope (frequency of 50 kHz) for measurement in-situ is not easy. The density measurement of the rays is the very important parameter before having been verified. This density measurement should be uniform and sufficient. For this, it is better to install the sensors in the symmetric form and around the structures sides. Table 5.5 shows the best conditions to improve of the tomographic image for models tested in this work.
### TABLE 5.5. CONDITIONS OF THE BEST IMAGES FOR MODELS TESTED

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>THE BEST CONDITION FOR:</th>
<th>GENERAL ADVISOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>MODEL I</strong></td>
<td><strong>MODEL II</strong></td>
</tr>
<tr>
<td>Pixel size</td>
<td>10 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>Influence radius</td>
<td>10 cm</td>
<td>5 cm</td>
</tr>
<tr>
<td>Ray type</td>
<td>straight</td>
<td>curved</td>
</tr>
<tr>
<td>Frequency</td>
<td>50 kHz</td>
<td>50 kHz</td>
</tr>
<tr>
<td>Measurement configuration</td>
<td>complete</td>
<td>complete</td>
</tr>
<tr>
<td>Measurement step</td>
<td>5 cm</td>
<td>5 cm</td>
</tr>
</tbody>
</table>
6.1 Introduction

Sonic tomography tests were used for the auscultation of concrete in Italy (Bertochi and al. (1991) [34]; ENEL-DPT and al. (1991) [35]), England (Smith and Dyer (1990) [36]) and Canada (Kharrat and al. (1995)[13]).

Rapidity, easiness and economy are three important advantages of the sonic tomography test on the field. If the conditions of measurement (chapter 5) are satisfied, this method could be used for any concrete structure.

Chapter 5 explained the results of the laboratory test. The following step in this research is to use these results to improve the tomographic image for the real structure (in service). One of the best example for demonstrating this is a hydraulic dam. For the first example, a hydraulic dam in service was auscultated by this method and for the second example a fissured slab concrete was chosen. This slab is constructed for a grouting test; therefore, it is possible to control the grouting operation by sonic tomography.

6.2 Diagnostic of a pillar

6.2.1 Specification of the pillar

This pillar is the first drainage of the Hemmings hydroelectric central fall in Drummondville.

This pillar has the form of a gravity wall of 19 m in height, 3 m in thickness and 17 m in width at the bottom (figure 6.1 and picture 6.1).
It was constructed in 1920 and was repaired a few years ago. The downstream face of this pillar was repaired due to a superficial degradation of the concrete as a result of the hard climatic conditions. It has been proven that durability of these repairs is limited. Indeed, the scaling of the concrete has appeared on certain pillars (picture 6.2); for the others pillars, it is the quality of the contact between the new concrete and the structure which is weakened. The surveyed pillar does not suffer from scaling problem and the state of the concrete surface is globally plane except for a restricted zone on top of the inclined surface (picture 6.3). Therefore, the auscultation was fixed as an objective of diagnosing the intern state of the pillar, and, as a second objective, testing the faculty of the technique to differentiate the repaired concrete from the old concrete. The boundary
between these two concretes is visible (picture 6.4). The contact between these two concretes was confirmed by the five drillings done some weeks after the auscultation. These drillings, perpendicular to the face endorsement of the pillar, are located between the elevation of 4 m and 14 m in relation to the top of the pillar. The repaired concrete is 0.25 m to 0.4 m thick and the maximum aggregate size of the new concrete is 20 mm whereas for the old concrete is 50 mm in diameter. The drilling indicates the presence of steel reinforcement of 20 mm in diameter.

The result of the laboratory tests on the concrete sample for measuring the compressive strength, elastic modulus, and sonic velocity test are shown in table 6.1. These results show that the old concrete has a better compressive strength than the new concrete.

### TABLE 6.1 MECHANICAL TEST RESULTS OF THE PILLAR

<table>
<thead>
<tr>
<th>DRILLING</th>
<th>POSITION (m)</th>
<th>LOCALIZATION OF THE CORES* (m)</th>
<th>E (GPa)</th>
<th>f'_c (MPa)</th>
<th>V (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>0 - 0.40</td>
<td>14.4</td>
<td>18</td>
<td>3980</td>
</tr>
<tr>
<td>B</td>
<td>7</td>
<td>0.27 - 0.55</td>
<td>24.6</td>
<td>35.6</td>
<td>3980</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.55 - 0.86</td>
<td>23.5</td>
<td>42.3</td>
<td>4700</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>0 - 0.33</td>
<td>19.6</td>
<td>25.7</td>
<td>4300</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.43 - 0.73</td>
<td>18.4</td>
<td>34.0</td>
<td>5000</td>
</tr>
<tr>
<td>D</td>
<td>14</td>
<td>0 - 0.40</td>
<td>21.6</td>
<td>26.37</td>
<td>4300</td>
</tr>
</tbody>
</table>

* Location of the cores in the drilling

6.2.2 Field measurement

a) **Measurement pattern**

A vertical section of the pillar was chosen to perform sonic measurement (figure 6.1).

An existing water borehole along the upstream face was used to lower the source which we moved at 1 m intervals along the depth of concrete and the position of each source is the
downstream side (picture 6.5). This procedure was repeated along a crossing path with different elevations of receivers. In this way, 225 seismic rays passed through the surveyed section in every direction, as shown in figure 6.2(a). Figure 6.2(b) shows some signals that were saved between the sources and receivers. The quality of these signals shows the global quality of the concrete interior.

b) Equipment

The equipment used for this test contains the accelerometer system with six channel data-acquisitions and a high sampling frequency which can reach 10 MHz for each channel. The receiving devices were B&K Deltatron accelerometers with a resonant frequency of 28 kHz. The signals from these sensors were routed to the 6-channel data-acquisition system. The main task of the first sensor placed close to each transmitting point is to trigger data acquisition on the 5 other channels. Acoustic waves were generated by detonators.

6.2.3 Image reconstruction

Chapters 4 and 5 showed that the rays can be determined for most reasonable velocity models and source-receiver configurations. At least three parameters must be evaluated: the type of data being considered, the geometry of the model, including the source-receiver locations and the velocity heterogeneity.

An effort should be made to determine a pixel size small enough to recover the velocity features at the optimal resolution of the problem, i.e., with the best possible resolution. Therefore, a pixel size of 50 cm × 50 cm seems to be the optimal size and the influence zone that was chosen is 50 cm.

Positions of the source-receiver (figure 6.2) show that the most of the rays deviate much from the horizontal line, in addition, the medium of this pillar is heterogeneous, therefore it is better to use the curved-ray for image reconstruction.
6.2.4 Image analysis

Figure 6.3 shows the final tomographic image of the chosen section of this pillar. The scale of wave velocity covers the band of 2000-4500 m/s. The mechanic quality of concrete is rated good and excellent \((V > 3650 \text{ m/s})\) at the basis of the pillar, and rated poor and doubtful \((2000 \leq V \leq 3650 \text{ m/s})\) at top of the pillar.

The first zone demonstrates the superficial damaging of concrete (picture 6.3). This damage has negative effects which stretch in depth to the second zone which is situated on the same level of the water in the reservoir (in winter time, the reservoir lowered of 2 mm). Therefore, it is certainly associated to seasonal variation effects of the marling zone, particularly with the frost and thaw of the water.

The boundary between the repaired concrete and the old concrete is not clearly identified on the tomographic image. This problem could be attributable to a low density of rays going through the interface and could indicate a good quality contact which is in agreement with the verification during the drilling. Moreover, the results of the laboratory test also concord with the tomographic results concerning the face endorsement of the pillar: the quality of the repaired concrete is not as good as that of the old concrete.
Figure 6.2.(a) Density measurement of the pillar
Figure 6.2(b) Example of the signals of figure 6.2(a)
Picture 6.1. Hemmings central fall

Picture 6.2. Scaling of the new concrete
Picture 6.3. damaged zone of the pillar

Picture 6.4. The boundary between new and old concertes
Picture 6.5. Position of the receivers on the pillar
Figure 6.3:

Tomographic image of the pillar

drilling

19 m

0

17 m

frequency: 10 kHz
m.s.: 100 cm
i.r.: 50 cm
p.s.: 50 cm
ray: curved

V-m/s
4500.
3666.
2833.
2000.
6.3 Monitoring grouting of a slab concrete

This section describes the ability of the sonic tomography method to assess the quantity of improvement after rehabilitation by injecting the grout in a fissured slab.

6.3.1 Specification of the slab

This slab was constructed according to these dimensions: 264×142×40 cm. To obtain a plane and horizontal fissure in the middle of the slab, it was formed of three layers of concrete. The upper and lower layers are made of reinforced concrete which has a compressive strength of 30 MPa. The steel of reinforcement has 25 mm in diameter and the distance between the bars is 15 cm. The central layer has a weakened compressive strength. It is made of the non reinforced concrete with a compression resistance of 15 MPa. To weaken this layer, a fissure initiator made of fiber cardboard with a thin layer (6 mm) of coarse sand was installed in its middle. Moreover, a PVC waterproof sheet is installed around 12 cm from the edges of the slab (figure 6.4).

After many months, this slab was fractured by the pressure of water (1300 kPa). In this step, one plane fissure was made up in middle of the slab. Figure 6.5 shows the approximate position of the fissure.

6.3.2 Measurement

Tomographic measurement was carried out before grouting of slab as well after grouting in order to determine how the concrete slab has been rehabilitated. By comparing the results of before grouting with those of after grouting one can see the interior changes of the slab.

a) Measurement pattern

To obtain suitable coverage of the portion of the slab that had been grouted, tomographic measurement were taken of different cross-section of the slab. Figure 6.4 shows the position of
these sections. The sonic measurements were taken in each cross-section both before and after the grouting in order to assess the changes that grouting had affected in the slab.

The important point that is necessary to explain is the measurement configuration. For section 1 there was no problem for measurement configuration and the sensors were installed on the four sides. For this section 15 source positions and 31 receiver positions were chosen, and 350 seismic rays were measured in all directions. Figure 6.5(a) and 6.6(a) show the position of the sensors and the density of the rays. For section 2 there is a limitation because the bottom of the slab was not available. Therefore, the sensors could be installed only on three sides and the density of the rays is poorer than in section 1 (155 rays). Figures 6.5(b) and 6.6(b) show the position of the sensors and density of the rays for this section.

b) Equipment

The equipment of this test was the accelerometer with 10 channels data-acquisition of 2 MHz sampling frequency. The receivers were B&k Detlatron accelerometer with a resonance frequency of 28 kHz. The main task of the first sensor placed close to each transmitting point is to trigger data acquisition on the nine other channels. For generating the acoustic wave, the air gun with 2 mm marble (chapter 5) was used.

6.3.3 Image reconstruction

As explained in chapter 5, the reconstruction parameters have to be chosen with due attention to the dimensions and conditions of the structure survey. A pixel size of 10 cm x 10 cm was used with regard to the frequency of the seismic pulse and the number of ray path. This slab is not homogeneous, therefore, it is better to use the curve ray path and the radius influence as big as the pixel size (10 cm). The image obtained did, however, allow us to observe variations along the vertical and horizontal axes of the section 1 and thus to assess the overall variations in the internal quality of medium, as well as the internal changes brought about by grouting.
6.3.4 Image analysis

a) Section 1:

The position of this section is illustrated in figure 6.4; its tomographic image is illustrated in figures 6.7 (before grouting) and 6.8 (after grouting). Figure 6.7 shows that the velocity of most stress waves varies from 2500 to 4000 m/s. In the middle of the slab, an horizontal weak zone is seen. The existence of this weak zone is not unexpected because as explained in section 6.2, this zone was constructed poorly, just for the grouting test, and figures 6.7 shows the poor zone clearly. Figure 6.8 shows this slab after grouting. This medium of the slab is homogenous, which means that the grouting was done completely.

To get a better idea of the changes that have resulted from grouting the slab, the distribution of the percentage of velocity increase is illustrated in figure 6.9. The information provided in this figure is very important because the differences observed in the tomographic images depend on the interval selected between pairs of velocity isolation. Generally, a difference of more than 10% would be very significant [31]. This chart gives a better idea about the order of magnitude of the changes; the majority volume is between 40% and 80% in the poor zone.

b) Section 2:

The results of this section were illustrated in figures 6.10, 6.11 and 6.12. Figure 6.10 shows the tomographic image of this section, before grouting. In the middle of this image, a strength zone \((v \geq 4000 \text{ m/s})\) is seen. The existence of this zone is not normal, because we are sure that there are the poor part and the horizontal fissure in middle of the slab. Therefore this image is not real. In chapter 5, in the images of the laboratory models, the same problem was seen where the ray density of the one part was too poor. Figure 6.6 (b) shows the density measurement of this section. It is clear that in the middle, the density measurement is poorer than the sides and that this density is also too poor to illustrate an acceptable image. There is exactly the same problem, for this section, after grouting (figure 6.11). However, for this section, the intensity of this problem is less,
because after grouting the medium is more homogeneous and the velocity volume distribution is very small, meaning that it is not worthy of attention to consider differences between the value of the ray-velocities. Therefore the information of each nod collected by influence radiuses is approximately the same.

In each side (about 40 cm of each side) the density of the rays is enough (figure 6.6 (b)). Therefore, the zones that are illustrated in figure 6.10 and 6.11 are nearer to reality. This part is weak before grouting (velocity varies from less than 2000 to 3500 m/s); however, after grouting it is stronger (velocity varies from less than 3500 to 4000 m/s).

Figure 6.12 shows the percentage of the velocity increase. This image is able to show the changes of the parts that have acceptable results. As shown in these figures, the percentage of vertical increase does not exceed 20% in centre of the slab, and 60% to 80% in the sides; however, the result of the central part is not acceptable.

c) **Signal quality:**

The comparison of the signals, before and after grouting is a good reference to verify the result of the tomographic image of fissuring medium. When the wave passes through a fissured medium, both the energy and the frequency of the received signals change. This element may help us to verify the grouting test. Figure 6.13 shows two signal examples. Signal 1 is the signal between E1 and R10 before grouting and Signal 2 shows the same position after grouting. Signals 3 and 4 are between E27 and R10 before and after grouting. In the both examples, the quality of the signals, after grouting is improved (the travel times are decreased and the amplitudes are creased).
Figure 6.4:
The horizontal section of the slab
Figure 6.5:
Vertical sections of the slab and source-receiver locations a) section 1, b) section 2
figure 6.6:
Density measurement of the slab a) section 1, b) section 2
Figure 6.7:

Tomographic image of the slab

Section 1 - before grouting

frequency: 10 kHz  m.c.: complete  m.s.: 10 cm  i.r.: 10 cm  p.s.: 10 cm  ray: curved

V-m/S

5000.
4500.
4000.
3500.
3000.
2500.
2000.
Figure 6.8:
Tomographic image of the slab
Section 1 - after grouting

frequency: 10 kHz  m.c.: complete  m.s.: 10 cm  i.r.: 10 cm  p.s.: 10 cm  ray: curved

V-m/s
5000.
4500.
4000.
3500.
3000.
2500.
2000.
Figure 6.9:
Percentage of the velocity increase

Section 1

%  
100.0  
80.00  
60.00  
40.00  
20.00  
0.00
Figure 6.10:

Tomographic image of the slab

Section 2 - before grouting

frequency: 10 kHz  m.c.: complete  m.s.: 10 cm  i.r.: 10 cm  p.s.: 10 cm  ray: curved
Figure 6.11:
Tomographic image of the slab
Section 2 - after grouting

frequency: 10 kHz  m.c.: complete  m.s.: 10 cm  i.r.: 10 cm  p.s.: 10 cm  ray: curved
Figure 6.12:

Percentage of the velocity increase

Section 2
Figure 6.13 Example of the signals (before and after grouting)
Picture 6.6. Slab concrete (before grouting)

Picture 6.7. Fissure of the slab and location of the receivers
Picture 6.8. Vertical section of the slab after cutting

Picture 6.9. Horizontal section of the slab after cutting (in position of the fissure)
CHAPTER 7

CONCLUSIONS

In this work, a number of problems related to the tomographic imaging have been addressed. In this study the principal theory of the sonic tomography was discussed and laboratory studies were designed to verify the capacities and limits of this method. Generally, these problems were discussed theoretically [17, 16]; however, these problems were verified practically in concrete structures and their results were interesting. In general, this method is one of the best nondestructive tests for scanning concrete structures. A variety of different experiments were undertaken in the course of this work. These experiments all involved the tomographic imaging of concrete structures in the laboratory and that was also utilized in a real structures in-situ. These studies were successful in demonstrating the potential of tomographic techniques to verify interior of concrete structures.

With this method the global information of the interior of the concrete structures can be given, however, this is limited by measurement and reconstruction parameters. For illustrating the best tomographic image in field: 1) the acquisition frequency of the wave has to be chosen so that the waves can pass through the medium easily without reflection or refraction, 2) the density measurement has to cover all of the medium uniformly, 3) the raypath should be the smallest possible. Furthermore, for reconstruction of the image: 1) the pixel size has to be small enough to take a maximum resolution and be big enough to cover a maximum the uniform information, 2) for the heterogeneous media, it is better to use the curved-ray and for the homogeneous media to use the straight-ray, 3) the influence zone should be able to sweep maximum uniform information therefore; its size has to be chosen carefully.
This original method was developed for medical and geophysical applications. For both fields, there is a harmony between dimension of the medium scanned and the object or problem that is studied. However, in Civil Engineering and especially in the tomography of concrete structures, the medium which is usually scanned is big whereas the object of study may be so small (a fissure) for instance that it might be impossible for the medium osculated and the problem studied to compatibly co-exist. Therefore this method can not be used to present all the details.

The extension of this work can take two directions:

1) To improve the information of the signal saved, as explained in chapter 1 the only parameter of the wave that is used in this algorithm is arrival time of the P-wave (travel time); however, as explained in chapter 3 and section 6-3 the propagation of the stress wave in the elastic medium has many properties that can be analyzed. These properties may be analyzed with the amplitude of the waveform and may be parametrized by an intrinsic quality factor. In addition, filtered data may be studied to determine the frequency dependence of the quality factor. This frequency dependence is an important indicator of the actual dissipation mechanisms and serves to further illuminate of the concrete properties. The S-wave was successfully used in cross hole tomography [32] and it is possible to use the S-wave to improve the process of wave analysis.

2) To make use of this methodology with the other reconstruction algorithms. Chapter 4 illustrated that theoretically, SIRT is a strong algorithm for inversion in image processing. Chapter 5 goes on to prove that practically, this algorithm has a capability to reconstruct the image; however, this algorithm is limited by its parameters. Therefore, it is necessary to combine the methodology and the tests which were presented in this study with the use of other algorithms like LSQRT or ART and to compare these results with results of the SIRT algorithm.
REFERENCES


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APPENDIX I

TOMOGRAPHIC IMAGES OF THE BLOCS
Figure A1.1 Tomographic image
MODEL I

- Frequency: 10 kHz
- M.c.: complete
- M.s.: 5 cm
- I. r.: 10 cm
- P.s.: 1 cm
- Ray: curved

Figure A1.2 Tomographic image
MODEL I

- Frequency: 10 kHz
- M.c.: complete
- M.s.: 5 cm
- I. r.: 10 cm
- P.s.: 10 cm
- Ray: curved
Figure A1.3 Tomographic image
MODEL I

V-m/S

- 5000
- 4500
- 4000
- 3500
- 3000
- 2500
- 2000

Figure A1.4 Tomographic image
MODEL I

V-m/S

- 5000
- 4500
- 4000
- 3500
- 3000
- 2500
- 2000
Figure A1.5 Tomographic image
MODEL I

Figure A1.6 Tomographic image
MODEL I
Figure A1.7 Tomographic image
MODEL I

Figure A1.8 Tomographic image
MODEL I
poor concrete

Figure A1.9 Tomographic image
MODEL I

frequency: 10 kHz
m.c.: complete
m.s.: 10 cm
i. r.: 10 cm
p.s.: 10 cm
ray: straight

poor concrete
good concrete

5000
4500
4000
3500
3000
2500
2000

Figure A1.10 Tomographic image
MODEL I

frequency: 50 kHz
m.c.: complete
m.s.: 5 cm
i. r.: 10 cm
p.s.: 5 cm
ray: curved
Figure A1.11 Tomographic image
MODEL I

Figure A1.12 Tomographic image
MODEL I
Figure A1.13 Tomographic image
MODEL I

Figure A1.14 Tomographic image
MODEL I
Figure A1.15 Tomographic image
MODEL I

frequency: 50 kHz
m.c.: partial
m.s.: 5 cm
i. r.: 5 cm
p.s.: 5 cm
ray: curved

Figure A1.16 Tomographic image
MODEL I

frequency: 50 kHz
m.c.: partial
m.s.: 5 cm
i. r.: 10 cm
p.s.: 5 cm
ray: curved
Figure A1.17 Tomographic image
MODEL I

Figure A1.18 Tomographic image
MODEL I
Figure A1.19 Tomographic image
MODEL II

Figure A1.20 Tomographic image
MODEL II
Figure A1.21 Tomographic image
MODEL II

Figure A1.22 Tomographic image
MODEL II
Figure A1.23 Tomographic image
MODEL II

frequency: 10 kHz
m.c.: complete
m.s.: 5 cm
i. r.: 5 cm
p.s.: 15 cm
ray: curved

Figure A1.24 Tomographic image
MODEL II

frequency: 10 kHz
m.c.: complete
m.s.: 5 cm
i. r.: 5 cm
p.s.: 1 cm
ray: straight
Figure A1.27 Tomographic image

MODEL II

frequency: 10 kHz
m.c.: partial
m.s.: 5 cm
i. r.: 5 cm
p.s.: 1 cm
ray: curved

Figure A1.28 Tomographic image

MODEL II

frequency: 10 kHz
m.c.: partial
m.s.: 5 cm
i. r.: 5 cm
p.s.: 5 cm
ray: curved
Figure A1.29 Tomographic image
MODEL II

Figure A1.30 Tomographic image
MODEL II
Figure A1.31 Tomographic image
MODEL II

- frequency: 10 kHz
- m.c.: complete
- m.s.: 10 cm
- i. r.: 5 cm
- p.s.: 1 cm
- ray: curved

Figure A1.32 Tomographic image
MODEL II

- frequency: 10 kHz
- m.c.: complete
- m.s.: 10 cm
- i. r.: 5 cm
- p.s.: 5 cm
- ray: curved
Figure A1.33 Tomographic image
MODEL II

- Frequency: 10 kHz
- M.C.: Complete
- M.S.: 10 cm
- I.R.: 5 cm
- P.S.: 5 cm
- Ray: Straight

Figure A1.34 Tomographic image
MODEL II

- Frequency: 10 kHz
- M.C.: Complete
- M.S.: 5 cm
- I.R.: 5 cm
- P.S.: 1 cm
- Ray: Curved
Figure A1.35 Tomographic image
MODEL II

Figure A1.36 Tomographic image
MODEL II
Figure A1.37 Tomographic image

MODEL II

- frequency: 50 kHz
- m.c.: partial
- m.s.: 5 cm
- i. r.: 5 cm
- p.s.: 1 cm
- ray: curved

Figure A1.38 Tomographic image

MODEL II

- frequency: 50 kHz
- m.c.: partial
- m.s.: 5 cm
- i. r.: 10 cm
- p.s.: 1 cm
- ray: curved
Figure A1.39 Tomographic image
MODEL II

Figure A1.40 Tomographic image
MODEL II
Figure A1.41 Tomographic image  
**MODEL II**

- Frequency: 50 kHz
- M.C.: complete
- M.S.: 10 cm
- I. R.: 5 cm
- P.S.: 5 cm
- Ray: straight

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Figure A1.42 Tomographic image  
**MODEL III**

- Frequency: 10 kHz
- M.C.: complete
- M.S.: 5 cm
- I. R.: 5 cm
- P.S.: 1 cm
- Ray: curved
**Figure A1.43 Tomographic image**

**MODEL III**

- Frequency: 10 kHz
- M.C.: Complete
- M.S.: 5 cm
- I.R.: 5 cm
- P.S.: 2.5 cm
- Ray: Curved

**Figure A1.44 Tomographic image**

**MODEL III**

- Frequency: 10 kHz
- M.C.: Complete
- M.S.: 5 cm
- I.R.: 5 cm
- P.S.: 5 cm
- Ray: Curved
Figure A1.45 Tomographic image
MODEL III

Figure A1.46 Tomographic image
MODEL III

frequency: 10 kHz
m.c.: complete
m.s.: 5 cm
i. r.: 10 cm
p.s.: 1 cm
ray: curved

frequency: 10 kHz
m.c.: complete
m.s.: 5 cm
i. r.: 10 cm
p.s.: 10 cm
ray: curved
Figure A1.47 Tomographic image
MODEL III

frequency: 10 kHz
m.c.: complete
m.s.: 5 cm
i. r.: 5 cm
p.s.: 1 cm
ray: straight

Figure A1.48 Tomographic image
MODEL III

frequency: 10 kHz
m.c.: complete
m.s.: 5 cm
i. r.: 5 cm
p.s.: 5 cm
ray: straight
Figure A1.49 Tomographic image
MODEL III

Figure A1.50 Tomographic image
MODEL III
Figure A1.51 Tomographic image
MODEL III

Figure A1.52 Tomographic image
MODEL III
Figure A1.55 Tomographic image
MODEL III

frequency: 50 kHz
m.c.: complete
m.s.: 5 cm
i. r.: 10 cm
p.s.: 1 cm
ray: curved

Figure A1.56 Tomographic image
MODEL III

frequency: 50 kHz
m.c.: complete
m.s.: 5 cm
i. r.: 10 cm
p.s.: 10 cm
ray: curved
Figure A1.57 Tomographic image
MODEL III

frequency: 50 kHz
m.c.: complete
m.s.: 5 cm
i. r.: 5 cm
p.s.: 1 cm
ray: straight

Figure A1.58 Tomographic image
MODEL III

frequency: 50 kHz
m.c.: complete
m.s.: 10 cm
i. r.: 5 cm
p.s.: 1 cm
ray: curved
Figure A1.59 Tomographic image
MODEL III

frequency: 50 kHz
m.c.: complete
m.s.: 10 cm
i. r.: 5 cm
p.s.: 1 cm
ray: straight
APPENDIX II

THEORY OF SIRT
A2.1 Principal of SIRT theory

In series expansion methods of reconstruction [1], the object plane undergoes immediate discretization. A Cartesian grid of square picture elements, or pixels, is constructed such that the entire object plane is confined within the extents of the grid. The reconstruction problem is then to determine an average value of the variable of interest for the image in each of the defined pixel areas. Figure A2.1 illustrates the reconstruction problem for a square object.

As can be seen in the diagram, the pixels are numbered in a regular fashion from \( j = 1 \) to \( n \), while the acoustic ray paths (assumed straight for the moment) are ordered from \( i = 1 \) to \( m \). A set of basis functions \( b_j(x,y) \) is associated with each pixel, usually the cubic basis:

\[
b_j(x,y) = \begin{cases} 
1 & \text{if } (x,y) \text{ contained by } j \text{ th pixel} \\
0 & \text{otherwise}
\end{cases} \quad \text{(A2.1)}
\]
At this stage, it is again assumed that one is interested in the variations in the acoustic slowness throughout the image from experimental time of flight data. Defining $S_j$ as the average slowness value within the $j$'th pixel, then the digitization of the slowness function $f(x,y)$ is given by:

$$f(x,y) = \sum_j S_j b_j(x,y) \quad \text{(A2.2)}$$

Now, a set of functional $\{R\}$ is defined which assign to any slowness function $f(x,y)$ the value of the line integral of the slowness along the $i$'th raypath (i.e. the time of flight). If $t_i$ represents the measurement of the $i$'th travel time, we can write:

$$t_i \equiv R_i f(x,y) \equiv \sum_j R_j S_j b_j(x,y) \equiv \sum_j S_j A_{ij} \quad \text{(A2.3)}$$

In matrix form, this simply becomes the equation 4.5 ($t = A S$), and the reconstruction problem has been reduced to solving a set of linear algebraic equations. The terms in this equation may be referred to as the data vector, $t$, the projection matrix, $A$, and the image vector, $S$, respectively. The projection matrix contains elements $A_{ij}$ describing the length of the segment of raypath $i$ contained within pixel $j$. It appears, at this point, that the problem has been completely solved. However, the solution of equation 4.5 is complicated by several factors. Firstly, the system is extremely large, in general, as the number of elements in the projection matrix is given by the product of the number of raypaths with the number of pixels in the model. This number can easily reach magnitudes in the hundreds of thousands. The projection matrix is also very sparse (that is, a large percentage of its entries are zero). Furthermore, the system may be under-determined due to lack of information, or degraded by inconsistent data. The first situation may arise if pixels in the image are traversed by no, or few, raypaths. The image slowness is then practically unconstrained at that point, and may vary wildly without creating any contradiction in the system. The second situation may arise from inaccuracies in the measurements of travel times and/or transducer locations, or, in fact, from deviations from the assumed linear raypaths (i.e. raybending). For the
above reasons, the problem becomes one of seeking the optimum solution, namely the one that best fits all available data. Traditionally, this occurs in a least squares sense. That is, the accepted solution is the one that minimizes the square of the discrepancies between experimental times of flight and those projected from the calculated image (referred to as the travel time residual).

One of the most popular techniques for dealing with these problems is a family of iterative algorithms collectively known as ART (algebraic reconstruction techniques). These are based upon a row-action method for solving large, sparse systems, developed by Kaczmarz in 1937 [2]. The reconstruction process begins by assigning an estimate to the image, often simply a constant value, and refining that estimate iteratively. Each iteration consists of a reduction of the travel time residual for a single raypath, or row of the projection matrix. In other words, the k'th iterative step refines the image only at the pixel; that raypath intersects. This process can be described by function $\alpha(k)$:

$$S^{k+1} = \alpha(k) \left\{ S^k, A_i, t_i \right\}$$ (A2.4)

where $A_i$ represents the i'th row vector of the projection matrix $A$.

The various iterative reconstruction methods differ in the way the correction functions $\alpha(k)$ are chosen. The original algorithm developed by Gordon, Bender and Herman [1] chose the correction in the least squares sense. The iterative updates used can be easily determined through the use of the technique of Lagrange multipliers, as is shown below.

To begin with, a small change in the image slowness will cause a first order change in the projected travel time of equation (A2.3) through the discretized medium to change by an amount

$$\Delta t_i = \sum_j \Delta S_j A_{ij}$$ (A2.5)

This equation must be solved in an iterative step, for the difference $\Delta t_i$ between a measured delay for a raypath and its projected delay from the current image. Now, a general norm of order $p$ can be written as:
\[ L_p = \sum_j \Delta S_j^p \]  

(A2.6)

which, when combined with the constraint of equation A2.5 and a constant Lagrange multiplier of \( \xi \) results in

\[ M = \sum_j \left[ \Delta S_j^p - \xi \Delta S_j A_{ij} \right] + \xi \Delta t_i \]  

(A2.7)

Minimizing equation (A2.7) with respect to \( \Delta S_j \) gives:

\[ \frac{\partial M}{\partial \Delta S_j} = P \Delta S_j^{p-1} - \xi A_{ij} = 0 \]  

(A2.8)

It is the solution of this equation that yields the update to the \( j \)'th pixel for the \( i \)'th ray as:

\[ \Delta S_j = \frac{\Delta t_i A_{ij}^{(p-1)}}{\sum_j A_{ij} A_{ij}^{(p-1)}} \]  

(A2.9)

Choosing a least squares norm (\( p=2 \)), the original correction function \( \alpha(k) \) of Gordon, Bender and Herman then results:

\[ \alpha(k) \left\{ S^k, A_i, t_i \right\} = S^k + \frac{t_i - \langle A_i, S^k \rangle}{\langle A_i, A_i \rangle} A_i \]  

(A2.10)

where \( \langle \cdot \rangle \) denotes the inner product of two \( n \) dimensional vector \( \langle A_i, S^k \rangle = \sum_j A_{ij} S_{ij}^k \).

In this equation, the term \( \Delta t_i = t_i - \langle A_i, S^k \rangle \) is the travel time discrepancy between the actual delay measurement \( t_i \) and its pseudo-delay projected from the image.
A useful geometric interpretation that aids in the visualization of this iterative procedure can be developed. A 'hyperplane' $H_i$ is defined which represents the set of all vectors that are solutions to equation A2.3, given by the $i$'th raypath. For example, if the dimension of $S$ is two, then the hyperplane is just a line in two-dimensional space, defined by each raypath should intersect at a common image vector $S$. Each iterative step is then represented by the orthogonal projection of the current estimate onto the hyperplane $H_i$. Because each correction to the slowness image ($\Delta S^k$) is perpendicular to the hyperplane, it can be seen to represent the smallest possible correction that satisfies equation A2.5. This interpretation is illustrated in figure A2.2(a) [3] for the ART correction of equation A2.10.

Noise can cause the system to fail to intersect at a common point and, when the algorithm is applied, successive iterations can oscillate about the best solution. To combat this, the applied correction is usually under-relaxed, which helps to enforce stability in convergence. That is, the $k$'th iteration moves only part of the way along the orthogonal projection to the $i$'th raypath. The relaxation parameter, $\lambda$, can be chosen as a constant value (usually between 0 and 1), or it can be defined dynamically as a function of the iteration number. Equation A2.10 then becomes:

$$\alpha(k) = S^k + \lambda(k) \frac{t_i - \langle A_i, S^k \rangle}{\langle A_i, A_i \rangle} A_i$$

(A2.11)

In addition, it can be helpful to apply a positivity constraint to each iteration, as the slowness of the object is usually known to be positive. This is done upon completion of an iterative step by replacing each point in the image vector with a slowness value below zero with a value of zero. There are in existence other types of ART type algorithms, each with different variations in the calculation of the correction function $\alpha(k)$. One of these, known as SIRT (simultaneous iterative reconstruction technique) [4], is the basis for the algorithm actually used for iterative reconstructions in the present work. The SIRT algorithm, considers all the expressions arising from equation 4.5 ($t = A S$) simultaneously. The correction function is calculated for each row (i.e.
each raypath), without updating the image. The \( k \)th iteration to the \( j \)th pixel is then the average of all corrections that correspond to all raypaths actually intersecting that pixel. Then, if \( n \) is defined as the number of raypaths intersecting the \( j \)th pixel, the correction function is given by:

\[
\alpha(k) = S^k + \frac{\lambda(k)}{n_j} \sum \frac{t_i - \langle A_i, S^k \rangle}{\langle A_i, A_i \rangle} A_i
\]  

(A2.12)

Averaging the corrections for all raypaths tends to produce a more stable convergence. For this reason, the relaxation parameter can be chosen as a larger value, or even discarded altogether, without affecting convergence adversely. The drawback is, of course, the additional storage space required for the correction factors. A geometric interpretation of the convergence of this technique is shown in figure A2.2(b) for comparison.

To reduce the number of calculations required to implement equation A2.12, a simplification can be made that makes the calculation of the elements of the projection matrix \( A \) unnecessary [5]. To begin with, note that if the minimax norm (\( p \to \infty \)) is chosen instead of the least squares norm (\( p = 2 \)) then equation A2.9 reduces to

\[
\Delta S_j = \frac{\Delta t_i \tilde{A}_{ij}}{L_i}
\]  

(A2.13)

where:

\[
\tilde{A} = \begin{cases} 
1 & \text{the corresponding elements of } A \text{ are non-zero} \\
0 & \text{otherwise}
\end{cases}
\]  

(A2.14)

and \( L_j \) is defined as the length of the \( i \)th raypath.

Now, if the slowness of each pixel intersected by the \( i \)th raypath, the residual \( \Delta t_i \) becomes:

\[
\Delta t_i = t_i - \frac{L_i}{N_i} \sum_j S_j \tilde{A}_{ij}
\]  

(A2.15)
Figure A2.2 An illustration of the difference in convergence between (a) ART and (b) SIRT
(In this example, both the image vector $S$ and the data vector $t$ have a dimension of two)
where $N_i$ is defined as the number of pixels the $i$th raypath intersects. Then, equation A2.12 for the correction function becomes

$$\alpha(k) = S^k + \frac{\lambda(k)}{n_j} \sum_i \left[ \frac{t_i}{L_i} - \frac{\langle \tilde{A}_i, S^k \rangle}{N_i} \right] \tilde{A}_i$$

(A2.16)

This equation can indeed be seen to eliminate the use of the distance elements $A_{ij}$ For images sampled rapidly enough compared to their bandwidth, it provides images of similar quality to the unsimplified algorithm of A2.12. Equation A2.16 is the basis of the algorithm implemented for series expansion class reconstructions created in the present work.

A2.2 Arbitrary geometries

Because there are no prior requirements for a regular arrangement of raypaths in the SIRT algorithm, irregular geometries can be treated by relatively simple modifications. The primary requirement for a general geometry is a determination of the object boundary. This is important for two reasons. The first is that when the image is smoothed to reduce noise, distortion of the pixel values at the object edge tends to result; as the pixels are zero-valued at positions outside the object. Prior knowledge of the boundary location enables one to prevent this effect. The second reason, applicable when a raybending correction is incorporated, is that one obviously does not wish the raypaths to bend outside of the object.

To begin with, the transducer coordinates are scaled to fit within a preselected square grid size, with the travel times being scaled accordingly. Transducer locations are moved to the nearest pixel location on the grid, as is required by the algorithm. The travel times are scaled again, by the change in path length. Raypaths made redundant by these operations have their transit times averaged for a single input raypath. The object boundary is determined by sorting the set of all unique source-receiver locations by angle about a central origin. Assuming the object has a convex
surface, a working boundary can then be defined by determining secants between adjacent locations. Reconstruction can proceed in exactly the same manner as the basic algorithm.

A2.3 References of appendix II


